Advanced Dynamics
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Module No # 10
Lecture No # 50
Hamilton's Principle and Lagrange's Equation of Motion – III

In this lecture we are going to continue with Lagrange's equation of motion, and later discuss the Hamilton-Ostrogradski principle.

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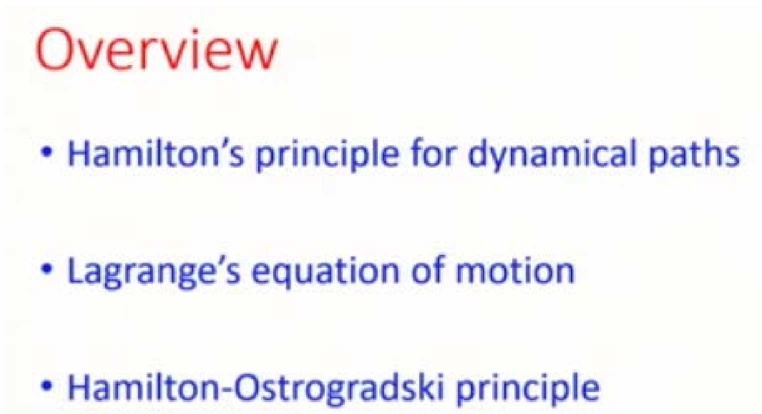

Hamilton-Ostrogradski principle is extension of Hamilton's principle where we also consider externally applied forces or non-conservative forces (for which we do not have a potential).

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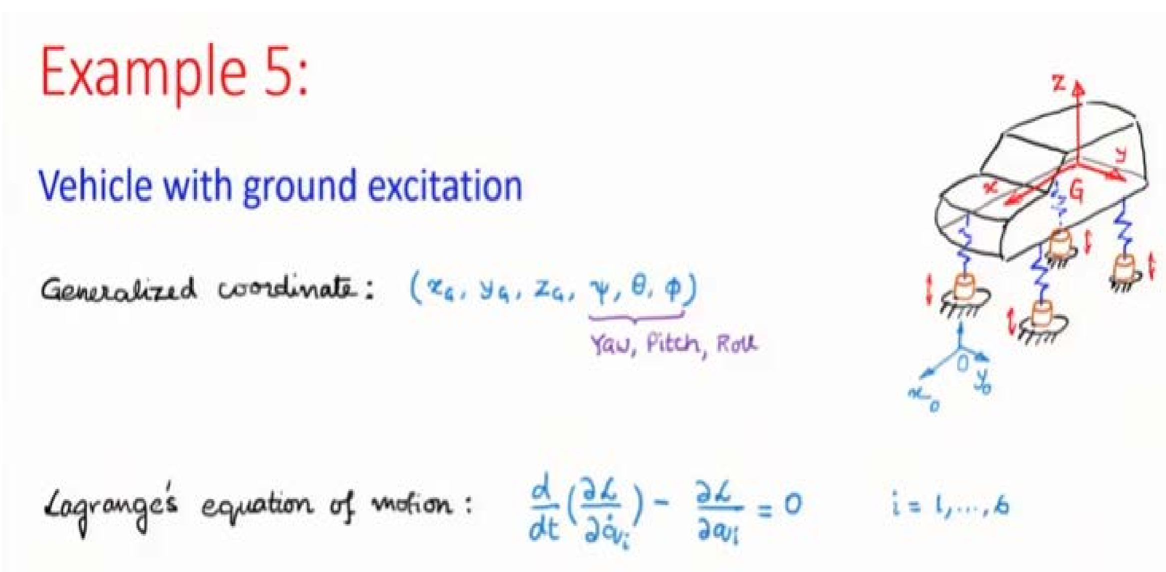

We first consider a vehicle with ground excitation as shown above.

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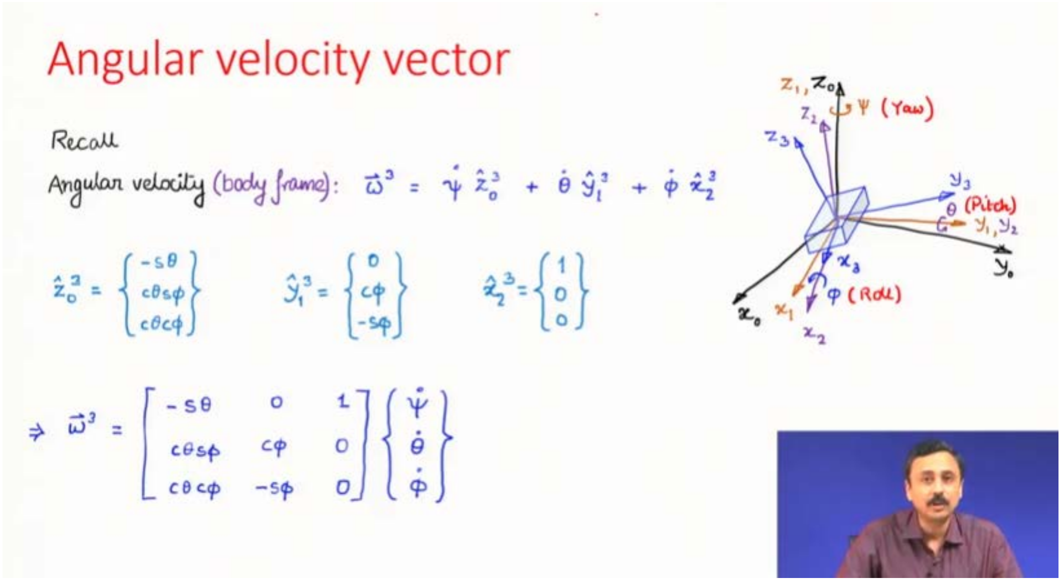

For the orientation part recall the discussion in our previous lecture where we derived the rotation matrix for the orientation of a frame with a given a yaw pitch and roll. Also recall the angular velocity vector written in the body frame in terms of yaw rate, pitch rate and roll rate, as shown in the slide above.

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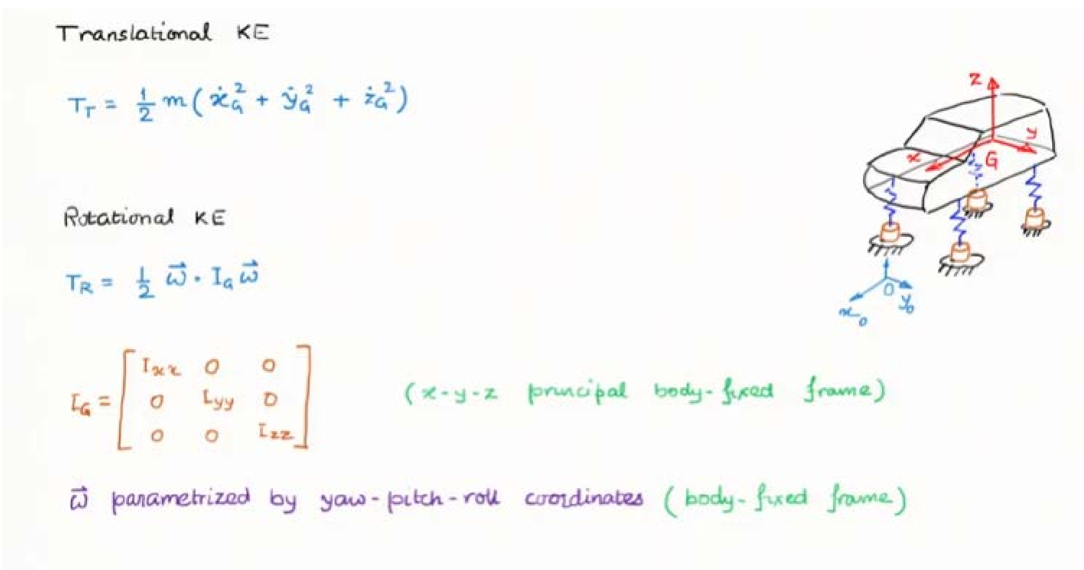

The structure of the translational and rotational kinetic energy expressions are presented in the slide above.

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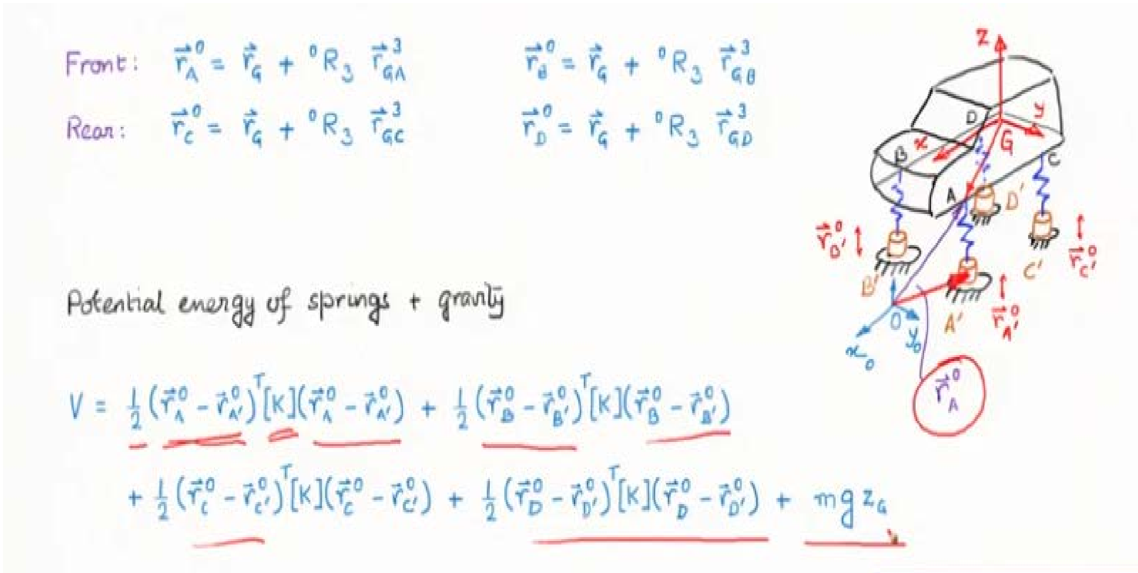

Next, we write down the potential energy contribution as presented in the above slide.

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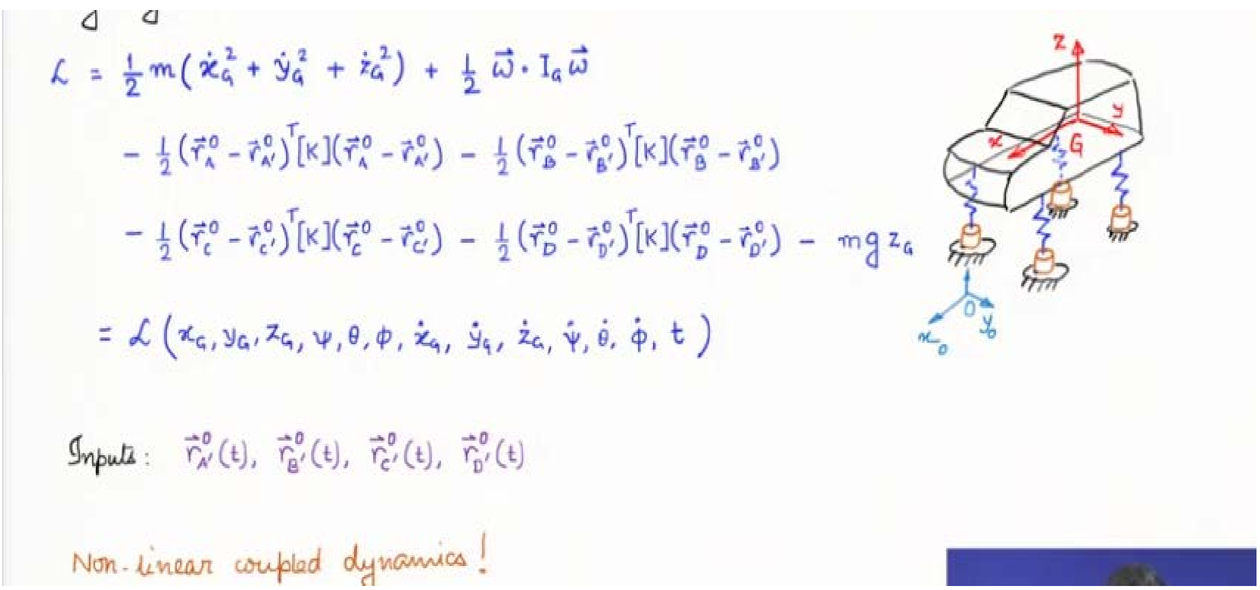

Finally we have the Lagrangian as kinetic minus potential energy as shown above. This will lead us to highly complex coupled second order ordinary differential equations which will be non-linear and extremely difficult to solve analytically. Therefore we have to take recourse to numerical methods for solution.

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Let us now look back and compare the Lagrangian approach with the Newtonian approach, as shown in the slide above. It may be noted that both the approaches lead us to the same equation of motion for a given mechanical system.

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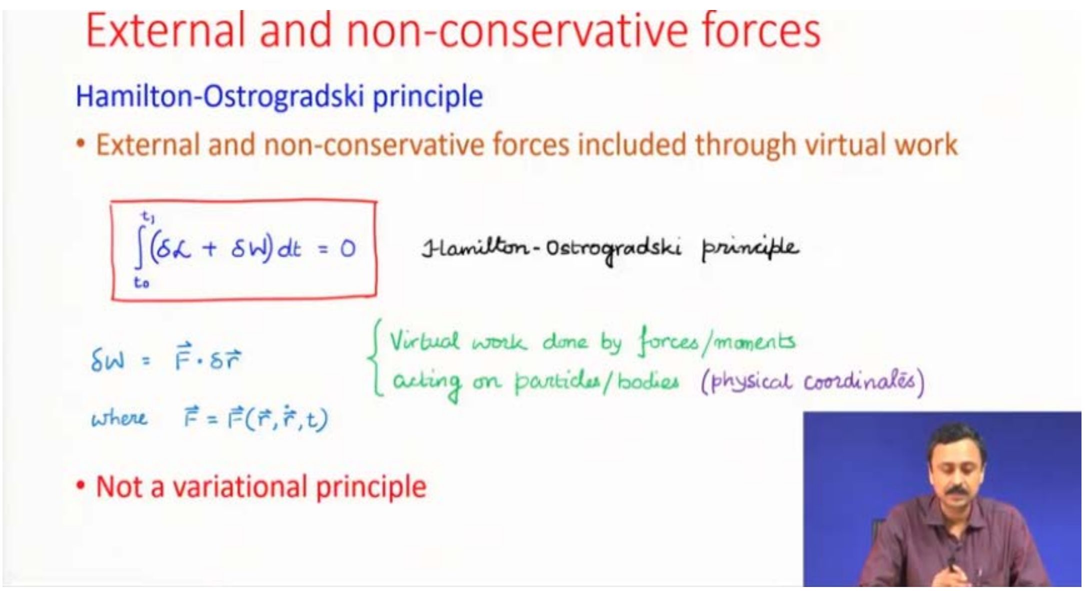

Next we consider systems with external forces or non-conservative forces. Till now we were looking at mechanical systems with potential forces. We now consider an approach to deal with non-potential external forces or non-conservative forces. For such systems, we have what is known as the Hamilton-Ostrogradski principle, or sometime known as extended Hamilton's principle. This is presented in the slide above.

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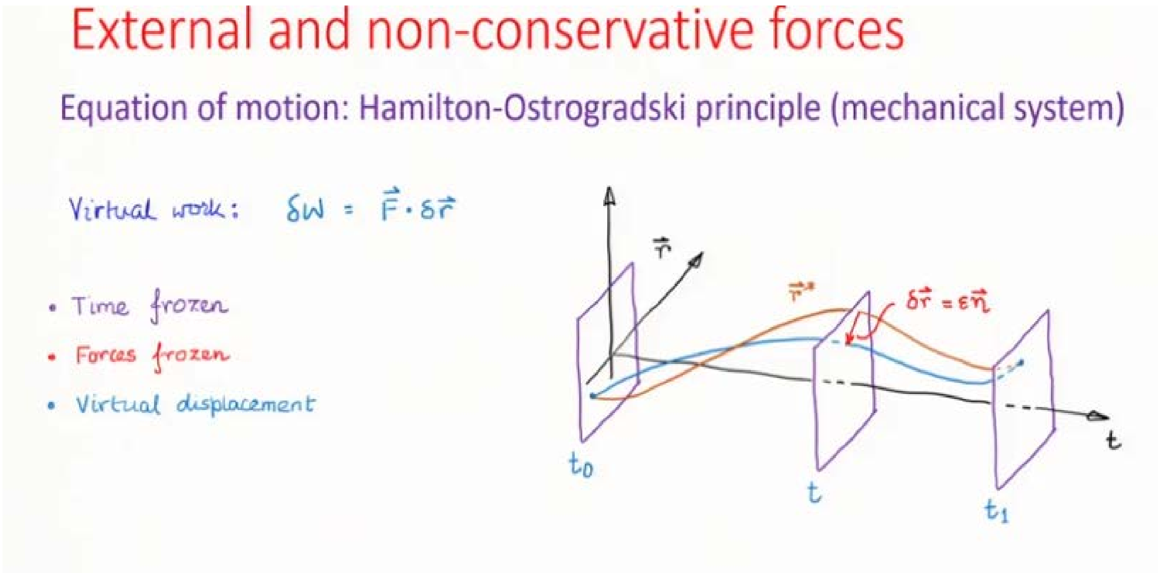

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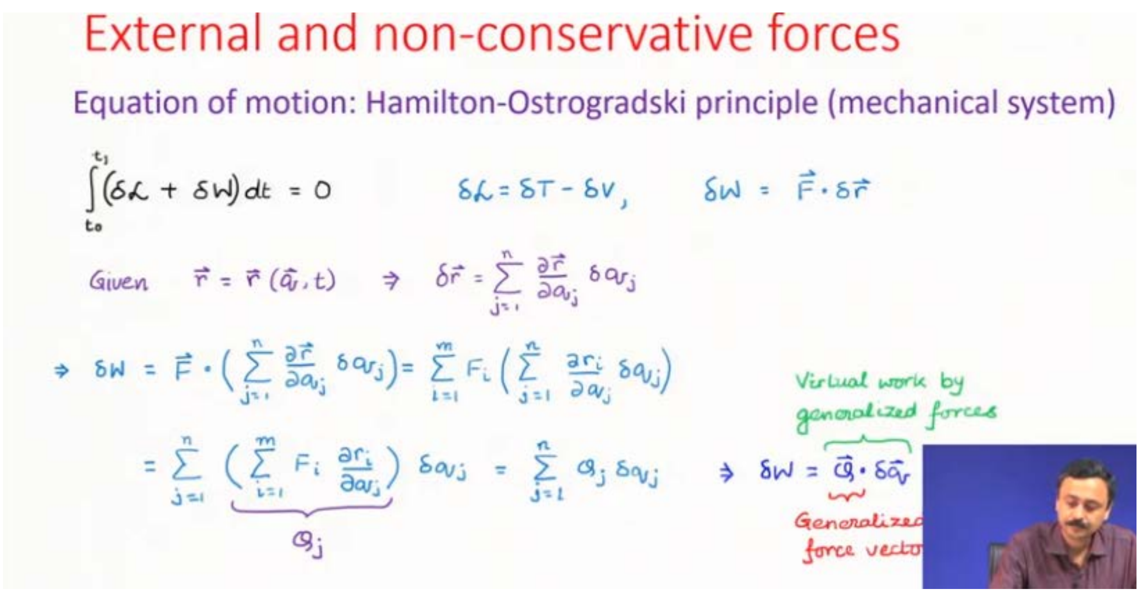

We first need to understand virtual work. The concept is as discussed in the 2 slides above.

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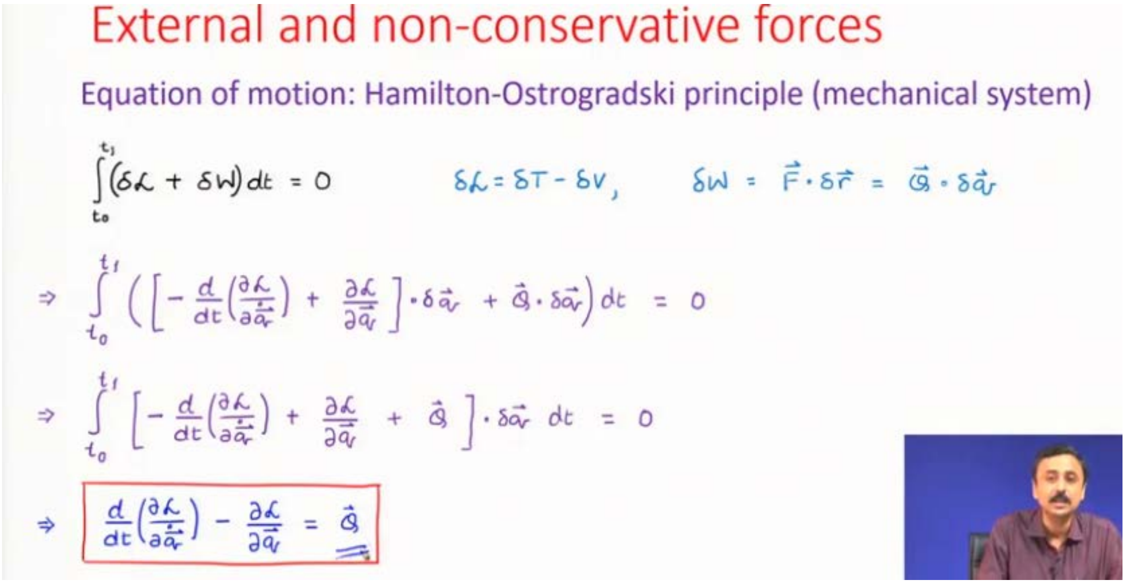

With the concept of virtual work, we can now use the Hamilton-Ostrogradski principle to obtain the equation of motion as presented in the slide above.

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