Advanced Dynamics
Prof. Anirvan DasGupta
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture – 55
Systems with Constraints – IV

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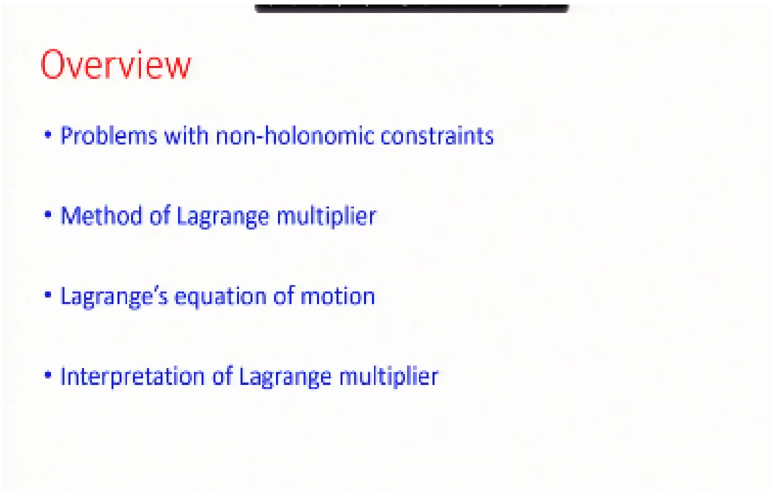

We will discuss systems with constraints a little further. And in this lecture, I am going to introduce non-holonomic constraints and discuss the derivation of the equation of motion using the method of Lagrange multipliers.

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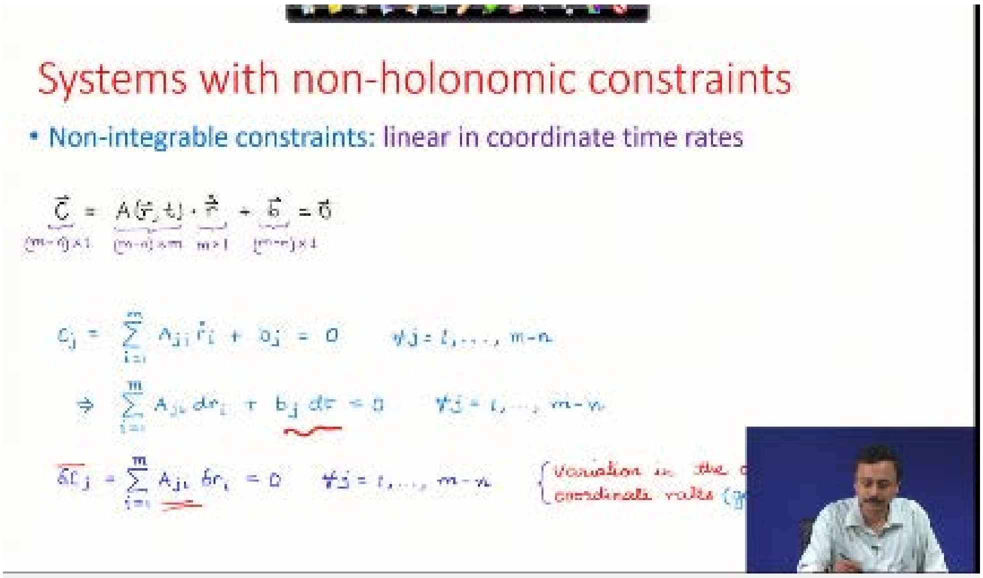

What are non-holonomic constraints? Constraints that come with derivatives of the coordinates and which we cannot integrate to obtain a constraint relating only the coordinates. We consider non-holonomic constraints that are linear in the generalized velocities. The above slide shows how non-holonomic constraints relate the variations of the coordinates.

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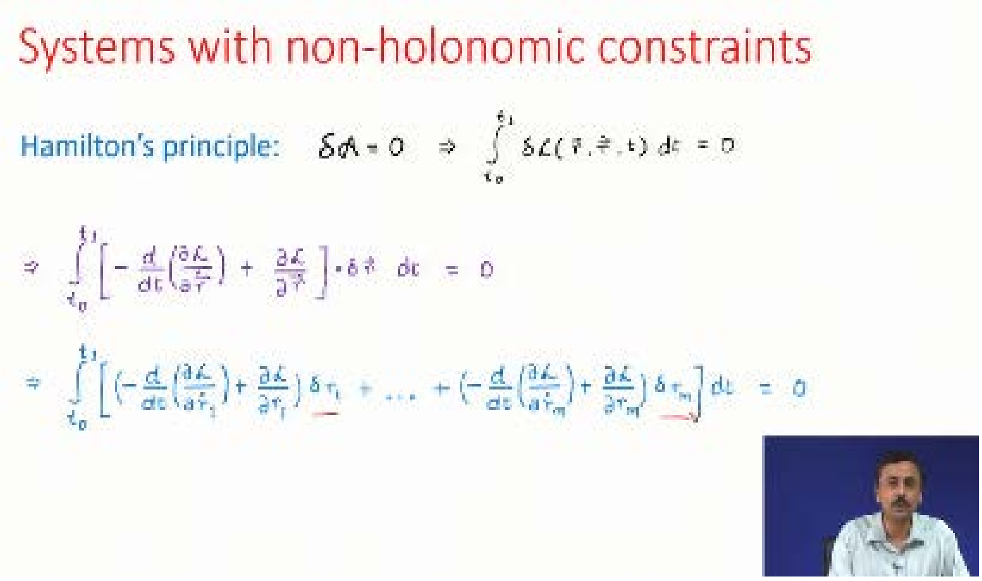

The variational statement obtained from the Hamilton's principle is presented in the above slide.

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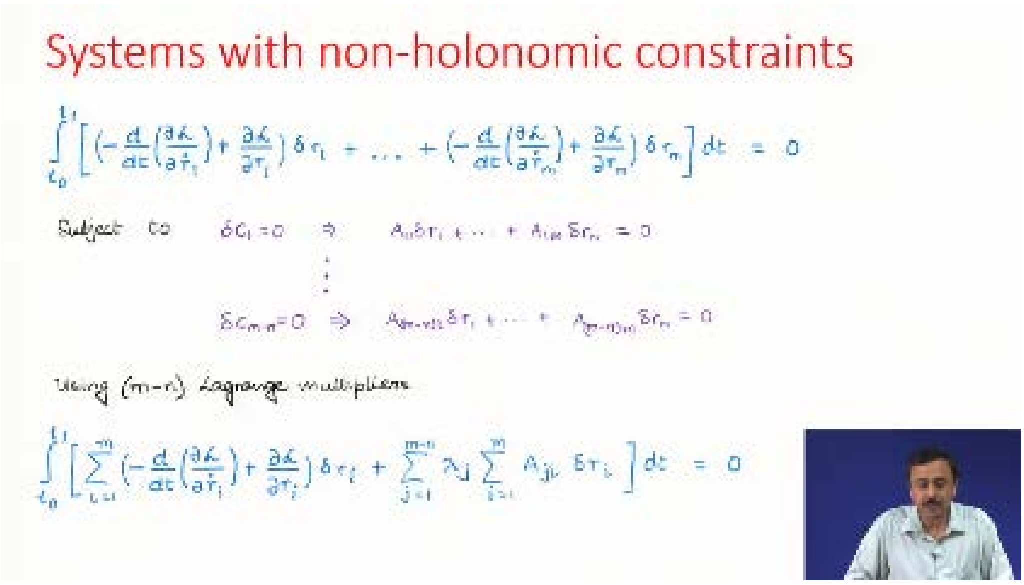

Next, we introduce the non-holonomic relations into the variational statement using the Lagrange multipliers, as shown above.

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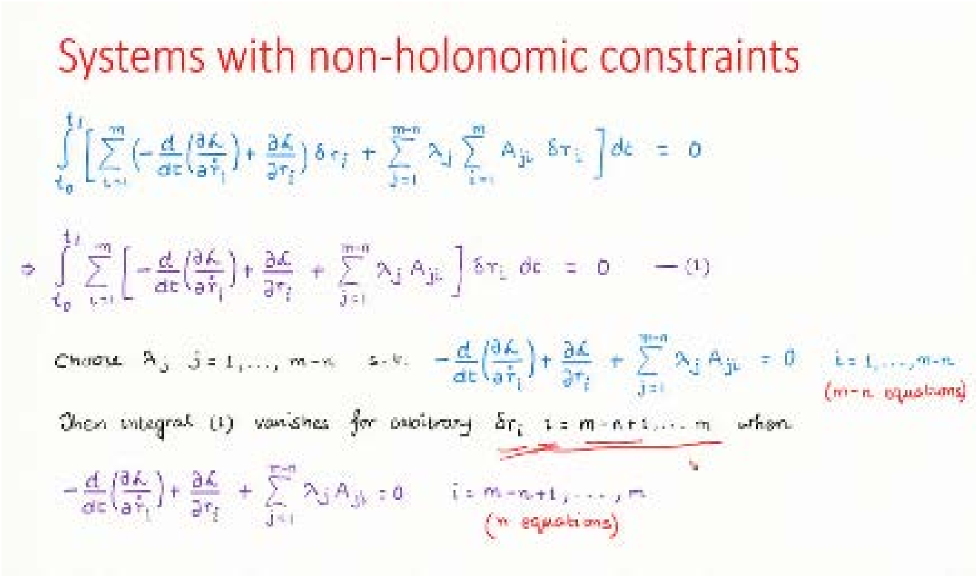

Using the arguments discussed previously, we obtain the equations of motion as presented in the slide above.

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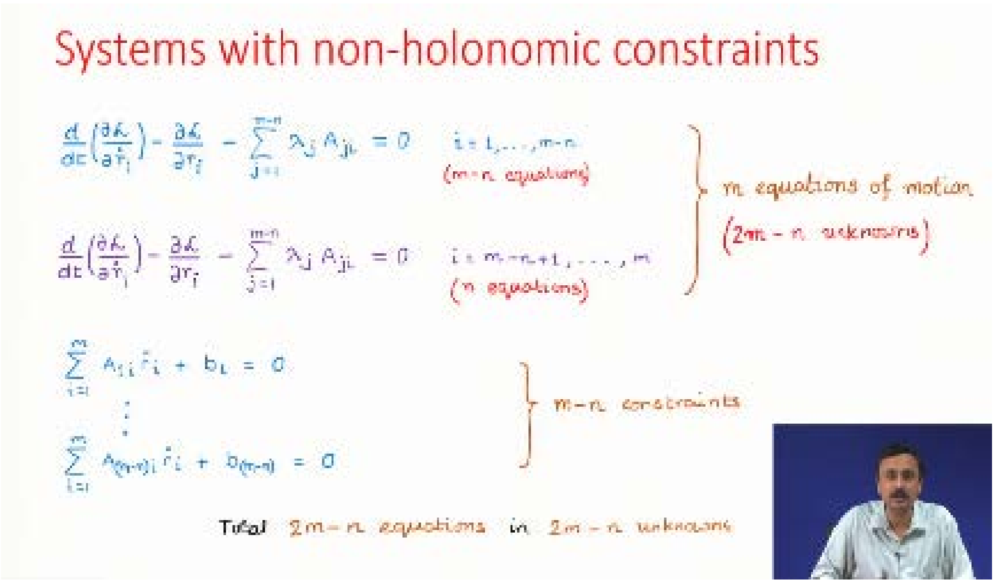

The complete set of equations involve the obtained equations of motion and the non-holonomic constraints as presented above.

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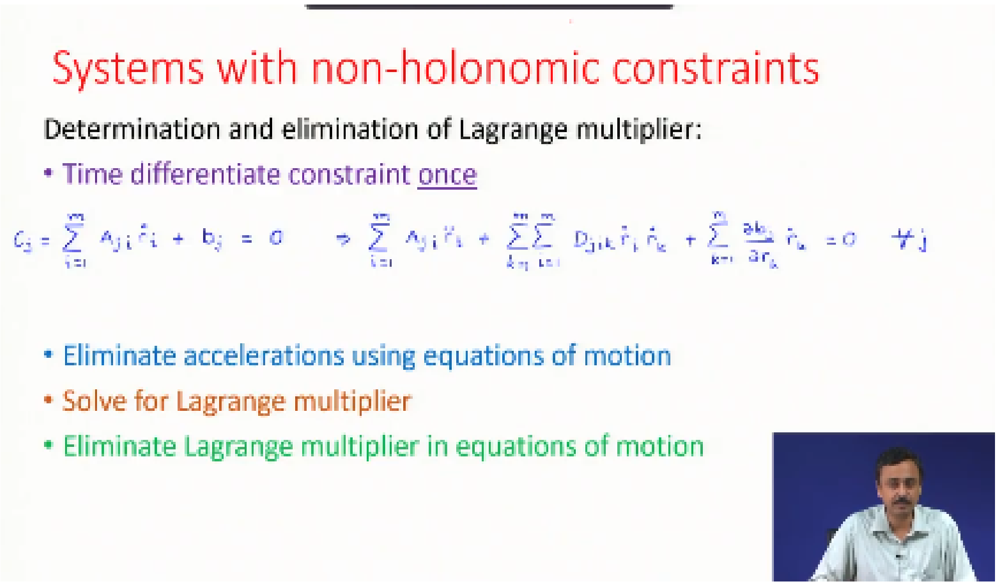

We eliminate the Lagrange multipliers following the steps outline in the above slide.

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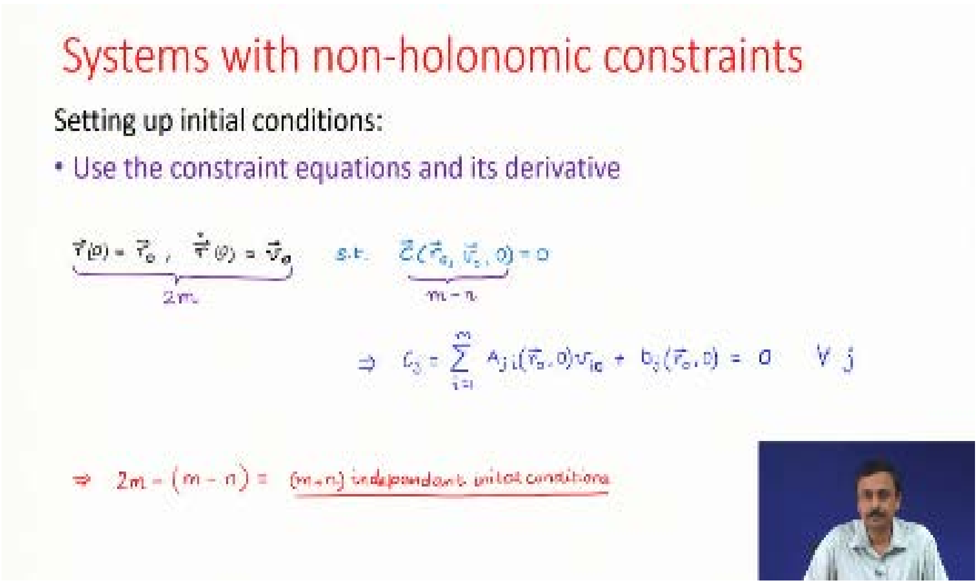

The above slide discusses the setting-up of the initial conditions.

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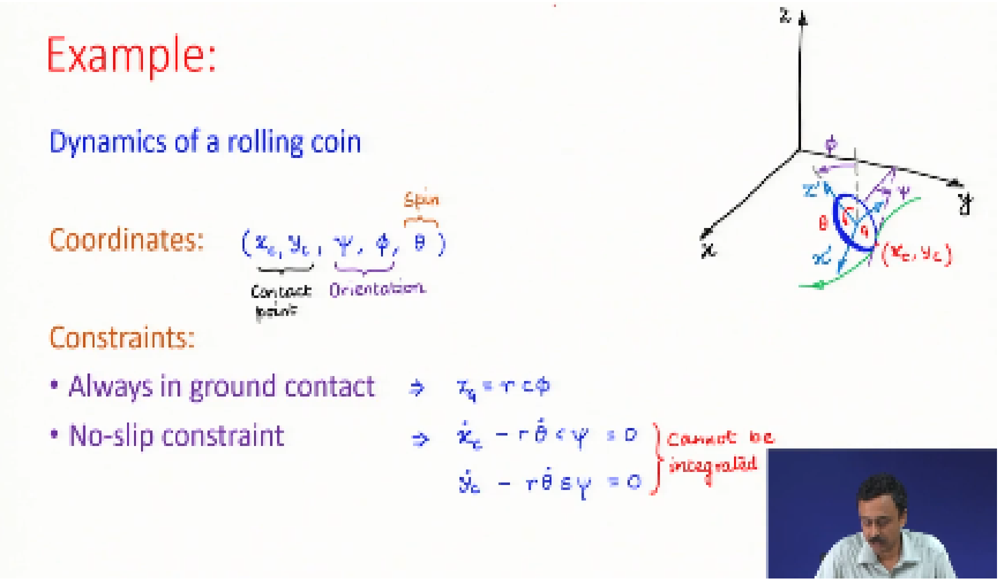

Now, let us look at this example of the dynamics of a rolling coin.

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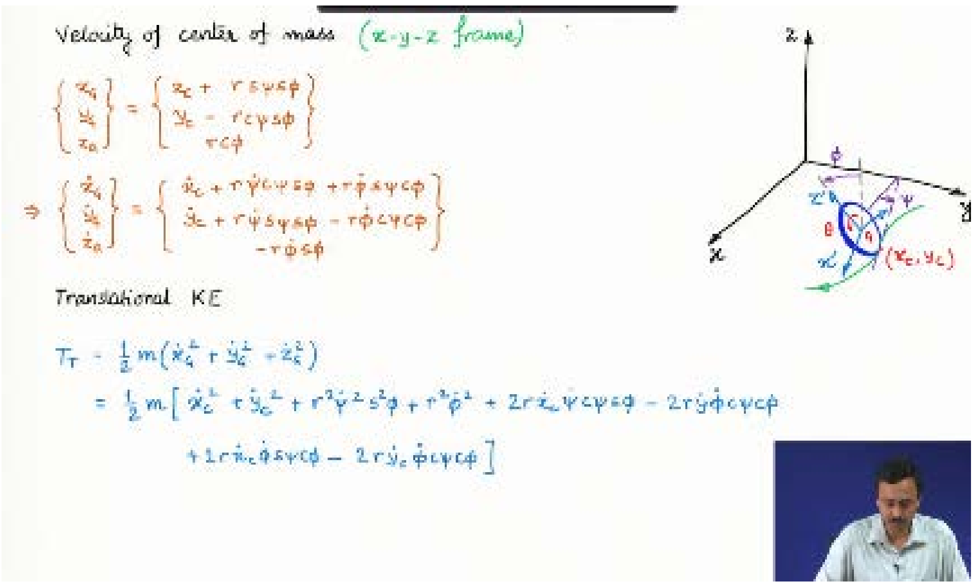

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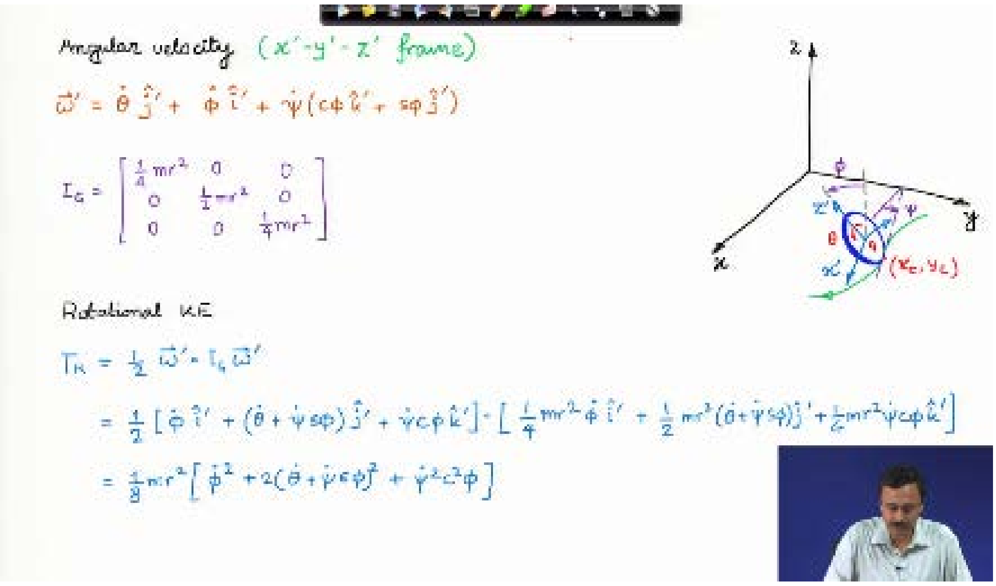

The translational and rotational kinetic energy terms are presented in the 2 slides above.

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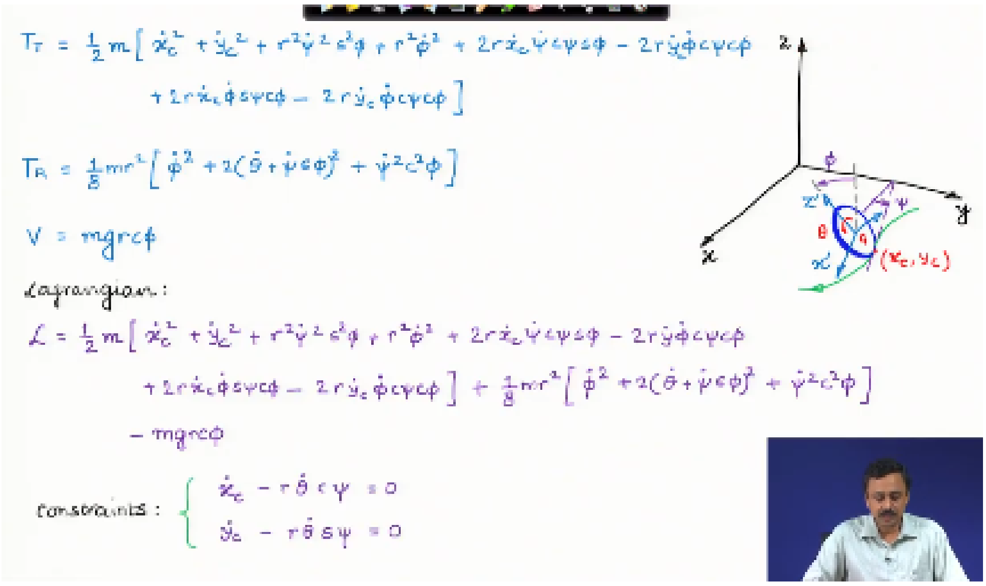

The Lagrangian and the 2 non-holonomic constraints are presented in the above slide.

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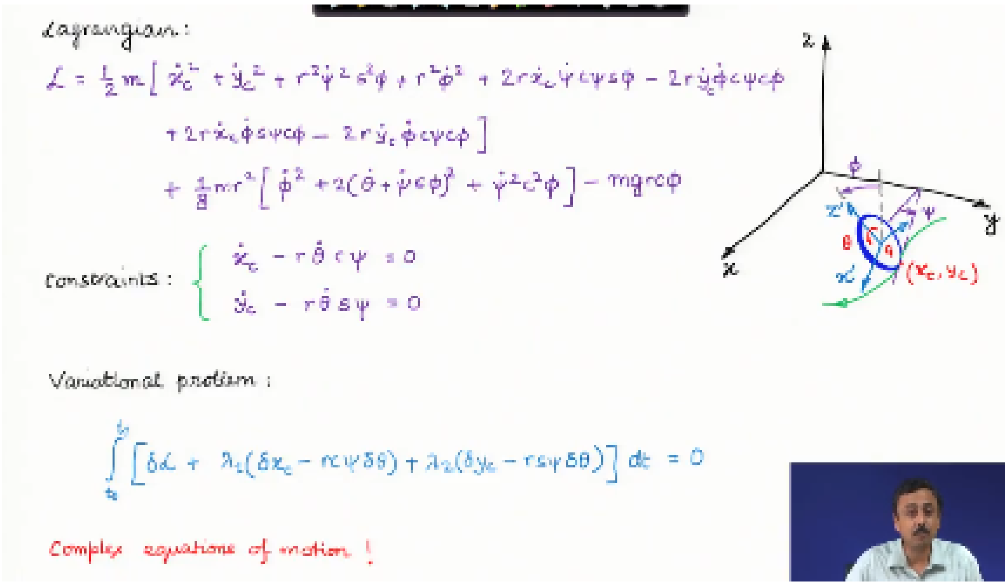

We can derive the equations of motion but they are nonlinear, coupled and extremely complex.

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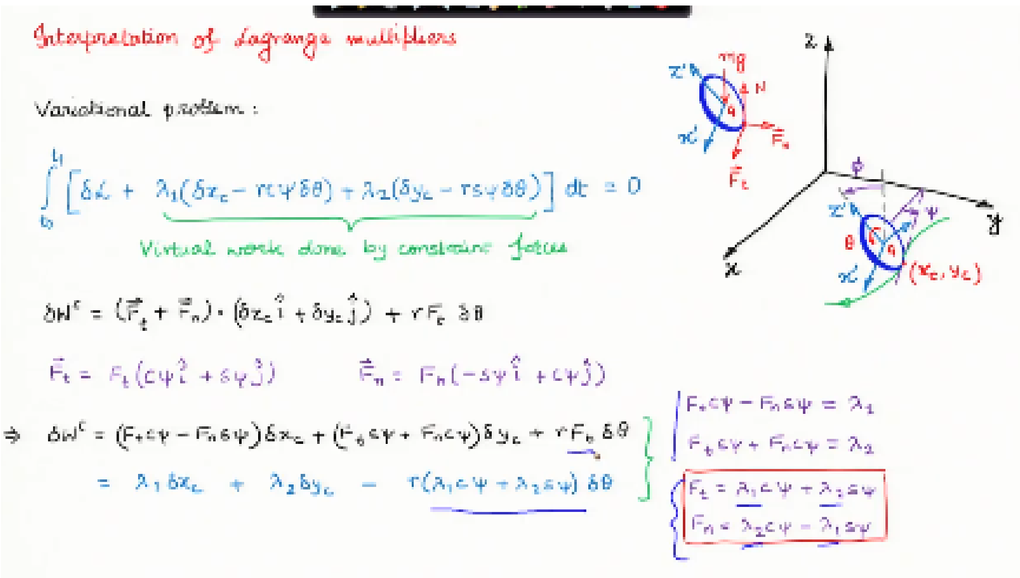

The interpretation of the Lagrange multipliers is presented in the above slide.

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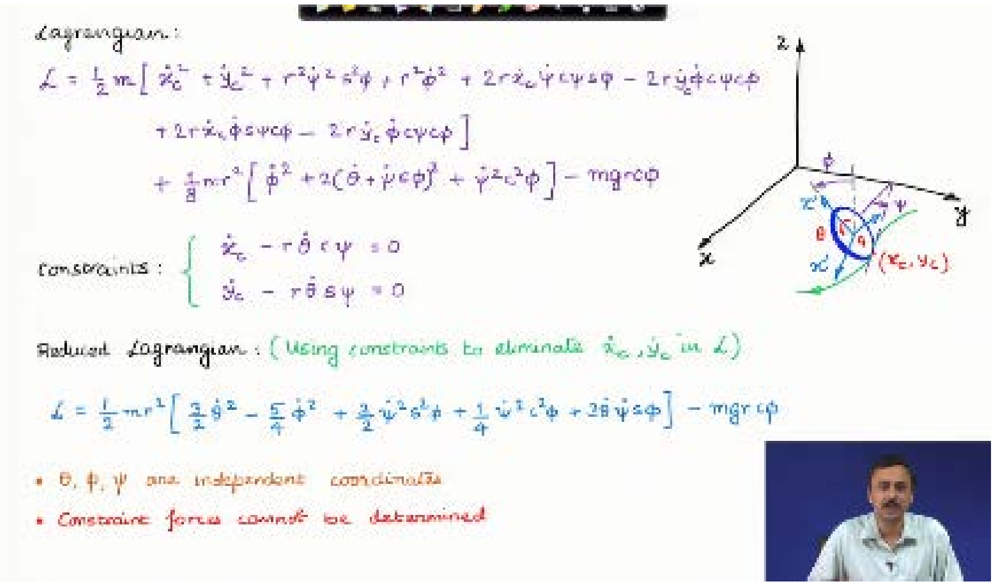

In the above slide, we obtain a reduced Lagrangian involving only the orientational coordinates. This is done by using the non-holonomic constraints to eliminate the contact point coordinate rates.

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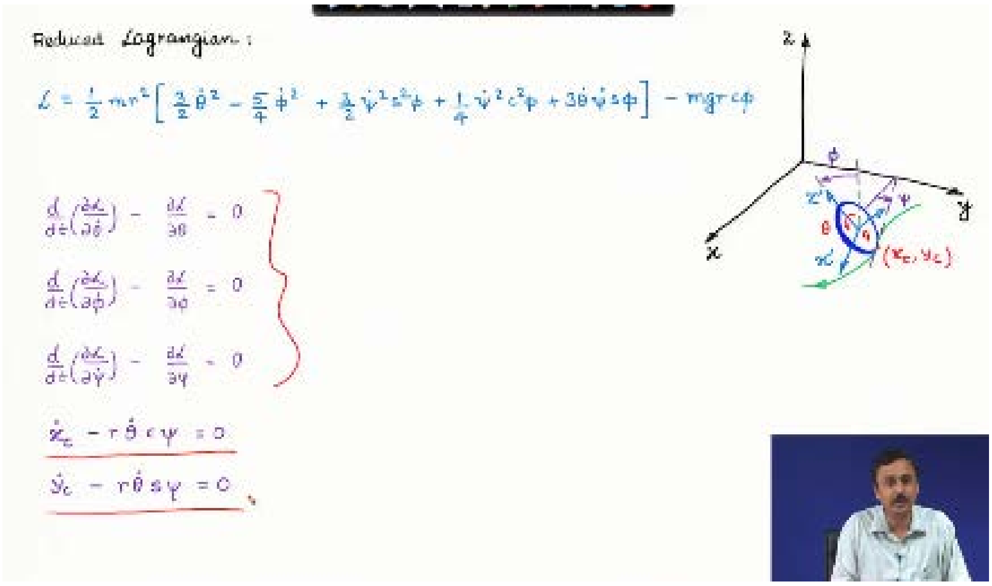

The equations of motion are now obtained, and the evolution of the contact point is obtained by integrating (numerically) the non-holonomic constraints.

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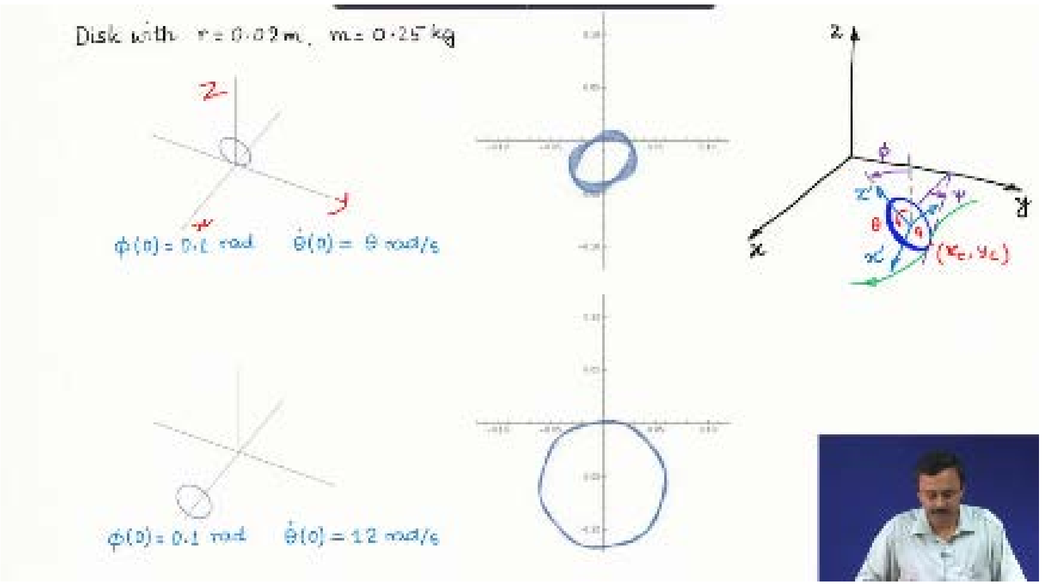

Snapshots of two simulations are presented in the above slide.

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