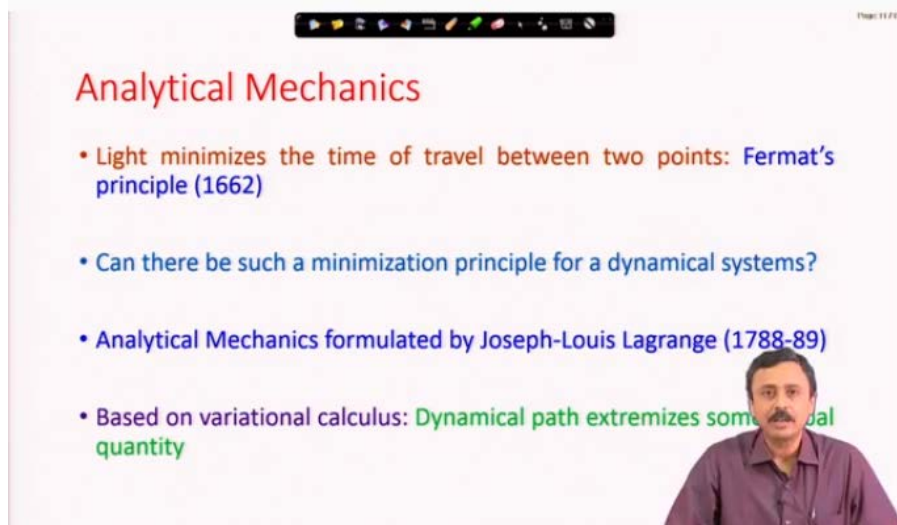


Advanced Dynamics
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Module No # 10
Lecture No # 46
Introduction to Analytical Dynamics's: Generalized Coordinates-I

In this lecture I am going to introduce the analytical dynamics formulation using the analytical mechanics developed by Lagrange.

(Refer Slide Time: 00:29)



The slide is titled "Analytical Mechanics" in red. It contains four bullet points:

- Light minimizes the time of travel between two points: Fermat's principle (1662)
- Can there be such a minimization principle for a dynamical systems?
- Analytical Mechanics formulated by Joseph-Louis Lagrange (1788-89)
- Based on variational calculus: Dynamical path extremizes some scalar quantity

A small video inset in the bottom right corner shows Prof. Anirvan Dasgupta, a man with a mustache wearing a purple shirt.

We will look at the introduction to analytical dynamics define what is the configuration space of a dynamical. Define the generalized coordinates to represent the configuration of a dynamical system on talk little bit about constraints. Analytical mechanics was formulated by Joseph Louis Lagrange around 1788-89. This is based on the calculus of variations.

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Configuration space

- A coordinate space in which a point fixes the configuration of a dynamical system
- Every point in the configuration space, specified by a set of **generalized coordinates**, represents a unique configuration of the system
- **Generalized coordinates**: The minimum number of independent variables required to uniquely fix the configuration of a dynamical system
- Change in generalized coordinates is governed by the dynamics, and represented by a dynamical path in the configuration space

The definition and characteristics of configuration space is described in the slide above.

In the following slides, examples of some dynamical systems are considered and the corresponding configuration space geometry is discussed.

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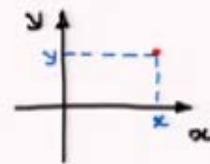
Configuration space: examples

A point in a plane

$$\mathcal{L} : \{(x, y)\} \quad (\text{physical coordinates})$$

Generalized coordinates: (x, y)

Degree of freedom: 2



Configuration space: \mathbb{R}^2

The configuration space of a point on a plane is discussed above.

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Configuration space: examples

Simple pendulum

$\mathcal{L} : \{(x, y) \mid x^2 + y^2 = l^2\}$ (physical coordinates)

$x = l \cos \theta \quad y = -l \sin \theta$

$\mathcal{L} : \{\theta\}$

Generalized coordinates: (θ)

Degree of freedom: 1

R^1 Configuration space: S^1

Let us now look at the configuration space of a mathematical pendulum as shown above.

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Configuration space: examples

A car in a plane

$\mathcal{L} : \{(x, y, \theta)\}$ (physical coordinates)

Generalized coordinates: (x, y, θ)

Degree of freedom: 3

Constraint on motion direction
(Admissible/dynamical path)

Configuration space: examples

Conical pendulum

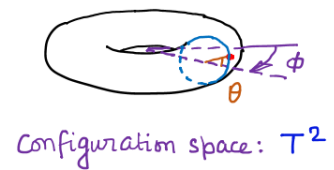
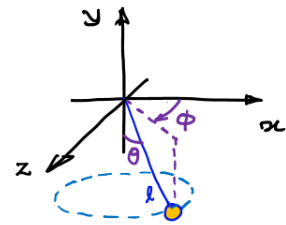
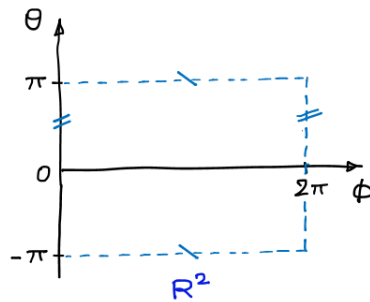
$$\mathcal{L} : \{ (x, y, z) \mid x^2 + y^2 + z^2 = l^2 \} \text{ (physical coordinates)}$$

$$x = l \sin \theta \cos \phi \quad y = -l \sin \theta \sin \phi \quad z = l \cos \theta$$

$$\mathcal{L} : \{ (\theta, \phi) \}$$

Generalized coordinates: (θ, ϕ)

Degree of freedom: 2



Configuration space: T^2

Conical pendulum example is discussed above.

Configuration space: examples

Bead on a rotating circular hoop

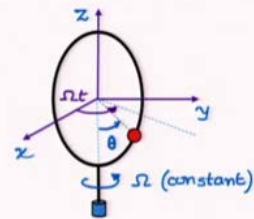
$$\mathcal{L} : \{ (x, y, z) \mid x^2 + y^2 + z^2 = R^2 \} \text{ (physical coordinates)}$$

$$x = R \sin \theta \cos \Omega t \quad y = R \sin \theta \sin \Omega t \quad z = R \cos \theta$$

$$\mathcal{L} : \{ \theta \}$$

Generalized coordinates: θ

Degree of freedom: 1



Configuration space: S^1

A bead on the rotating circular hoop this is a circular hoop on which a bead can slide is considered in the above slide.

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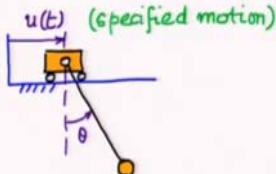
Configuration space: examples

Pendulum on a cart with support motion

$\mathcal{L} : \{(\mathbf{x}, \theta) \mid \mathbf{x} - u(t) = 0\}$ (physical coordinates)

Constraint: $\mathbf{x} - u(t) = 0 \Rightarrow \mathbf{x} = u(t)$

Configuration space: S^1



Generalized coordinates: θ

Degree of freedom: 1

Pendulum on a cart is considered above.

(Refer Slide Time: 25:34)

Configuration space: examples

Rolling coin

Coordinates: $(x_a, y_a, z_a, \psi, \phi, \theta)$

Center: (x_a, y_a, z_a) Orientation: (ψ, ϕ, θ) (Spin)

Constraints:

- Always in ground contact $\Rightarrow z_a = r \cos \phi$
- No-slip constraint $\Rightarrow \dot{x}_a - r \dot{\phi} \sin \phi \sin \psi - r(\dot{\theta} + \dot{\psi} \cos \phi) \cos \psi = 0$
 $\dot{y}_a + r \dot{\phi} \cos \phi \cos \psi - r(\dot{\theta} + \dot{\psi} \sin \phi) \sin \psi = 0$ *cannot be integrated*

Generalized coordinates: $(x_a, y_a, \psi, \phi, \theta)$

Degree of freedom: 5

We discuss the configuration space of a rolling coin in the slide above.

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Summary

- Introduction to Analytical Dynamics
- Configuration space
- Generalized coordinates
- Constraints