

Advanced Dynamics
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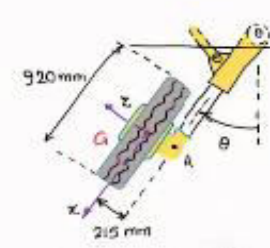
Lecture - 36
Spatial Kinetics of Rigid Bodies - II

In this lecture we are going to continue our discussions on spatial kinetics of rigid bodies. We will discuss some problems based on the equations of motion that we have developed in the previous lecture.

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Problem 1:

An aircraft landing gear viewed from front is being retracted immediately after take-off with θ increasing at a rate of 30 deg per second, and the wheel is spinning at the rate corresponding to the take-off speed of 200 km/h. The 45 kg wheel has a radius of gyration about its z axis of 370 mm. Neglect the thickness of the wheel and calculate the angular momentum of the wheel about G and about A.



Source: Dynamics, Meram and Shigle

Consider the above problem. The detailed solution parts are given in the 3 slides below.

(Refer Slide Time: 01:35)

Coordinate frame $x-y-z$

$$\vec{\Omega} = -\dot{\theta} \hat{j} \quad \dot{\theta} = \frac{\pi}{6} \text{ rad/s}$$

$$\vec{p} = -p \hat{k} \quad p = \frac{500}{9(0.46)} = 120.77 \text{ rad/s}$$

Angular velocity of wheel

$$\vec{\omega} = \vec{\Omega} + \vec{p} = -\frac{\pi}{6} \hat{j} - 120.77 \hat{k} \text{ rad/s} = \begin{Bmatrix} 0 \\ -\pi/6 \\ -120.77 \end{Bmatrix} \text{ rad/s}$$

(a) Angular momentum about G

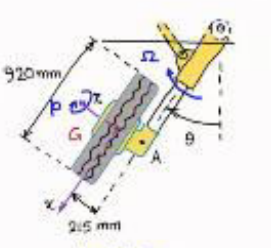
$$[I_G] = \begin{bmatrix} \frac{mk^2}{2} & 0 & 0 \\ 0 & \frac{mk^2}{2} & 0 \\ 0 & 0 & mk^2 \end{bmatrix}$$

$$I_{zz} = \frac{1}{2} mr^2 = mk^2$$

perpendicular axis theorem

$$I_{xx} + I_{yy} = I_{zz} \quad I_{xx} = I_{yy}$$

$$\Rightarrow I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{1}{2} mk^2$$



$k = 0.37 \text{ m}$
 $m = 45 \text{ kg}$

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Coordinate frame $x-y-z$

$$\vec{\Omega} = -\dot{\theta} \hat{j} \quad \dot{\theta} = \frac{\pi}{6} \text{ rad/s}$$

$$\vec{p} = -p \hat{k} \quad p = \frac{500}{9(0.46)} = 120.77 \text{ rad/s}$$

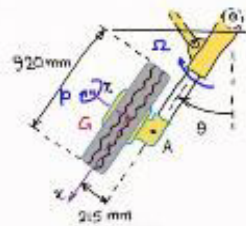
Angular velocity of wheel

$$\vec{\omega} = \vec{\Omega} + \vec{p} = -\frac{\pi}{6} \hat{j} - 120.77 \hat{k} \text{ rad/s} = \begin{Bmatrix} 0 \\ -\pi/6 \\ -120.77 \end{Bmatrix} \text{ rad/s}$$

(a) Angular momentum about G

$$[I_G] = \begin{bmatrix} \frac{mk^2}{2} & & \\ & \frac{mk^2}{2} & \\ & & mk^2 \end{bmatrix}$$

$$\Rightarrow \vec{H}_G = \frac{(45)(0.37)^2}{2} \left(-\frac{\pi}{6}\right) \hat{j} + (45)(0.37)^2 (-120.77) \hat{k} \text{ kg m}^2/\text{s}$$

$$= \underline{-1.613 \hat{j} - 744 \hat{k} \text{ kg m}^2/\text{s}}$$


$k = 0.37 \text{ m}$
 $m = 45 \text{ kg}$

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(b) Angular momentum about A

$$\vec{H}_A = \vec{H}_G + \vec{AG} \times m \vec{v}_G \quad \vec{AG} = 0.215 \hat{k}$$

$$\vec{v}_G = \vec{v}_O + \vec{\Omega} \times \vec{OG}$$

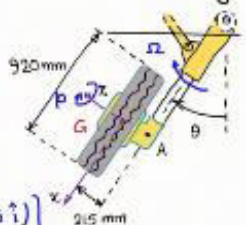
$$= \frac{500}{9} \hat{j} - \frac{\pi}{6} \hat{j} \times (0.4 \hat{i} + 0.4 \hat{k})$$

$$= \frac{500}{9} \hat{j} - \frac{\pi}{6} (-0.4 \hat{k} + 0.4 \hat{i})$$

$$\Rightarrow \vec{H}_A = -1.613 \hat{j} - 744 \hat{k} + 0.215 \hat{k} \times \left[\frac{500}{9} \hat{j} - \frac{\pi}{6} (-0.4 \hat{k} + 0.4 \hat{i}) \right]$$

$$= \underline{-1.94 \hat{i} - 2.7 \hat{j} - 744 \hat{k} \text{ kg m}^2/\text{s}}$$

$$\vec{H}_A^{\text{rel}} = \vec{H}_G + \vec{AG} \times m \vec{v}_{G/A} = \vec{H}_G + \vec{AG} \times (m \vec{\Omega} \times \vec{AG})$$

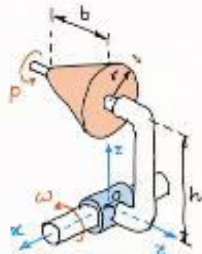
$$= \underline{-2.7 \hat{j} - 744 \hat{k} \text{ kg m}^2/\text{s}}$$


$k = 0.37 \text{ m}$
 $m = 45 \text{ kg}$

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Problem 2:

A solid right-circular cone of mass m , length b , and base radius r spins at an angular rate p about its axis of symmetry. Simultaneously, the bracket and attached shaft axis revolve at a rate ω about the x -axis. Determine the angular momentum \mathbf{H}_O of the cone about the point O , and its kinetic energy T .



Source: Dynamics, Merian and Kraige

The next problem is shown above. The solution steps are shown in detail in the following 2 slides.

(Refer Slide Time: 13:51)

Coordinate frame $x-y-z$

$$\vec{H}_O = \vec{H}_G + \vec{r} \times m\vec{v}_G \quad \vec{H}_G = I_G \vec{\omega} \quad \vec{\omega} = \omega \hat{i} + p \hat{j}$$

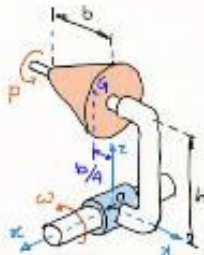
$$I_G = \begin{bmatrix} \frac{3}{20}mr^2 + \frac{3}{80}mb^2 & 0 & 0 \\ 0 & \frac{3}{10}mr^2 & 0 \\ 0 & 0 & \frac{3}{20}mr^2 + \frac{3}{80}mb^2 \end{bmatrix}$$

$$\vec{H}_G = \left(\frac{3}{20}mr^2 + \frac{3}{80}mb^2 \right) \omega \hat{i} + \frac{3}{10}mr^2 p \hat{j}$$

$$\vec{v}_G = \omega \hat{i} \times \left(h \hat{k} - \frac{b}{4} \hat{j} \right) = -\omega h \hat{j} - \frac{\omega b}{4} \hat{k} \quad \vec{r} = h \hat{k} - \frac{b}{4} \hat{j}$$

$$\vec{r} \times m\vec{v}_G = m\omega \left(h^2 + \frac{b^2}{16} \right) \hat{i}$$

$$\vec{H}_O = m\omega \left[\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2 \right] \hat{i} + \frac{3}{10}mr^2 p \hat{j}$$

$$T = \frac{1}{2} \vec{\omega} \cdot \vec{H}_O = \frac{1}{2} m\omega^2 \left[\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2 \right] + \frac{3}{20}mr^2 p^2$$


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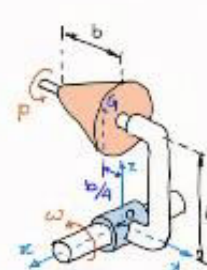

Coordinate frame $x-y-z$ (alternate calculation)

$$T = \frac{1}{2} m \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} \vec{\omega} \cdot \vec{H}_G \quad \vec{H}_G = I_G \vec{\omega} \quad \vec{\omega} = \omega \hat{i} + p \hat{j}$$

$$\vec{H}_G = \left(\frac{3}{20} m r^2 + \frac{3}{80} m b^2 \right) \omega \hat{i} + \frac{3}{10} m r^2 p \hat{j}$$

$$\vec{v}_G = \omega \hat{i} \times (h \hat{k} - \frac{b}{4} \hat{j}) = -\omega h \hat{j} - \frac{\omega b}{4} \hat{k}$$

$$\vec{v}_G \cdot \vec{v}_G = \omega^2 \left(h^2 + \frac{b^2}{16} \right)$$

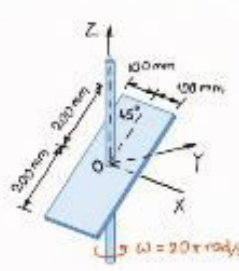
$$\Rightarrow T = \frac{1}{2} m \omega^2 \left[\frac{3}{20} r^2 + \frac{1}{10} b^2 + h^2 \right] + \frac{3}{20} m r^2 p^2$$



The next problem is shown in the slide below.


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Problem 3:

A rectangular uniform thin plate, with mass of 3 kg, is welded at 45 deg angle to a vertical shaft, which rotates with an angular velocity of 20π rad/s. Determine (a) the angular momentum \mathbf{H} of the plate about O, (b) the kinetic energy of the plate, and (c) the moment acting on the shaft due to the plate.



Source: Dynamics, Meriam and Kraige



The solution details are presented in the following 2 slides. In this solution, we use the principal frame x-y-z of the plate.

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Coordinate frame x-y-z (principal frame)

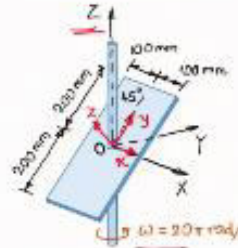

$$I_O = \begin{bmatrix} \frac{ml^2}{12} & 0 & 0 \\ 0 & \frac{mb^2}{12} & 0 \\ 0 & 0 & \frac{m}{12}(l^2 + b^2) \end{bmatrix} \quad \vec{\omega} = \frac{\omega}{\sqrt{2}}(\hat{j} + \hat{k})$$

$$\vec{H}_O = \frac{mb^2}{12} \frac{\omega}{\sqrt{2}} \hat{j} + \frac{m}{12}(l^2 + b^2) \frac{\omega}{\sqrt{2}} \hat{k}$$

In x-y-z frame $\hat{i} = \hat{I} \quad \hat{j} = \frac{1}{\sqrt{2}}(\hat{J} + \hat{K}) \quad \hat{k} = \frac{1}{\sqrt{2}}(-\hat{J} + \hat{K})$

$$\Rightarrow \vec{H}_O = \frac{3(0.2)^2}{12} \frac{20\pi}{2} (\hat{J} + \hat{K}) + \frac{3}{12}(0.4^2 + 0.2^2) \frac{20\pi}{2} (-\hat{J} + \hat{K})$$

$$= -0.4\pi \hat{J} + 0.6\pi \hat{K} \text{ kg m}^2/\text{s}$$

$$T = \frac{1}{2} \vec{\omega} \cdot \vec{H}_O = \frac{1}{2} (20\pi \hat{K}) \cdot (-0.4\pi \hat{J} + 0.6\pi \hat{K}) = 59.2 \text{ J}$$



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Calculation of moment acting on the shaft

In x-y-z frame

$$\vec{H}_O = \frac{3(0.2)^2}{12} \frac{20\pi}{2} (\hat{J} + \hat{K}) + \frac{3}{12}(0.4^2 + 0.2^2) \frac{20\pi}{2} (-\hat{J} + \hat{K})$$

$$= -0.4\pi \hat{J} + 0.6\pi \hat{K} \text{ kg m}^2/\text{s}$$

$$\vec{\omega} = 20\pi \hat{K} \text{ rad/s}$$

Moment on the plate

$$\vec{M}_O = \vec{H}_O = \frac{d\vec{H}_O}{dt} + \vec{\omega} \times \vec{H}_O$$

$$\Rightarrow \vec{M}_O = 20\pi \hat{K} \times (-0.4\pi \hat{J} + 0.6\pi \hat{K}) \text{ Nm}$$

$$= 8\pi^2 \hat{I} \text{ Nm} = 78.96 \hat{I} \text{ Nm}$$
