Advanced Dynamics

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Lecture - 57
Symmetries and Conservation Laws - II

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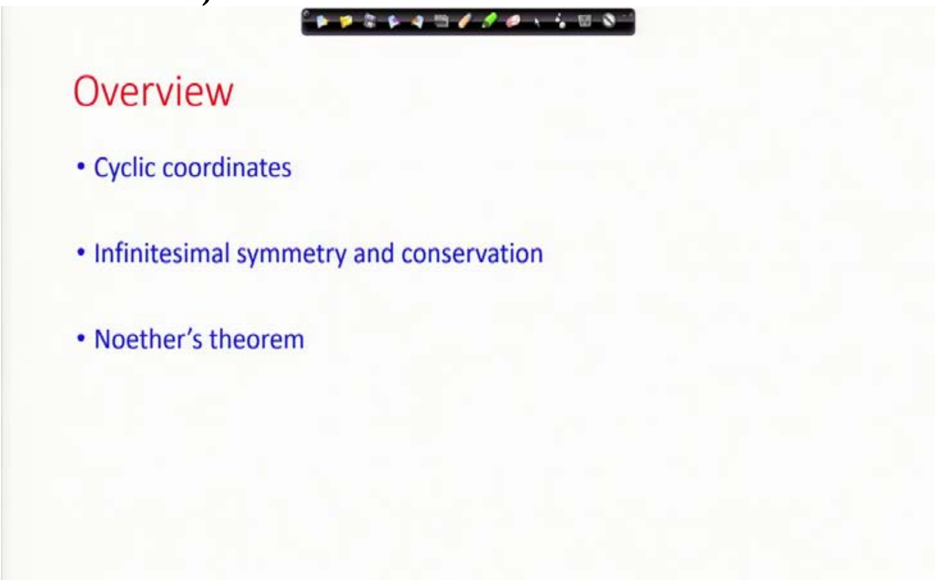

In this lecture, I am going to continue further with symmetries and conservation laws. This is the overview of what we are going to discuss in this lecture, we are going to introduce this concept of cyclic coordinates. Then we are going to discuss infinitesimal symmetry and the corresponding conservation which goes by the name Noether's theorem.

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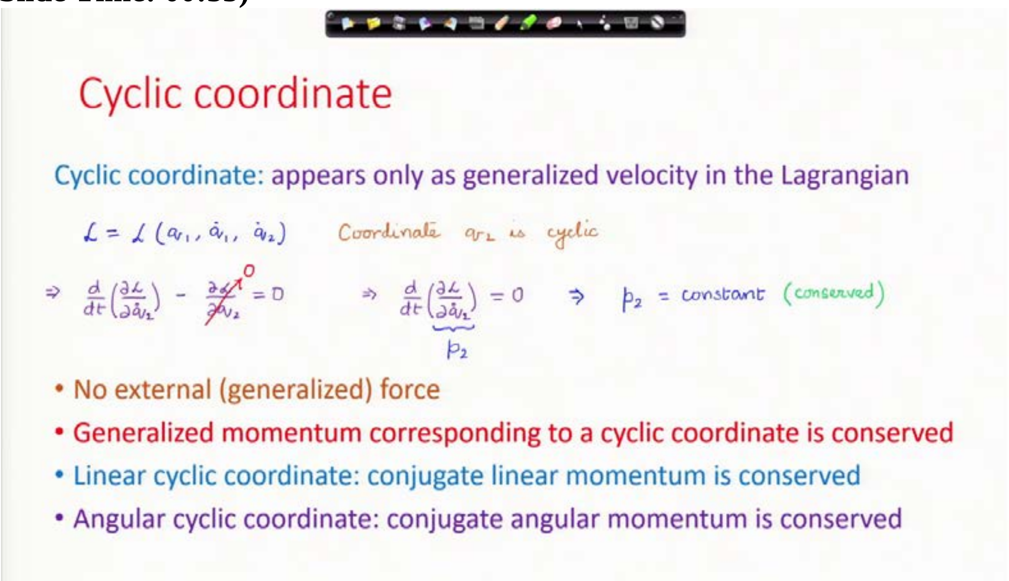
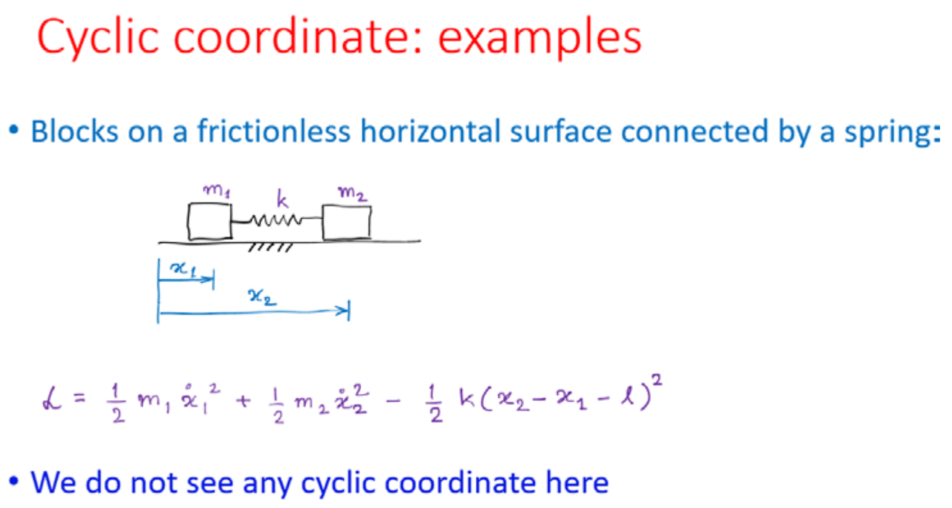


First we are going to look at cyclic coordinates. The concept is explained in the above slide. The consequences are also discussed.



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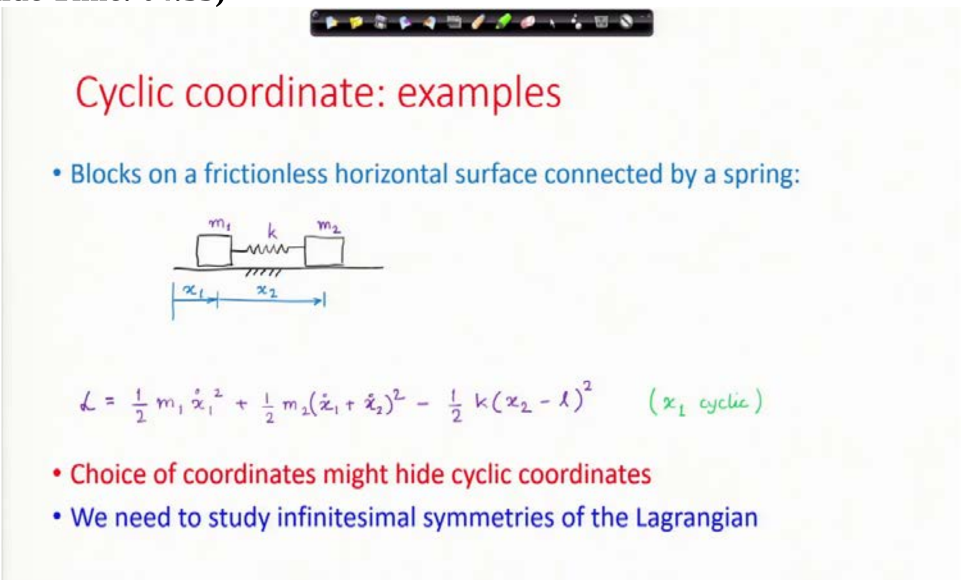

In the 2 slides above, we observe that choice of coordinates can hide cyclic coordinates. This is investigated using the concept of symmetry of the Lagrangian.

Infinitesimal symmetries of the Lagrangian

Infinitesimal (smooth) transformation of Lagrangian

$$\frac{d}{d\sigma}\left[\tilde{\mathcal{L}}(\vec{z}(t,\sigma),\dot{\vec{z}}(t,\sigma),t)\right]\Big|_{\sigma=0} = 0$$

$$\Rightarrow \left[\frac{\partial\tilde{\mathcal{L}}}{\partial\vec{z}}\cdot\vec{z}'(t,\sigma) + \frac{\partial\tilde{\mathcal{L}}}{\partial\dot{\vec{z}}}\cdot\dot{\vec{z}}'(t,\sigma)\right]_{\sigma=0} = 0 \qquad \left(\vec{z}' = \frac{\partial\vec{z}}{\partial\sigma}\right)$$

Lagrange's equation: $\frac{d}{dt}\left(\frac{\partial\tilde{\mathcal{L}}}{\partial\dot{\vec{z}}}\right) = \frac{\partial\tilde{\mathcal{L}}}{\partial\vec{z}}$ Also, $\frac{\partial\tilde{\mathcal{L}}}{\partial\vec{z}} = \frac{\partial\mathcal{L}}{\partial\vec{q}}$ $\frac{\partial\tilde{\mathcal{L}}}{\partial\dot{\vec{z}}} = \frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}$ at $\sigma = 0$

$$\Rightarrow \left[\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}\right)\cdot\vec{z}'(t,\sigma) + \frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}\cdot\dot{\vec{z}}'(t,\sigma)\right]_{\sigma=0} = 0$$

Infinitesimal symmetries of the Lagrangian

Infinitesimal (smooth) transformation of Lagrangian

$$\left[\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}\right)\cdot\vec{z}'(t,\sigma) + \frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}\cdot\dot{\vec{z}}'(t,\sigma)\right]_{\sigma=0} = 0 \quad \Rightarrow \left[\frac{d\vec{p}}{dt}\cdot\vec{z}' + \vec{p}\cdot\frac{d\vec{z}'}{dt}\right]_{\sigma=0} = 0$$

$$\Rightarrow \frac{d}{dt}\left[\vec{p}\cdot\vec{z}'\Big|_{\sigma=0}\right] = 0 \qquad \left(\vec{p} = \frac{\partial\mathcal{L}}{\partial\dot{\vec{q}}}\right)$$

$\Rightarrow \vec{p}\cdot\vec{z}'\big|_{\sigma=0}$ is conserved

($\vec{z}'|_{\sigma=0}$: Infinitesimal generator of the transformation)

$$\begin{cases} \vec{z} = \vec{q} + \sigma\vec{b} & (\vec{b}: \text{constant}) \\ \frac{d\vec{z}}{d\sigma}\Big|_{\sigma=0} = \vec{b} & (\text{Translation direction}) \end{cases}$$

The concept of infinitesimal symmetry of the Lagrangian is explained in the 2 slides above.

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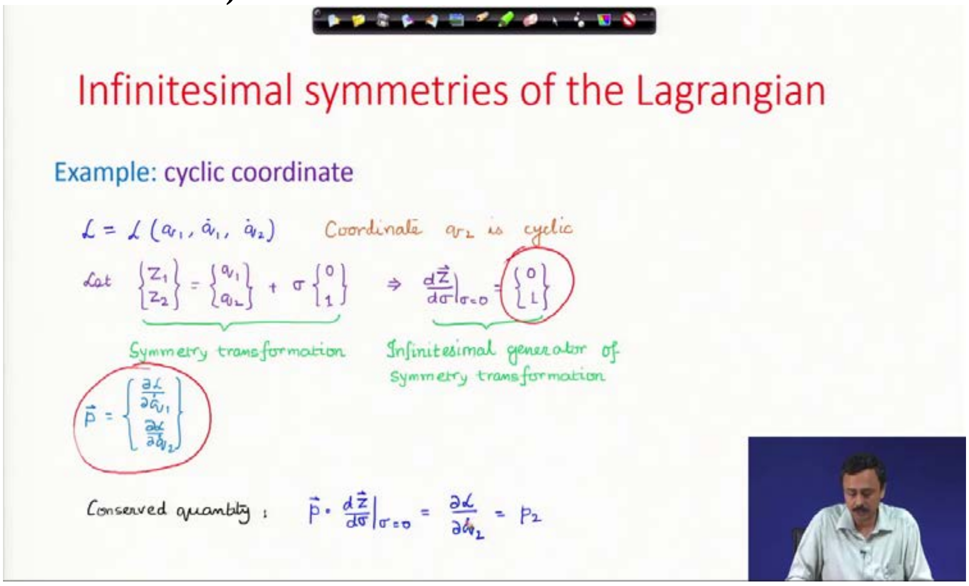

The application of infinitesimal symmetry in the case of cyclic coordinate is explained in the above slide.

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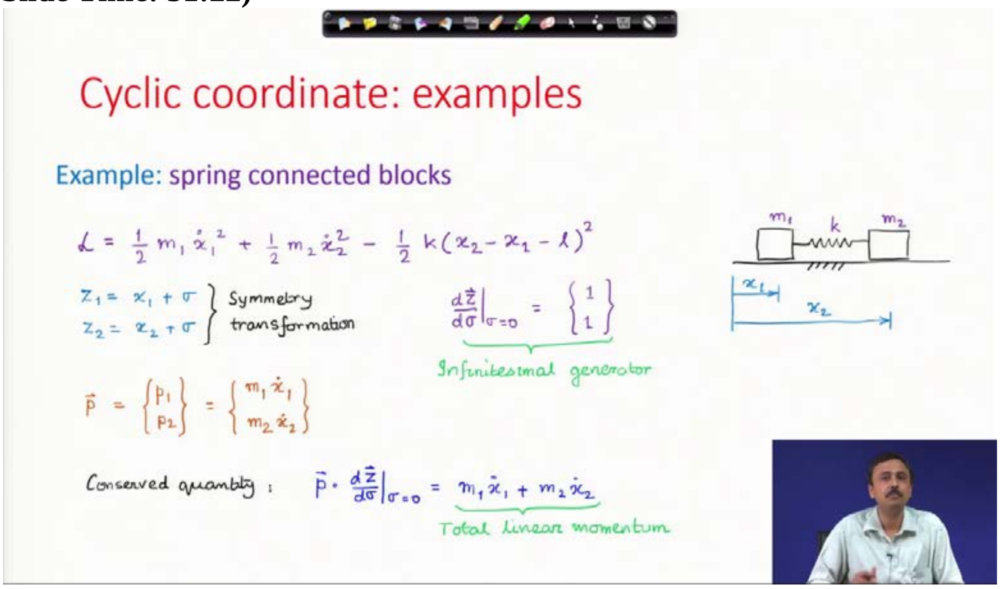

Now, we come back to that example, with our original scheme of coordinate position where we do not see that symmetry in a certain coordinatization scheme. It is shown how the concept of infinitesimal symmetry can be used to uncover the conserved quantity.

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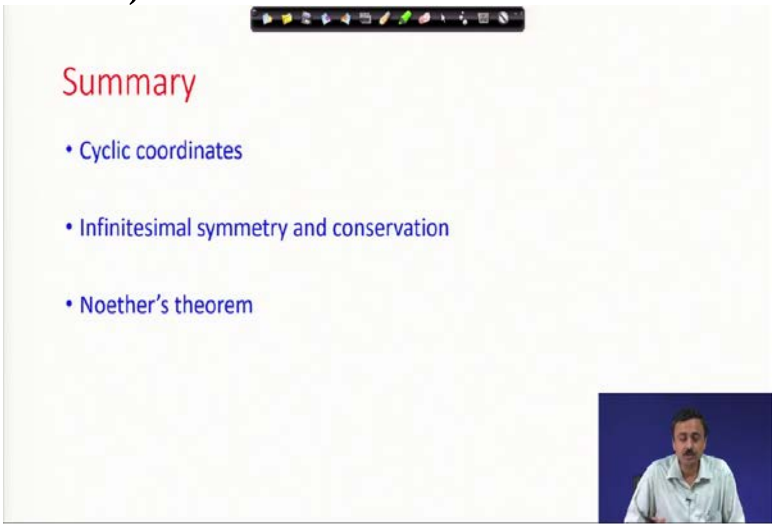

The above slide summarizes the discussions.