Advanced Dynamics
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Lecture - 41
Kinematics of Rotation - I

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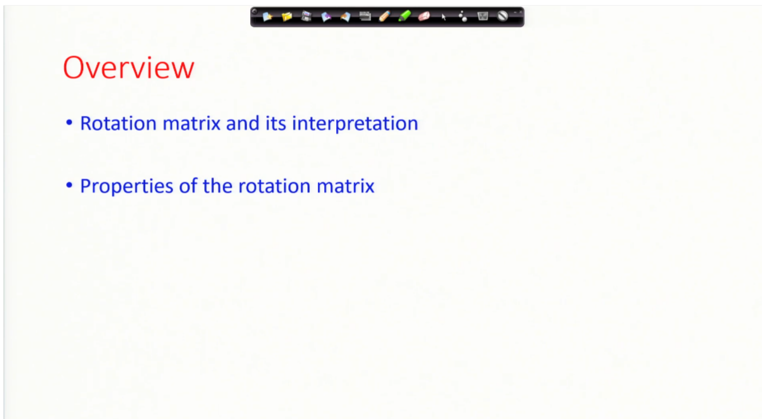

In this lecture I am going to start the topic of rotation, Kinematics of rotation. In this discussion today I am going to discuss about the rotation matrix and its interpretation and properties of the rotation matrix.

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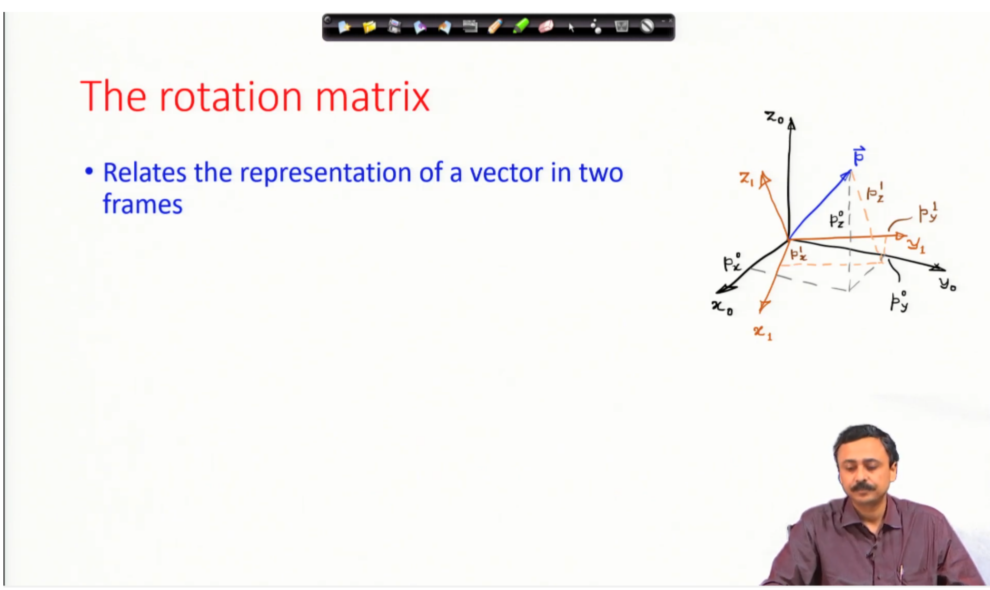

What is the rotation matrix? It is a matrix which relates the representation of a vector in two frames. For example, I have the black frame and the brown frame. The black frame is x_0 y_0 z_0 and the brown frame is x_1 y_1 z_1. Imagine a vector, p as shown. How do you represent p in different frames? We drop perpendiculars to the planes from p we drop perpendiculars to the plane. For example, if I drop it on the x_0 y_0 axis that perpendicular distance from the x_0 y_0 axis gives me z_0. And similarly, z_0 coordinate of p and similarly I can find out the x_0 coordinate y_0 coordinates etc. I have shown these ah these perpendiculars that I have dropped this thing that I have written p_x^0, that superscript 0 indicates in which frame the vector is represented. Vector is an abstract quantity with which exists by its own right its representation from one frame to another can be very different. For example, velocity if you have a particle which has a velocity, that velocity belongs to the particle. The particle is moving in a certain way now if I choose a particular frame, I will have particular representation of this velocity. If you choose another frame, you will have another representation of that velocity.

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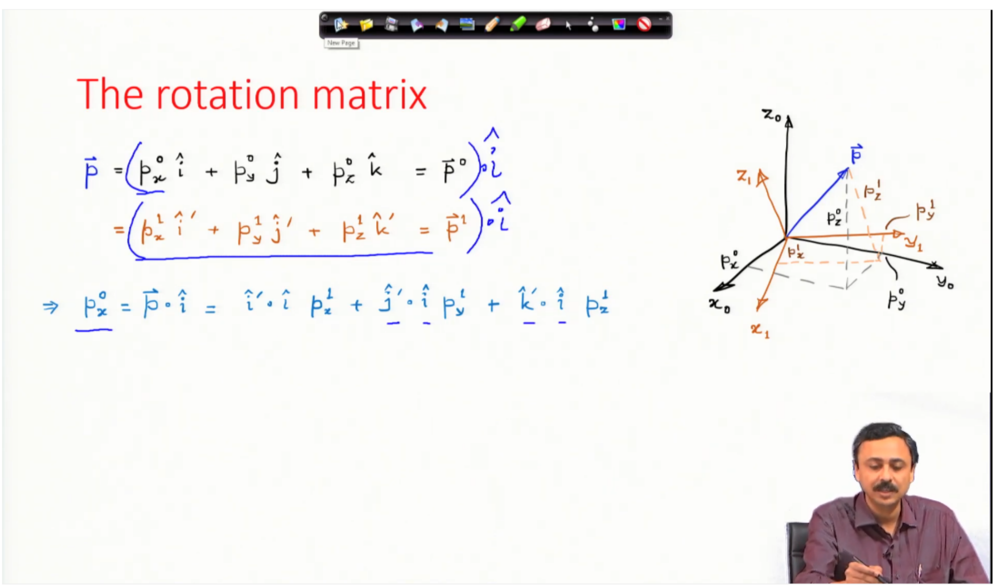

Therefore, this is the representation of that same vector p in x_0 y_0 z_0 the same vector and this I will write with superscript 0 to show the representation of p in frame 0. Similarly, I can write that same vector p in frame 1 as shown above.

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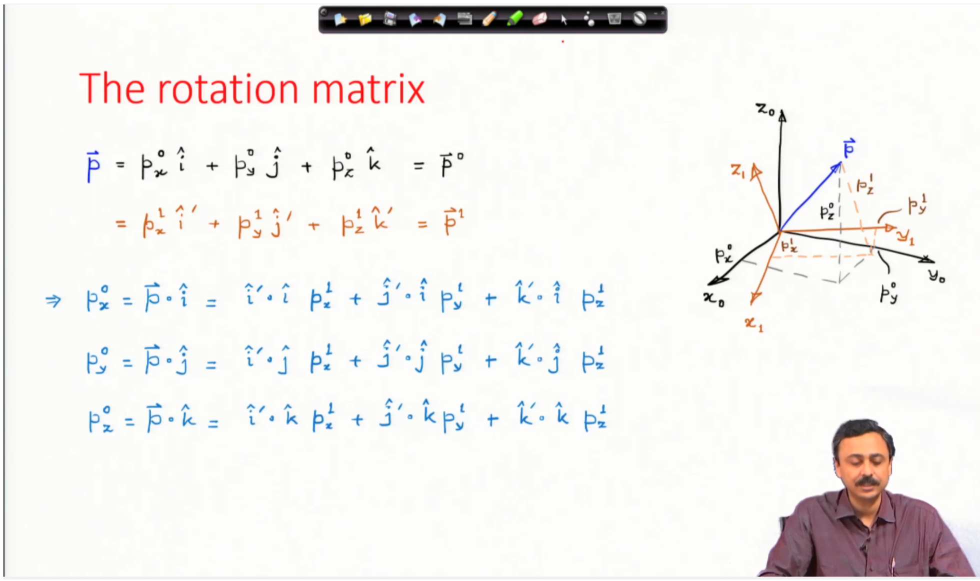

The determination of the components of the vector p using dot products is shown in the slide above.

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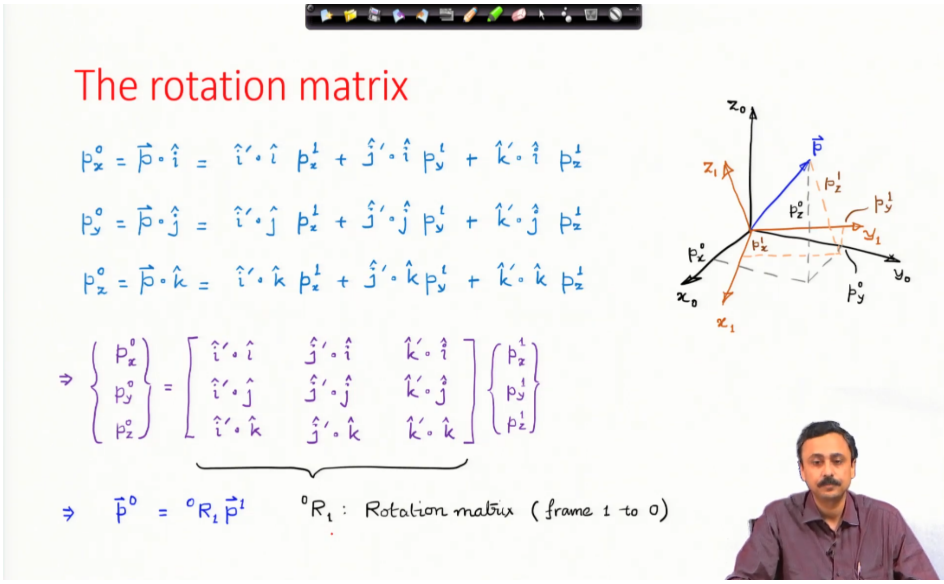

The components of the vector p in the two frames are related by the rotation matrix as presented above. The relation is from frame 1 to frame 0.

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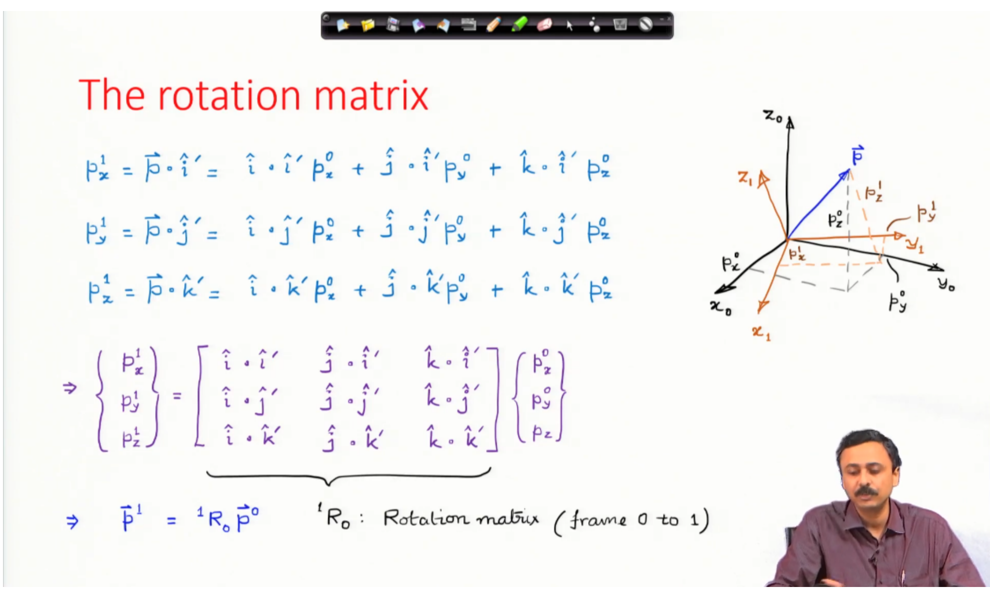

Now I do the reverse thing. The above relation is from frame 0 to frame 1.

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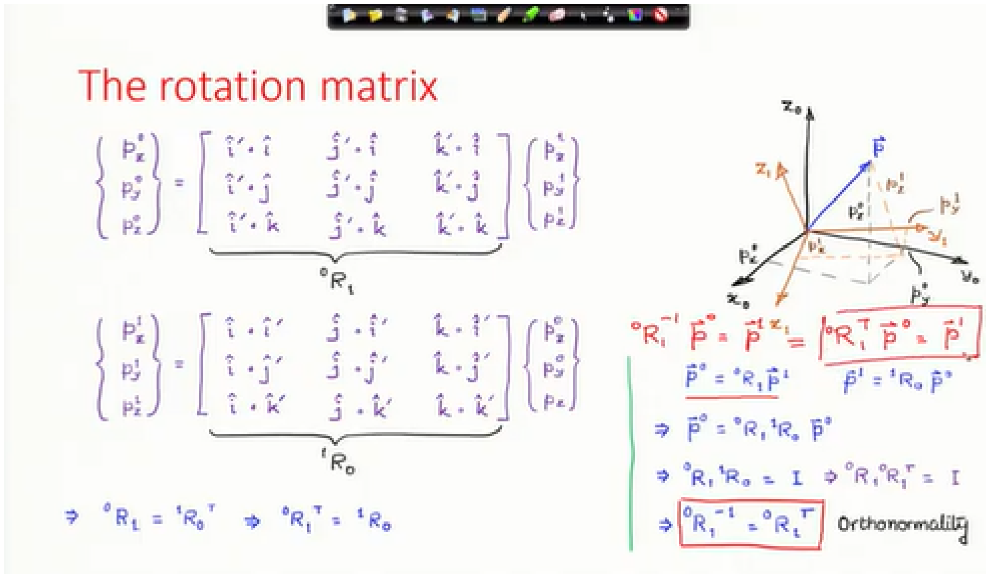

Let us look at these two matrices together. You will find that the column here is same as the row here. Every column in the first matrix is has become a row in the second matrix in other words the second matrix is a transpose of the first matrix. The consequences of this property are shown in the slide above.

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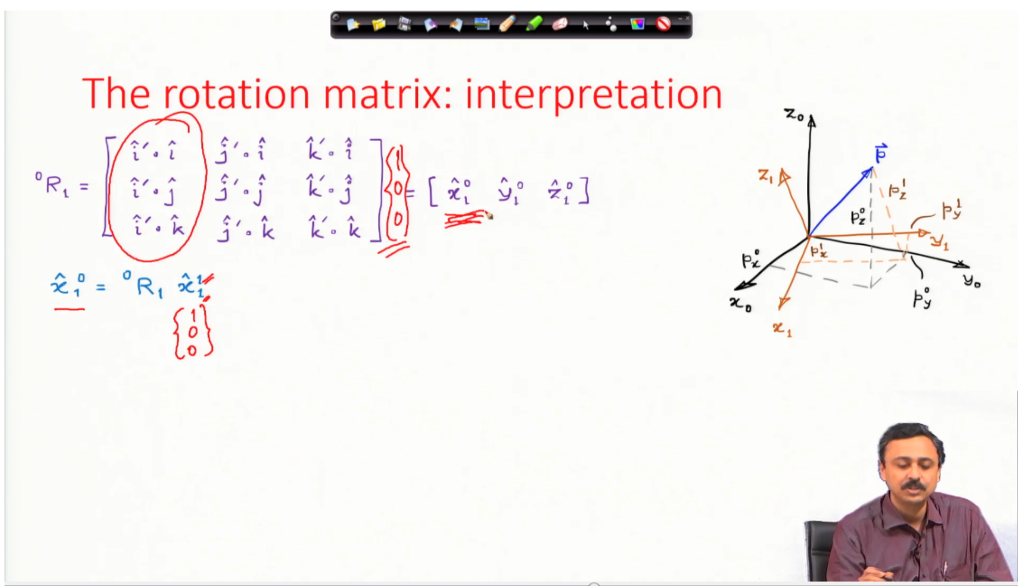

In the above slide, we interpret the columns and rows of the rotation matrix. The first column of ${}^0R_1$ is nothing but the representation of the unit vector x_1 in frame 0. Similarly, the second column is a representation of y_1 in frame 0, and the third column is a representation of the unit vector z_1 in frame 0.

The interpretation and properties of the rotation matrix are discussed in the following slides.

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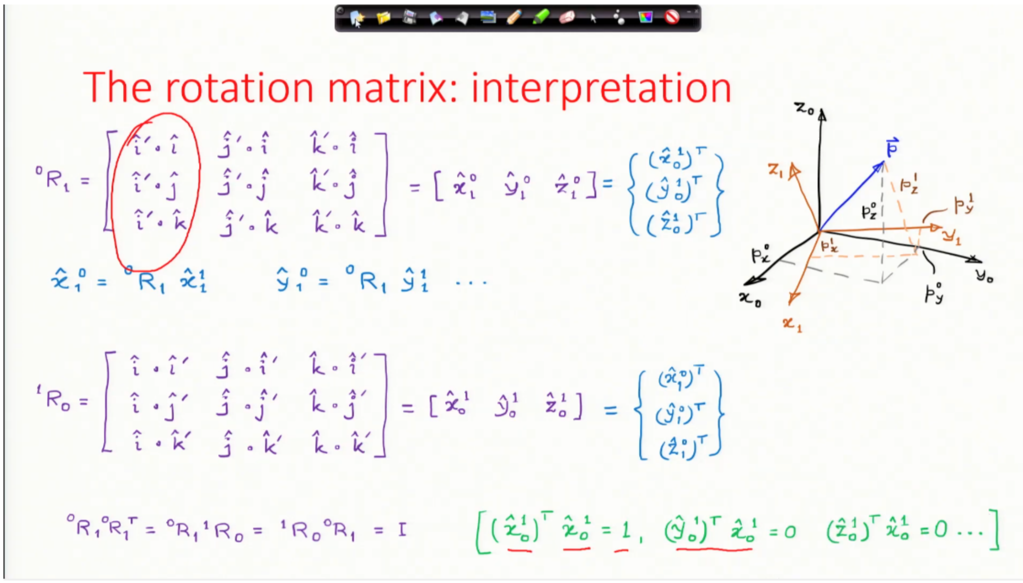

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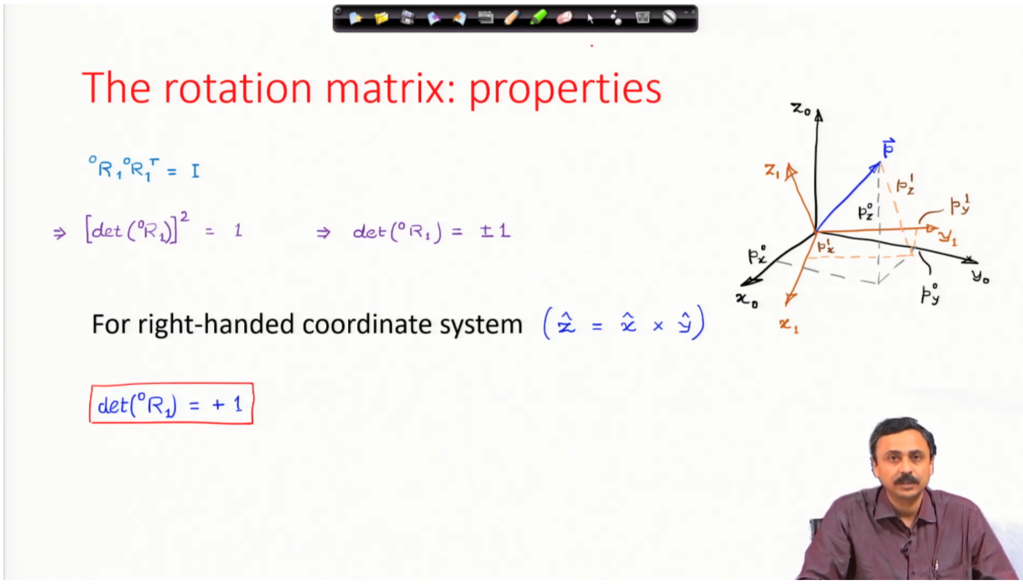

The rotation matrix for a right-handed coordinate system will have determinant +1, as shown above.

Some examples of simple rotation matrices are considered in the following slides.

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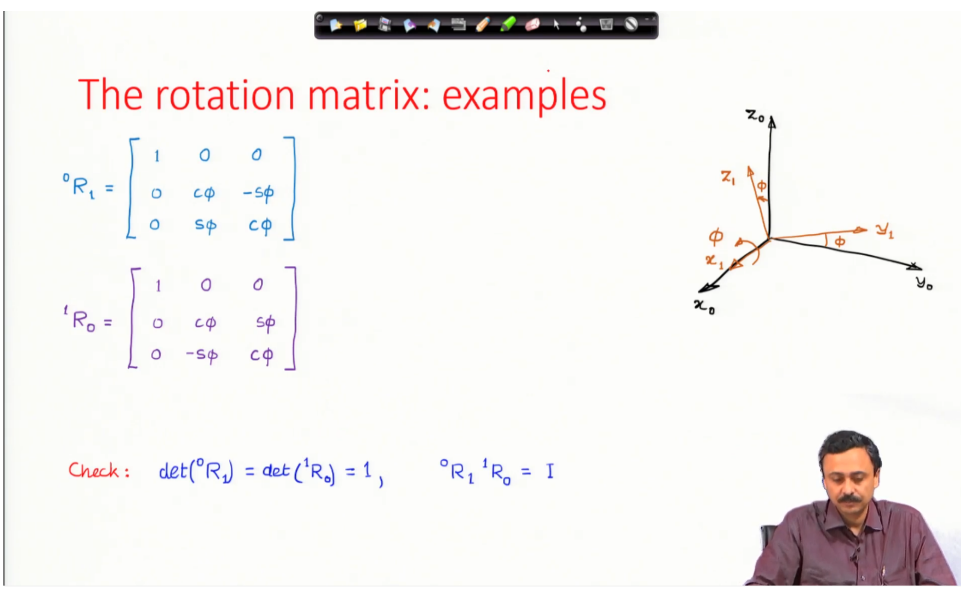

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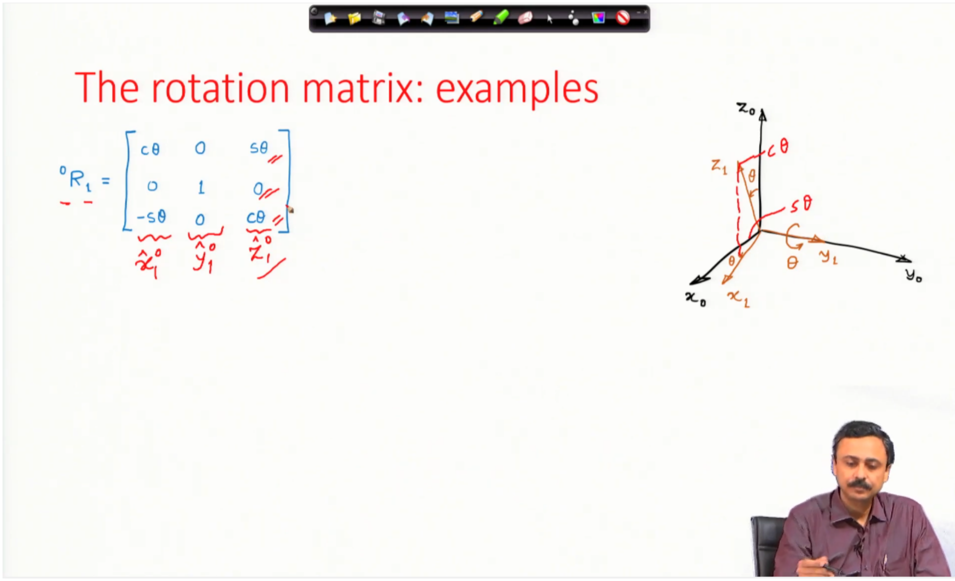

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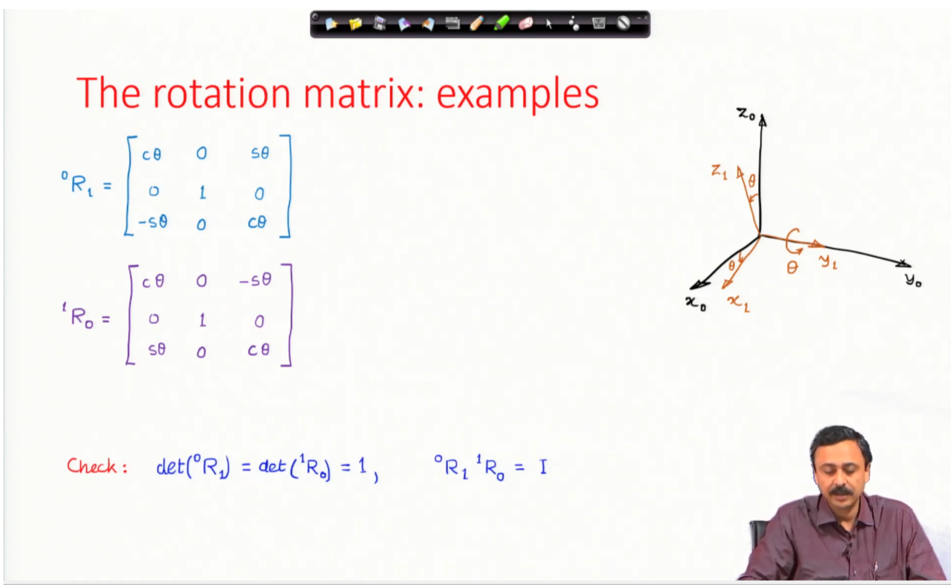

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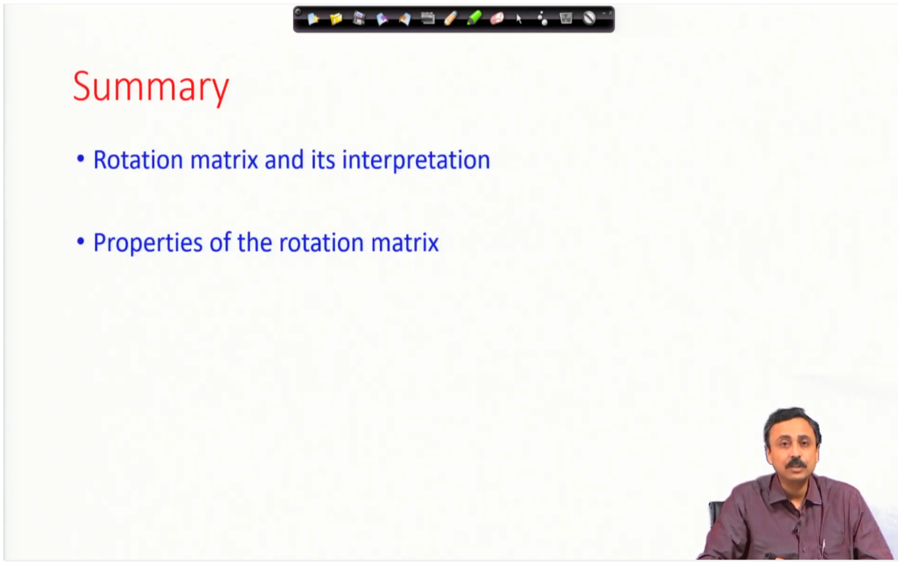

Summary is provided above.