


**Advanced Dynamics**  
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**Lecture – 39**  
**Gyroscopic Motion -- II**

We are going to continue our discussions on gyroscopic motion of rigid bodies. As noted in the previous lecture, we were looking at gyroscopic forces on spinning axisymmetric bodies and precession with and without external moment. And in this lecture I am also going to introduce the dynamics of a boomerang.

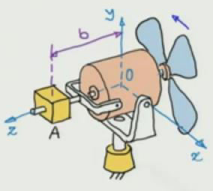
We consider the following problem.

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**Problem 1:**

A special purpose fan is mounted as shown. The rotor with the blades have a combined mass of 2.2 kg with radius of gyration of 60 mm. The axial position  $b$  of the 0.8 kg block A can be adjusted. With the fan turned off, the unit is balanced about the x-axis when  $b=180$  mm. The motor operates at 1725 rpm in the direction shown. Determine the value of  $b$  which will produce a steady precession of 0.2 rad/s about the positive y-axis.



Source: Dynamics, Meriam and Kraige



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Approximate analysis ( $p \gg \dot{\psi}$ )

$$\vec{H}_0 = I p \hat{k}$$

$$\vec{M}_0 = \vec{\Omega}_f \times \vec{H}_0 = \dot{\psi} \hat{j} \times I p \hat{k} \quad \dot{\vec{R}}_0 = \vec{M}_0$$

$$\Rightarrow \vec{M}_0 = I p \dot{\psi} \hat{i} = \Delta b (m_A g) \hat{i}$$

$$\Rightarrow \Delta b = \frac{2 \cdot 2 (0.06)^2}{0.8 (9.81)} 1725 \left( \frac{2\pi}{60} \right) (0.2) \text{ m}$$

$$= 0.0364 \text{ m}$$

Hence,  $b' = b + \Delta b = \underline{216.4 \text{ mm}}$

$I = 2 \cdot 2 (0.06)^2$   
 $p = 1725 \left( \frac{2\pi}{60} \right) / \text{s}$   
 $\dot{\psi} = 0$

The detailed solution is presented in the slide above.

Next, we consider the following problem.

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**Problem 2:**

The housing of an electric motor is freely pivoted about the horizontal x-axis, which passes through the center of mass of the rotor. If the rotor is turning at a constant rate  $p$ , determine the angular acceleration  $d^2\psi/dt^2$  which will result from the application of a moment  $M$  about the vertical shaft if  $d\psi/dt = d\psi/dt = 0$  at that instant. The mass of the frame and housing is considered negligible compared to the mass  $m$  of the rotor. Take the radius of gyration of the rotor about the z-axis to be  $k_z$  and that about the x and y axes to be  $k$ .

Source: Dynamics and Kraige

The detailed solution is presented in the following 2 slides.

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Using  $x-y-z$  frame

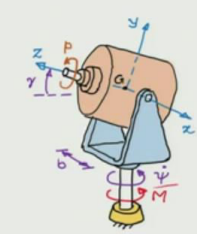
$$\vec{p} = p \hat{k} \quad \vec{\omega}_p = \dot{\psi} (\cos \gamma \hat{j} + \sin \gamma \hat{k})$$

$$\vec{\omega}_n = \dot{\gamma} \hat{i} \quad \vec{\omega}_f = \vec{\omega}_p + \vec{\omega}_n$$

$$\vec{\omega} = \vec{\omega}_f + \vec{p} = \dot{\gamma} \hat{i} + \dot{\psi} \cos \gamma \hat{j} + (\dot{\psi} \sin \gamma + p) \hat{k}$$

$$[I_a] = \begin{bmatrix} mk^2 & & \\ & mk^2 & \\ & & mk_z^2 \end{bmatrix}$$

$$\vec{H}_a = mk^2 \dot{\gamma} \hat{i} + mk^2 \dot{\psi} \cos \gamma \hat{j} + mk_z (\dot{\psi} \sin \gamma + p) \hat{k}$$

$$\dot{\vec{H}}_a|_{\dot{\gamma}=\dot{\psi}=0} = mk^2 \ddot{\psi} \cos \gamma \hat{j} + mk_z \ddot{\psi} \sin \gamma \hat{k} + \vec{r}_G \times \vec{H}_a \quad (\text{at } t)$$


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$$\dot{\vec{H}}_a|_{\dot{\gamma}=\dot{\psi}=0} = mk^2 \ddot{\psi} \cos \gamma \hat{j} + mk_z \ddot{\psi} \sin \gamma \hat{k}$$

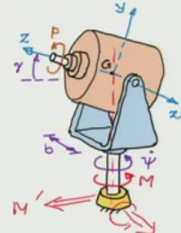
$$\dot{\vec{H}}_a = \underline{M} (\cos \gamma \hat{j} + \sin \gamma \hat{k}) + \underline{M_x} \hat{i} + M' (-\sin \gamma \hat{j} + \cos \gamma \hat{k})$$

(Foundation forces are zero)

$$\Rightarrow 0 = \underline{M_x}$$

$$mk^2 \ddot{\psi} \cos \gamma = M \cos \gamma - M' \sin \gamma$$


$$mk_z^2 \ddot{\psi} \sin \gamma = M \sin \gamma + M' \cos \gamma$$

$$\Rightarrow \ddot{\psi} = \frac{M}{m(k^2 \cos^2 \gamma + k_z^2 \sin^2 \gamma)}$$



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### Dynamics of a boomerang

- Aerodynamically designed tool that returns to the thrower
- Used by Aboriginal Australians for hunting: returns to thrower if target is missed



Source: Deutscher Boomerang Club e.V.

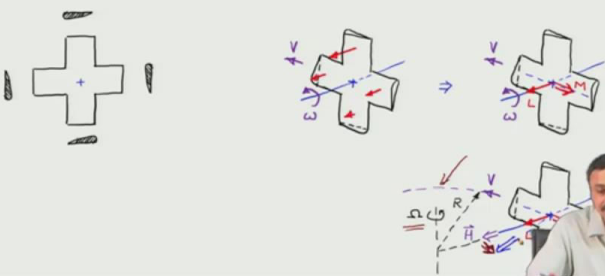


Next, we are going to look at the dynamics of a boomerang. These are aerodynamically designed tools that return to the thrower. They were typically used by the Australians aborigines during hunting. If the boomerang is thrown properly, and if it misses the target, then it returns back to the thrower. We can have different shapes of Boomerang as shown here. L shaped 4 armed, 3 armed. I have a boomerang which is a 3 armed one. We are going to discuss the dynamic analysis of boomerangs.


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### Dynamics of a boomerang

- Structure of boomerang



The diagram illustrates the structure of a boomerang, showing its cross-section and the forces acting on it during flight. It includes vectors for velocity ( $V$ ), angular velocity ( $\omega$ ), and forces ( $F$ ,  $M$ ,  $R$ ). The boomerang is shown in three different orientations, illustrating its curved path and the forces that cause it to return to the thrower.



I have considered for simplicity this 4 arm boomerang as shown in the slide above. There is this crossed structure with 4 arms having aerofoil shape and note carefully how the aerofoils are oriented.

If the boomerang is given only a spin about its axis as shown, then the 4 aerofoils will generate equal lift. But if I spin and throw it as shown, the upper arm will generate greater lift, while lower arm will generate the lower lift because of the combination of the spin and the throw velocity. We have the total lift which is the sum of these 4 lift forces, and there is a moment because of the moment arm. So, we have an overall moment which I am considering to be roughly vertical. This moment produces precession. As that happens, the boomerang is going to change its course and follow a curved path with a certain radius of curvature.

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**Dynamics of a boomerang**

$$L = \frac{\pi}{4} C_L \rho a^2 (\omega^2 a^2 + v^2)$$

$$M = \frac{\pi}{4} C_L \rho a^4 \omega v$$

$$m \vec{a}_g = \vec{F} \Rightarrow \frac{m v^2}{R} = L = \frac{\pi}{4} C_L \rho a^2 (\omega^2 a^2 + v^2) \quad (1)$$

$$I_g \vec{a} + \vec{\omega} \times I_g \vec{\omega} = \vec{M} \Rightarrow I_g \omega = \frac{\pi}{4} C_L \rho a^4 \omega v$$

$$\Rightarrow \omega = \frac{\pi}{4} C_L \rho a^4 v \quad (2)$$

$$\text{Also, } v = \omega R \Rightarrow \boxed{R = \frac{4 I_g}{\pi C_L \rho a^4}} \quad (3)$$

To analyze this dynamics further, let us make some simplifications. I am going to consider that this effect of lift comes from a disk because ultimately we have analyzed only axisymmetric bodies. I considered that lift is being generated over a disk shaped object. This is not a very bad approximation considering that the boomerang is spinning at a very high rate and the blades are passing by every point at a very high rate. I consider a small element as I have shown and I will calculate the lift because of that. This has been shown in the slide above. Finally, we have calculated the lift  $L$  and moment  $M$ .

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### Dynamics of a boomerang

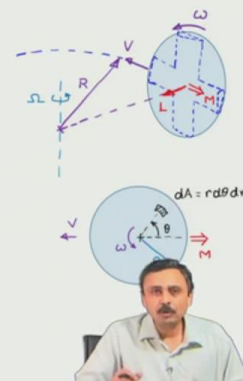
$$\frac{mV^2}{R} = \frac{\pi}{4} C_L \rho a^2 (\alpha^2 \omega^2 + V^2) \quad - (1) \quad \left\{ \begin{matrix} \omega \\ p \\ v \end{matrix} \right\}$$

$$\Omega = \frac{\pi}{4 I_G} C_L \rho a^2 V \quad - (2)$$

$$R = \frac{4 I_G}{\pi C_L \rho a^2} \quad - (3)$$

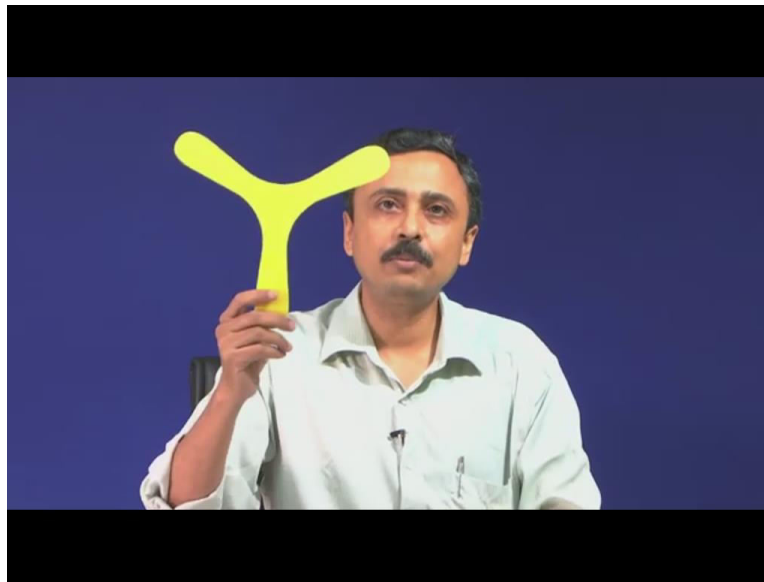
From (1) and (3)

$$mV^2 \frac{\pi C_L \rho a^2}{4 I_G} = \frac{\pi}{4} C_L \rho a^2 (\alpha^2 \omega^2 + V^2)$$

$$\Rightarrow \underline{V} \approx \underline{\alpha \omega} \sqrt{\frac{I_G}{m\alpha^2 - I_G}} \quad (\text{Relation between } V \text{ and } \omega)$$


Now, from Newton's second law for the center of mass, mass times acceleration of center of mass must be a net force acting on the boomerang. Because I am assuming that it will be moving on a circular path, the only acceleration term is because of the centripetal acceleration of the center of mass of the boomerang. This has been shown in the slide above. We can then relate the throw velocity and the spin angular speed required to follow the circular path. This is where the art of throwing boomerangs come in. When you throw a boomerang, you have to give an appropriate spin and throw velocity, which are related through the dynamic parameters of the boomerang.

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The lecture ends with a demonstration of the motion of a toy boomerang in the video.