Advanced Dynamics
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Lecture – 54
System with Constraints – III

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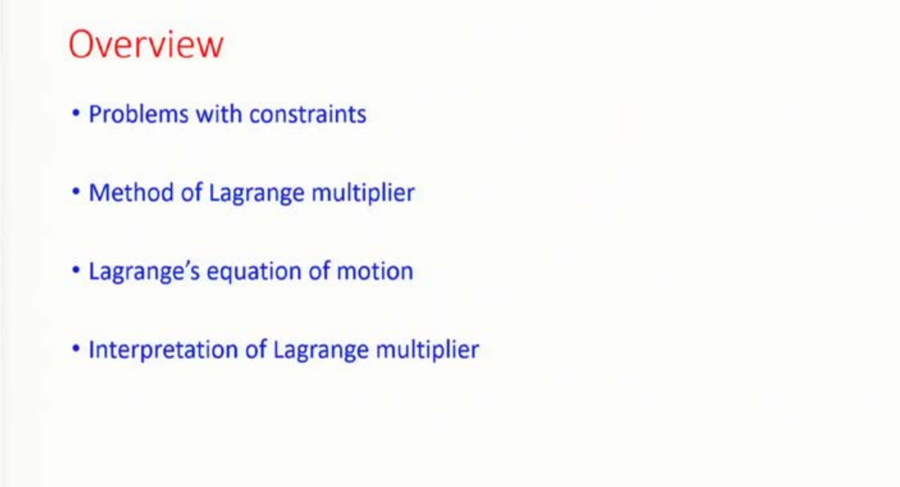

We are going to continue our discussions on systems with constraints. We had started discussing problems with constraints and used the method of Lagrange multiplier to derive the equations of motion. And in the last lecture, I had also discussed the interpretation of Lagrange multiplier and how that is used in finding out the constraint forces.

In this lecture, I am going to take a few more examples and show you the full power of this method of Lagrange multiplier in determining constrained forces. Recall that in order to determine constrained forces, we have to first break the constraint, and then introduce the constraint as a mathematical condition.

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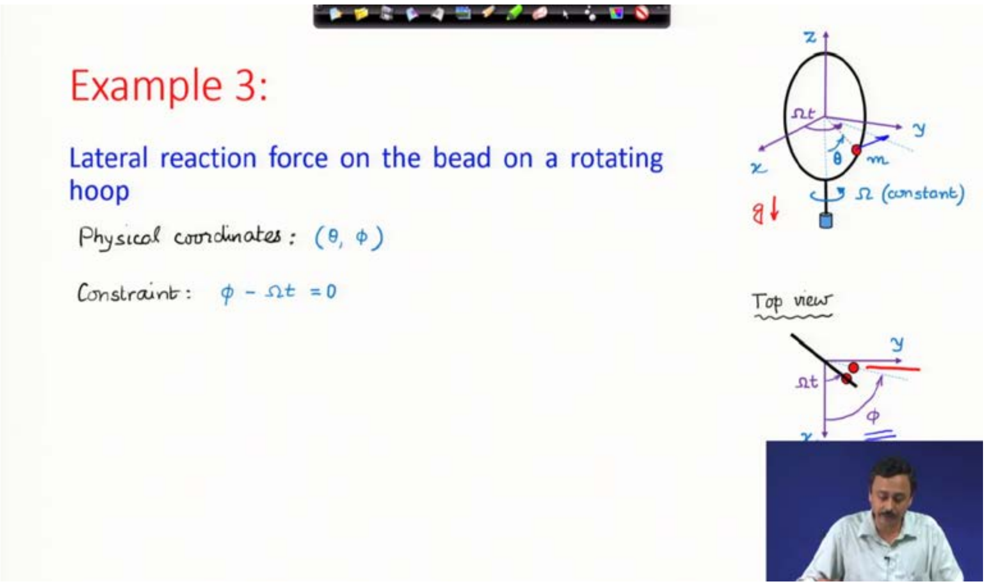

We will start with this example of bead on a hoop. The hoop is rotating at a constant angular velocity. I would like to find out the lateral reaction force on the bead from the hoop.

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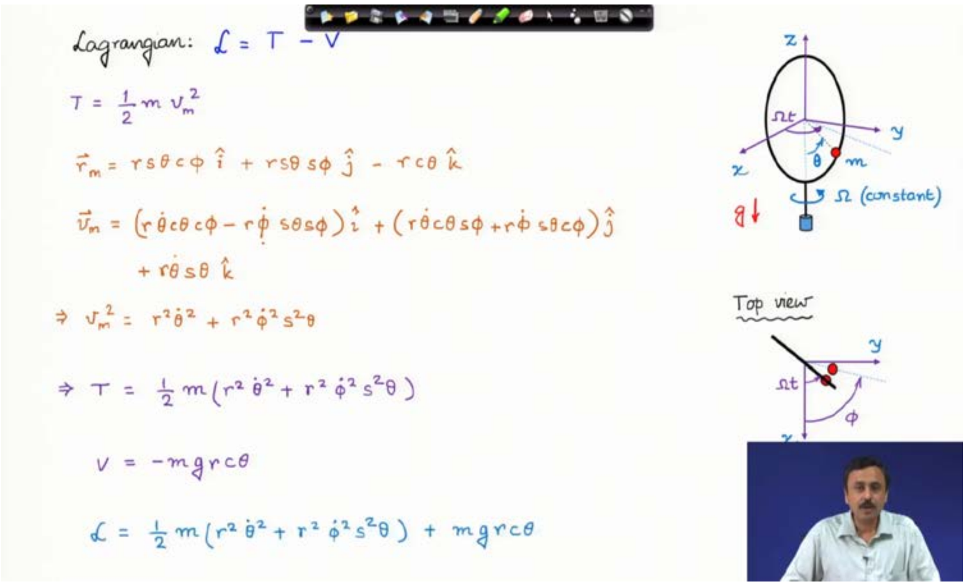

We introduce a new coordinate ϕ as shown above. Thus, we break the constraint that the bead always remains on the hoop.

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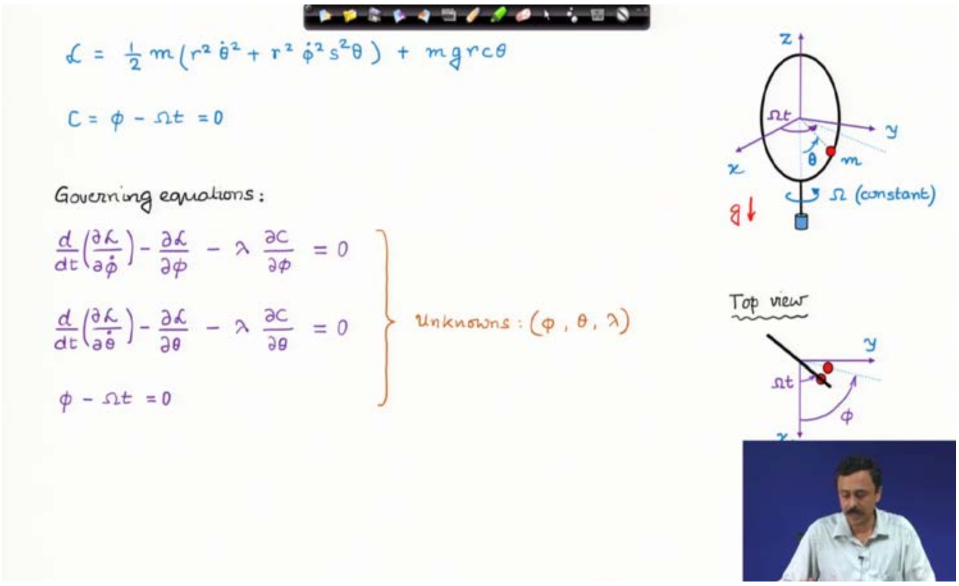

We introduce the constraint separately to enforce the condition that the bead actually moves on the hoop as shown above.

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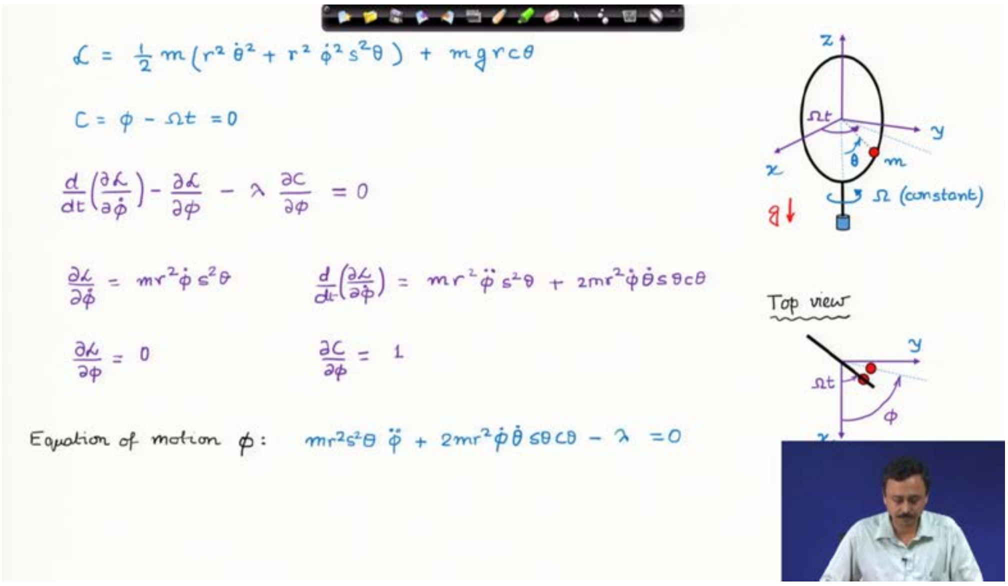

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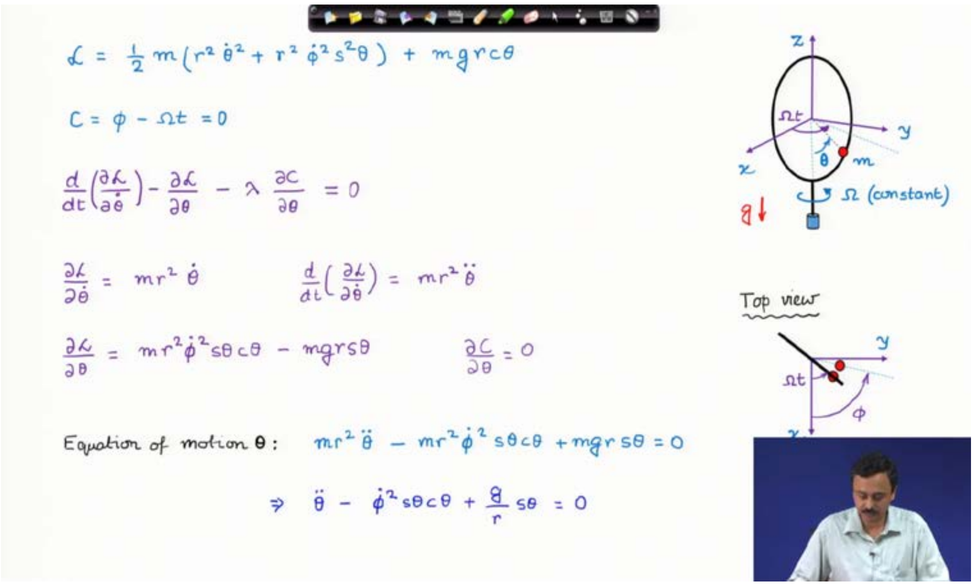

The equations of motion are then derives as shown in the 2 slides above. These equations involve the Lagrange multiplier.

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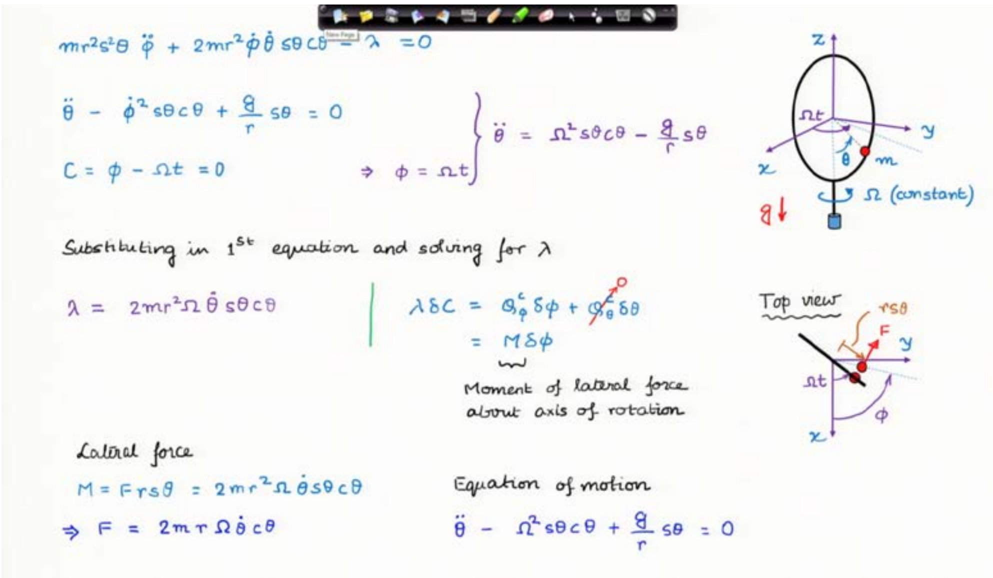

The above slide shows how the Lagrange multiplier is used to determine the lateral force and obtain the equation of motion.

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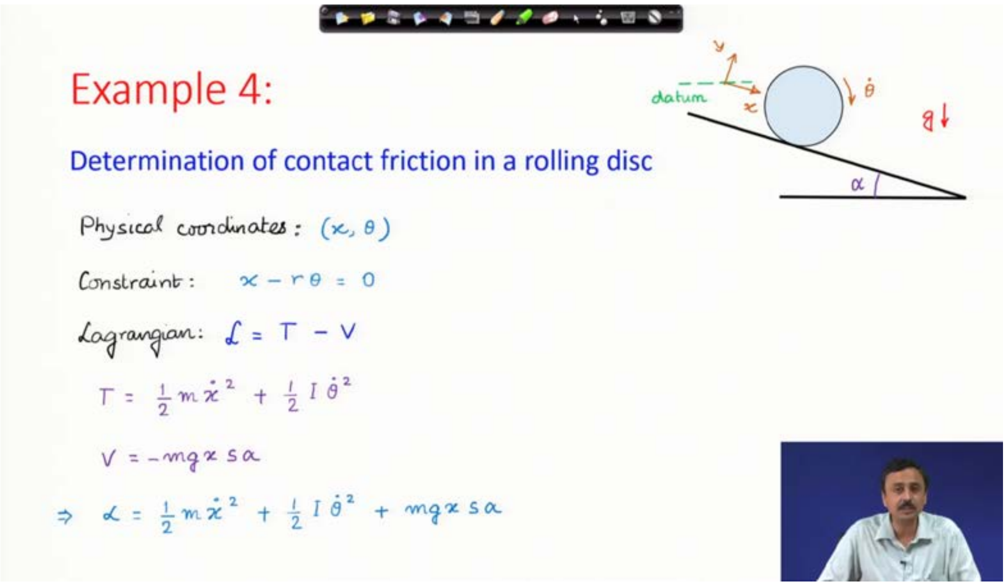

Now, we come to this rolling disc on an incline in a uniform gravitational field. We want to find out that friction force at the contact.

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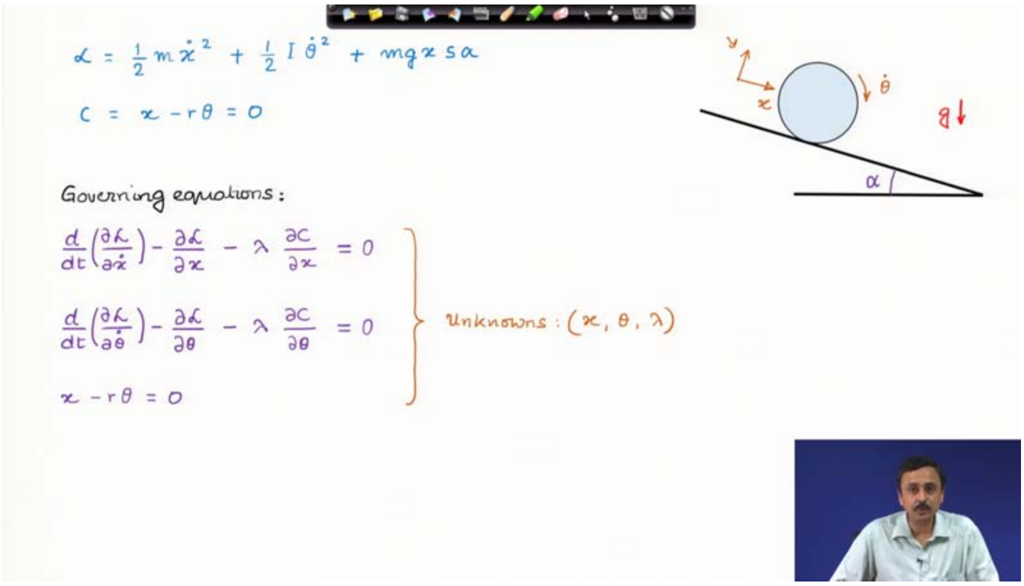

In order to do this, we break the rolling constraint and introduce it as a mathematical condition.

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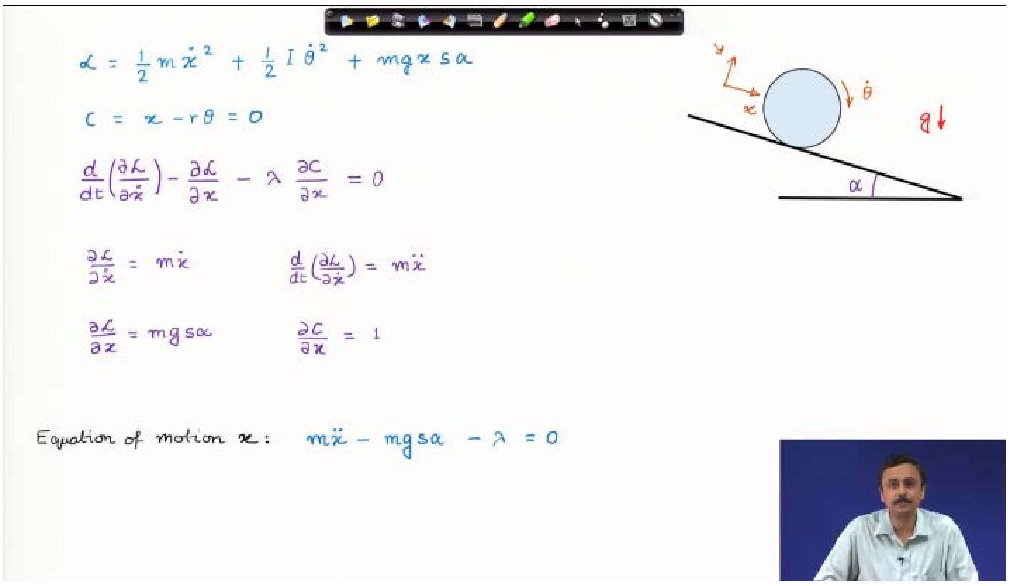

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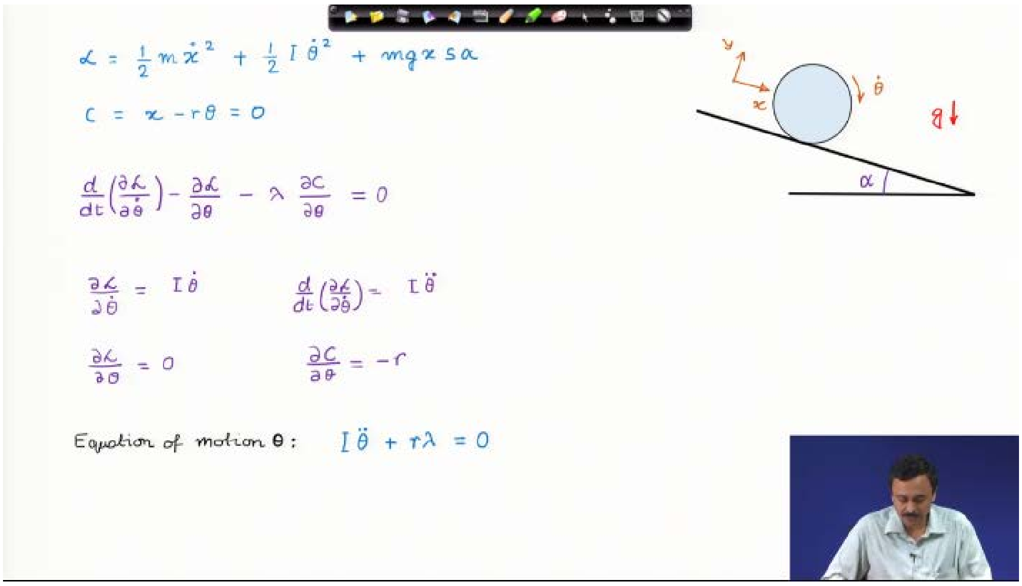

The equations of motion are derived in the above 2 slides. These equations involve the unknown Lagrange multiplier.

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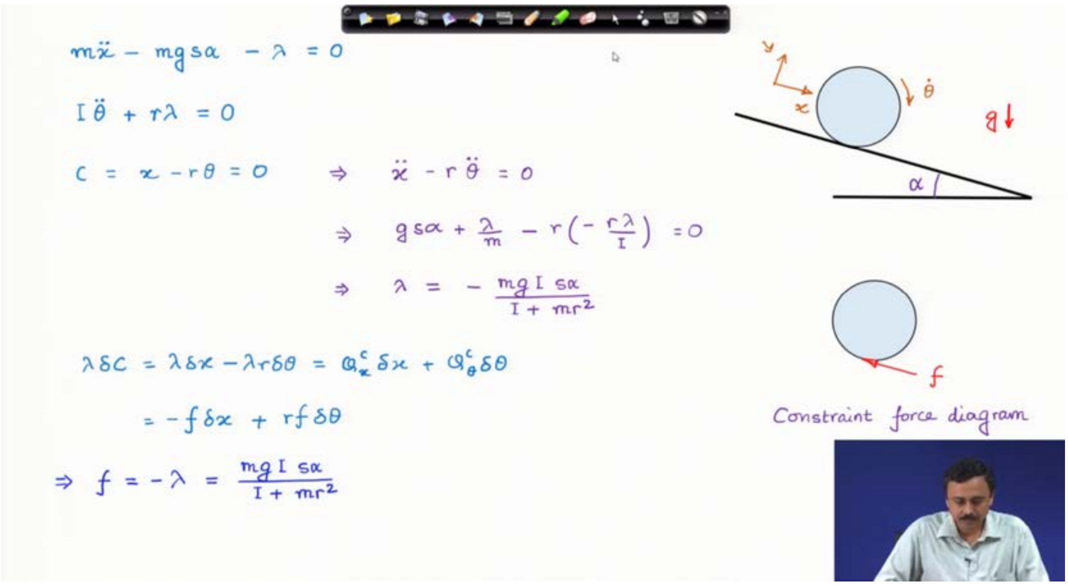

The above slide discusses the approach to determine the unknown friction force from the Lagrange multiplier.

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