Advanced Dynamics
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Lecture – 40
Gyroscopic Motion – III

In this lecture, we are going to first discuss the stability of rotation of a rigid body. Next, we are going to look at precession of the earth.

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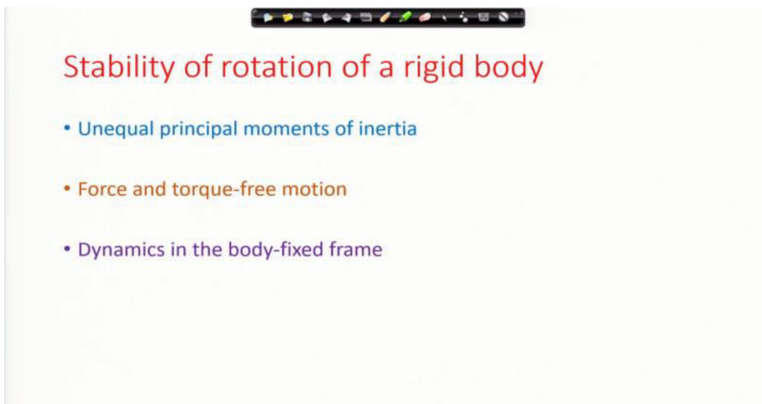

Suppose we have a body which is in general asymmetric. We will have in general 3 unequal principal moments of inertia. We will study torque free rotations of such bodies. We will study the dynamics of such a body in the body fixed frame. This gives us this simplification that the moment of inertia tensor becomes time invariant.

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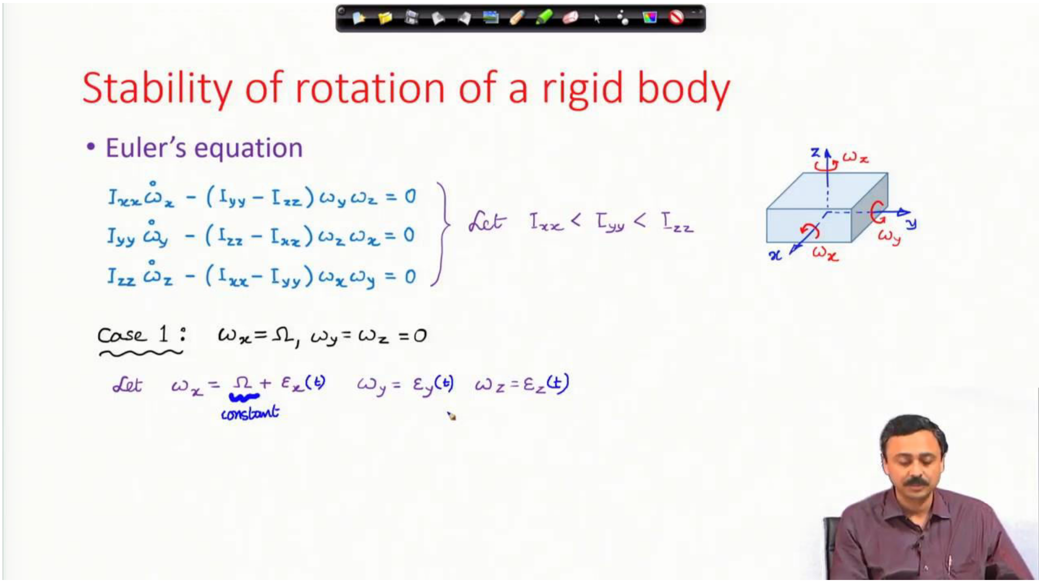

Consider a body as shown in the figure above. The Euler's equation for the rotational dynamics of the body can be written as shown. If the body is given a spin about the x-axis along with a small disturbance, the angular velocity vector components can we written as shown.

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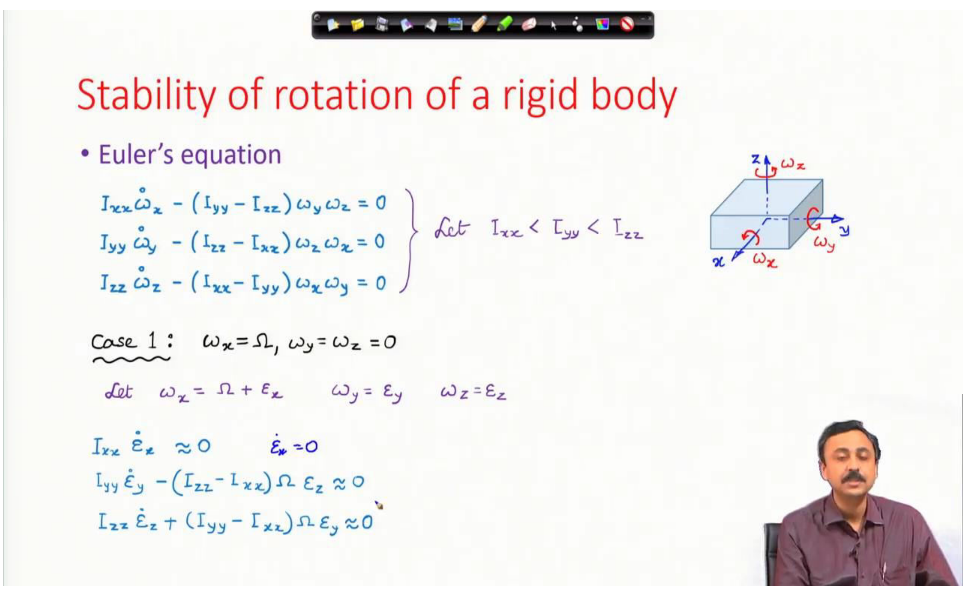

As shown above, the Euler's equations can be linearized considering the disturbance to be small, and dropping the second order small terms.

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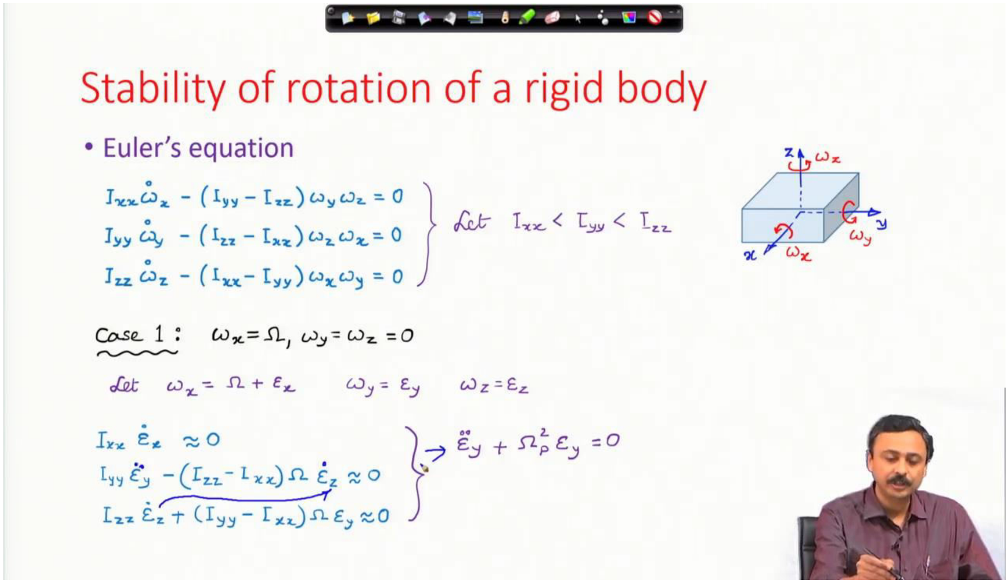

We time derivate the y-equation and use the z-equation to obtain the disturbance dynamics in the y-axis direction, as shown above.

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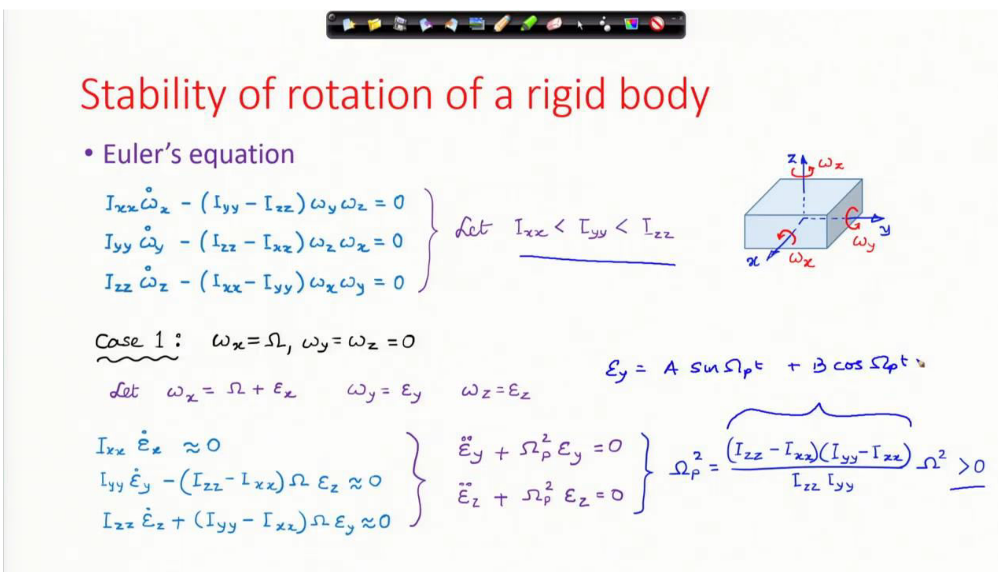

Similarly, we can also write the disturbance dynamics in the z-axis direction.

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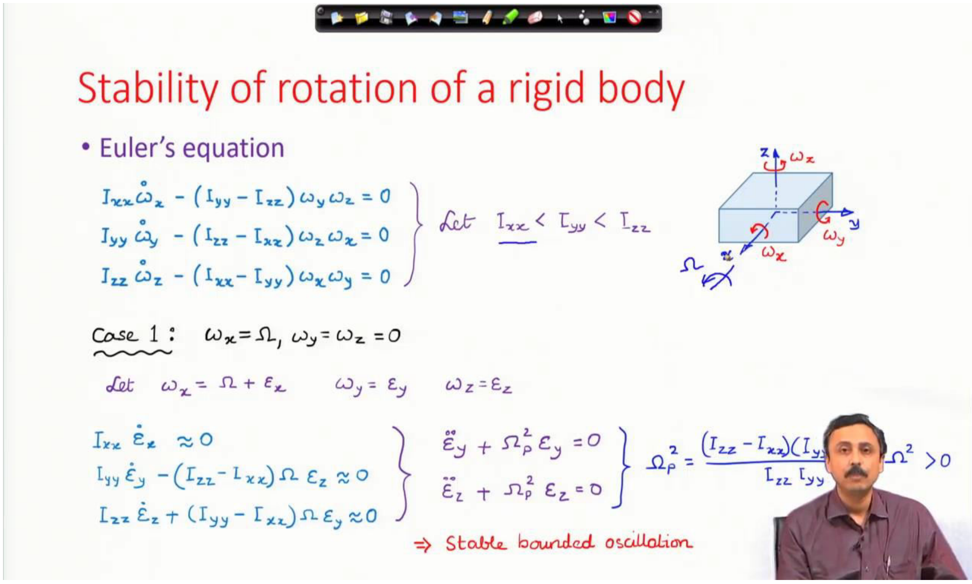

From the nature of solutions of the disturbance dynamics equations, we can conclude that if we give a spin about the smallest moment of inertia axis, the disturbances will remain bounded.

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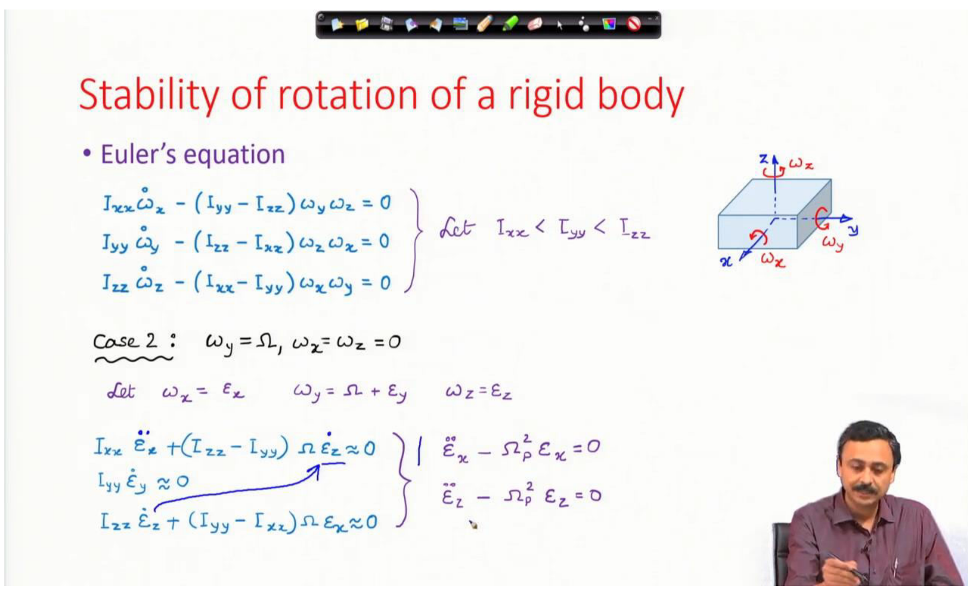

Let us move on to the second situation where we give the spin about the axis which has the intermediate moment of inertia.

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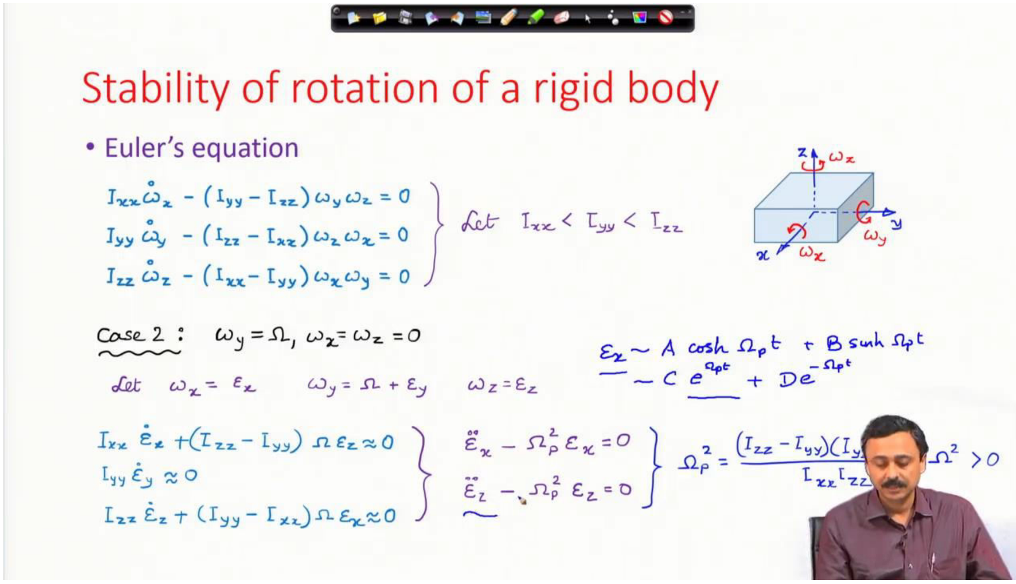

The disturbance dynamics equation in this case is presented above. From the nature of solution of these equations, we observe that one solution term represents exponential divergence. In other words, the disturbance in the x and z axes is going to exponentially diverge, implying that spin about the intermediate axis is unstable. This is sometimes known as the intermediate axis theorem.

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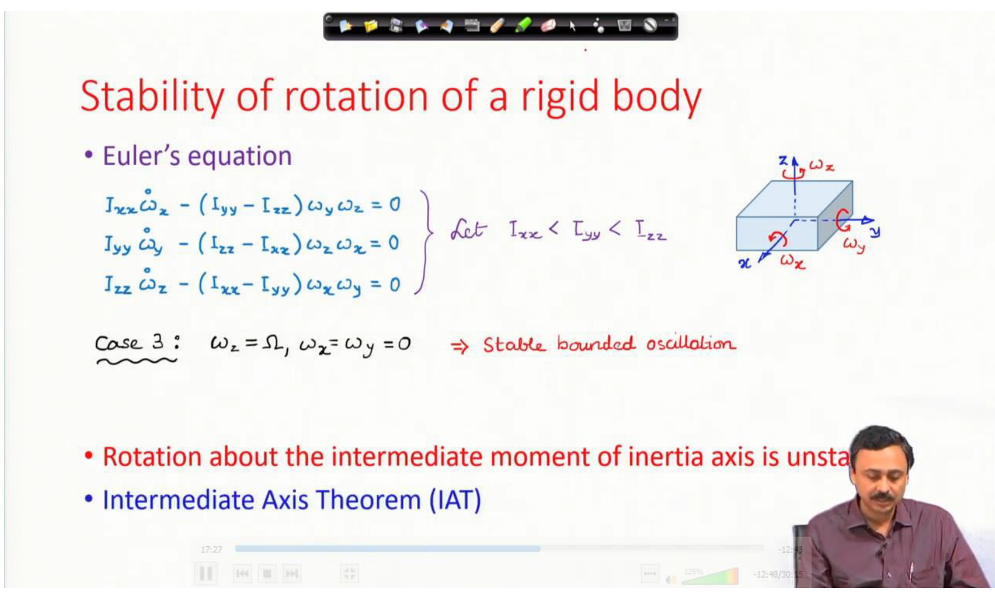

Following the same steps outlined in the last 2 cases, one can show that spin about the maximum principal moment of inertia axis is also stable.

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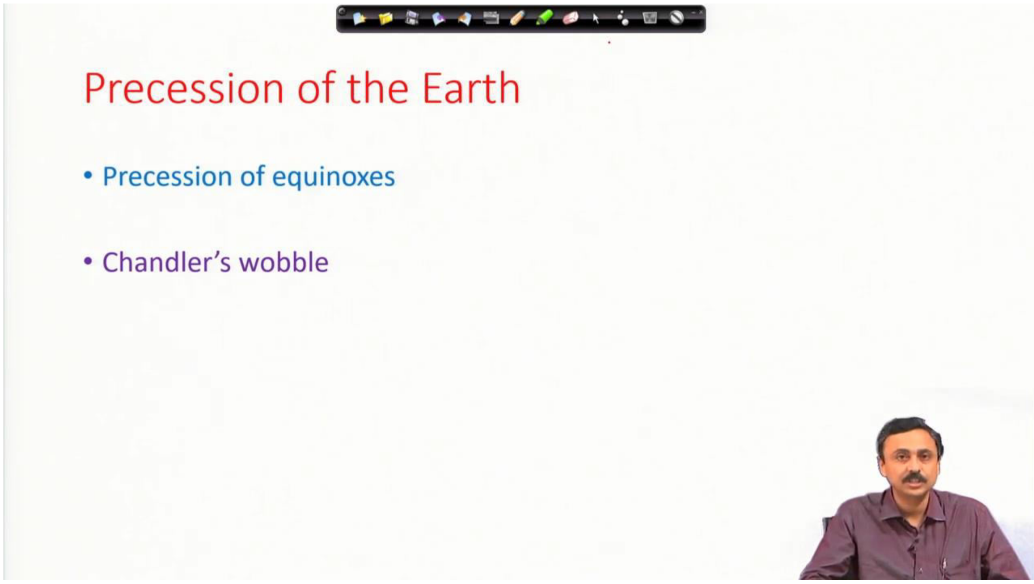

Next we look at the precession of the earth. The earth is not a perfect sphere but an oblate sphere. There are 2 kinds of precession of the earth. One is because of external torque that acts due to the sun, and the other is torque-free precession, also known as the Chandler's wobble.

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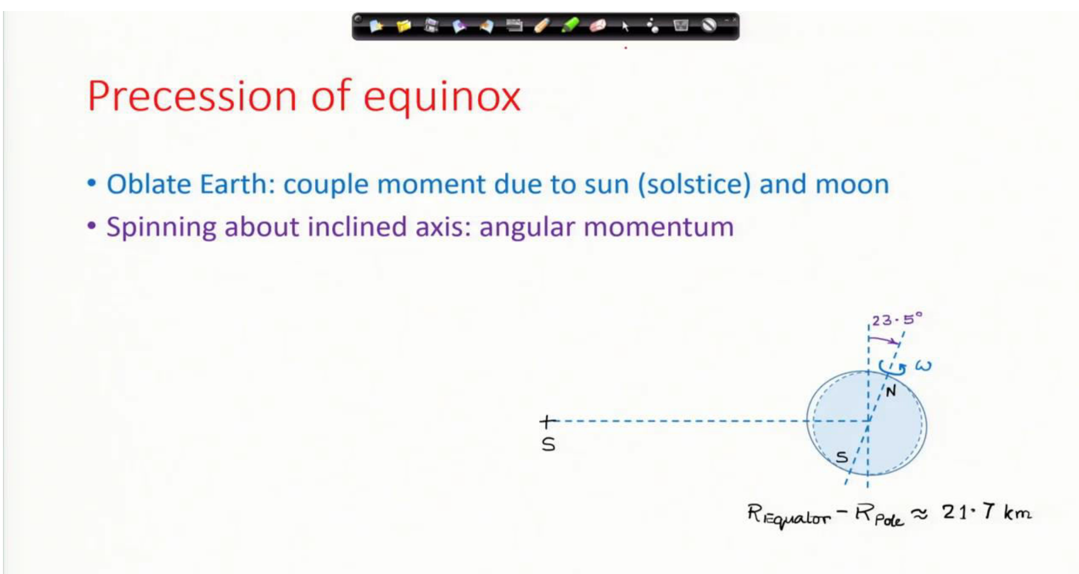

Now because of this oblateness of the Earth, we consider the effect of the additional mass around the equatorial region. This leads to additional attraction forces from the sun.

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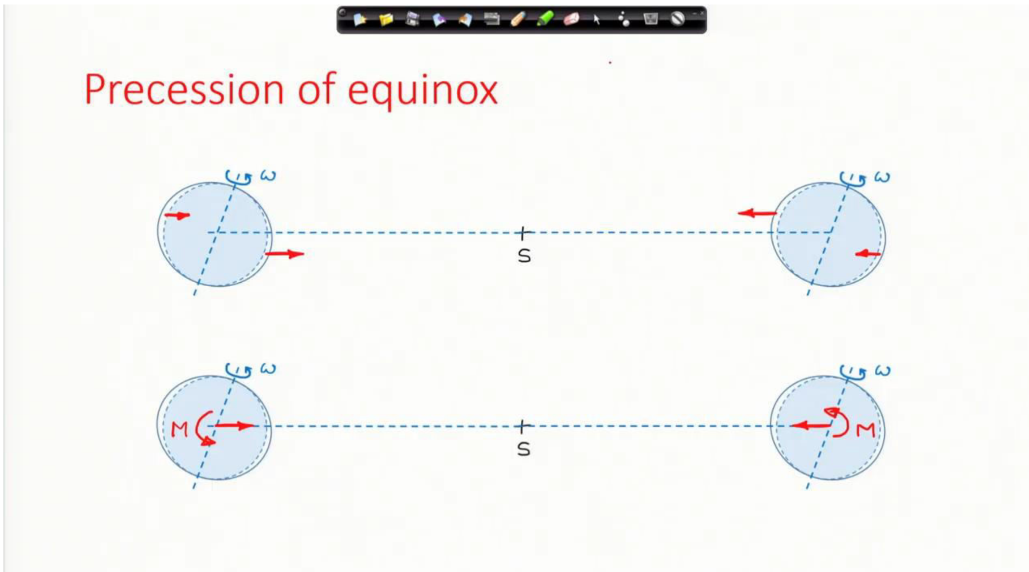

Because of the additional mass which is distributed around the equator we have a little bit of additional attraction force. The forces at the near-side of the sun is slightly higher that that at the far-side. Due to this small difference in forces and the inclination of the Earth, at the two solstice positions, this leads to a couple moment as shown above. A spinning body under action of a moment will precess as discussed previously.

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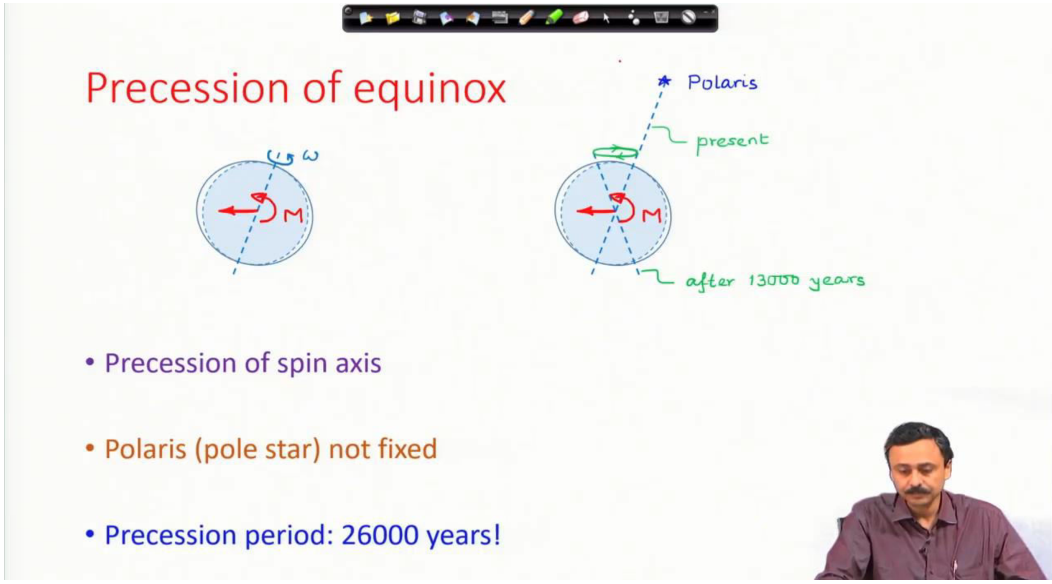

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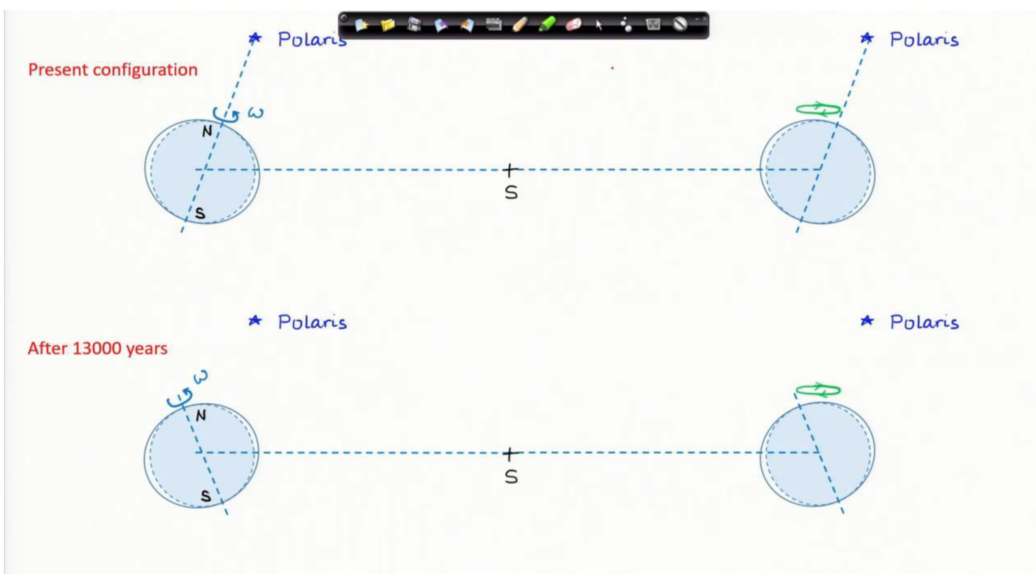

Because of this precession the spin axis is going to change orientation. Right now our north south axis is roughly oriented towards the Polaris or the pole star. Because of this precession, after a while (approximately 13000 years), the axis is going to point in a different direction as shown in the slides above. It may be noted that the spatial location where we have summer in the northern hemisphere today, after 13000 years, at this location we will have winter in the northern hemisphere. Similarly, the spatial location where we have spring now, we will have autumn.This means that the location of the equinoxes spring (vernal) equinox and autumnal equinox is rotating, or precessing. That is why it is called precession of equinoxes.

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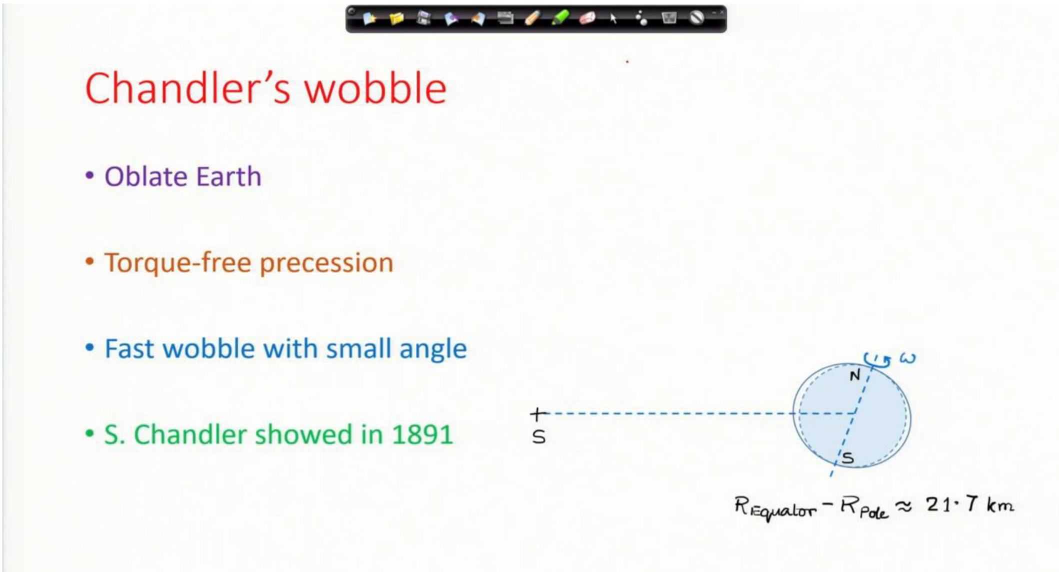

Next we are going to discuss Chandler's wobble which is a torque-free precession of the earth. Because of the oblateness of the Earth, just like the precession of a frisbee disc, we have torque-free precession of the Earth. Many people had predicted this wobble because oblateness of the Earth was known for a long time. Euler had calculated this period of this wobble but it was never found because that value was off from the actual value. Finally, Chandler showed from the existing data in 1891 that this wobble has a period which is substantially different from the predicted value because of various other factors.

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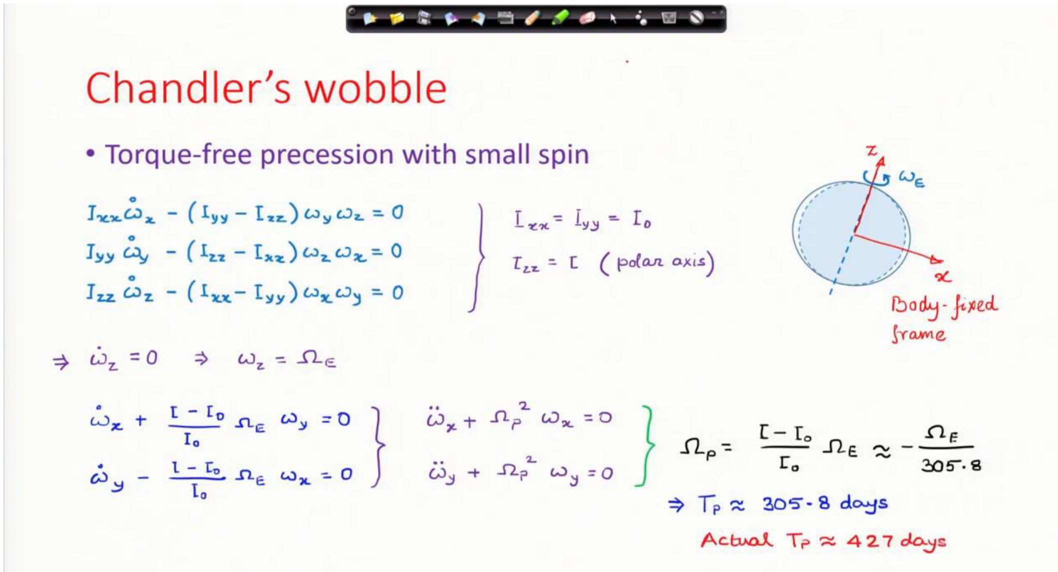

The equations of motion of torque-free rotational dynamics of an axisymmetric body is obtained from Euler's equations, as shown above. Using the known data of moment of inertia of the Earth, the precessional time period is estimated as 305.8 days. However, the actual value observed in 427 days.

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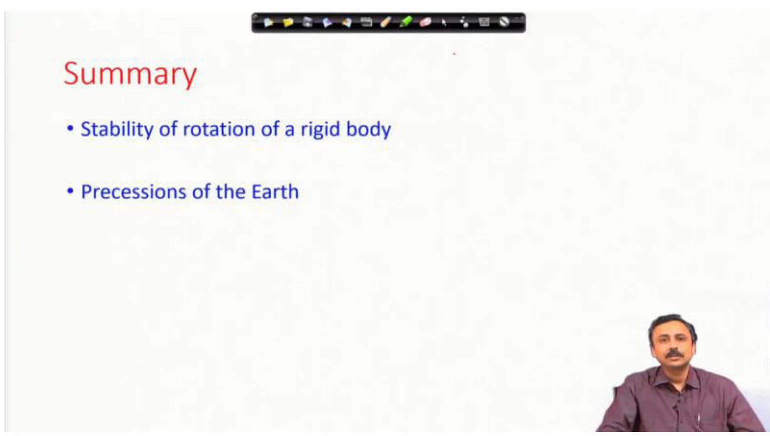

The above slide summarizes the discussions.