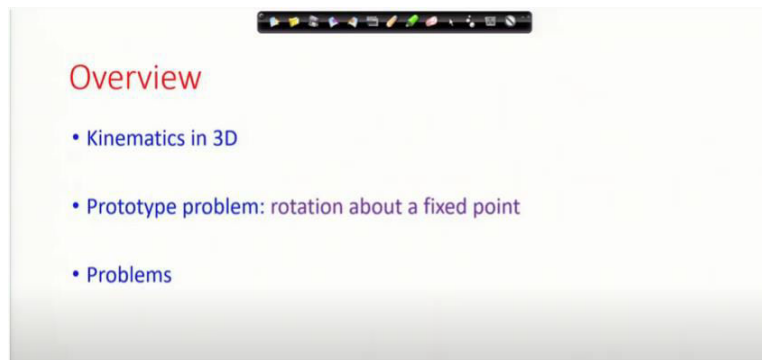


**Advanced Dynamics**  
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**Module No # 07**  
**Lecture No # 34**  
**Spatial Kinematics of Rigid Bodies – II**

We will continue our discussions on spatial kinematics of rigid bodies.

(Refer Slide Time: 00:19)



We will first recapitulate what we have discussed in the last lecture about kinematics in 3 dimensions and the prototype problem which represents rotation of an axisymmetric body about a fixed point. Then we are going to look at some problems.

(Refer Slide Time: 00:35)

**Spatial kinematics**

Prototype system

Angular velocity of x-y-z frame:  $\vec{\Omega}_f = \vec{\omega}_p + \vec{\omega}_n$

Total angular velocity of the body:  $\vec{\omega} = \vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s$

Angular acceleration:  $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{\Omega}_f \times$

- Body:  $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{\partial \vec{\omega}_p}{\partial t} + \frac{\partial \vec{\omega}_n}{\partial t} + \frac{\partial \vec{\omega}_s}{\partial t} + (\vec{\omega}_p + \vec{\omega}_n) \times \vec{\omega}_s$
- Frame:  $\vec{\alpha}_f = \frac{d\vec{\Omega}_f}{dt} = \frac{\partial \vec{\omega}_p}{\partial t} + \frac{\partial \vec{\omega}_n}{\partial t}$

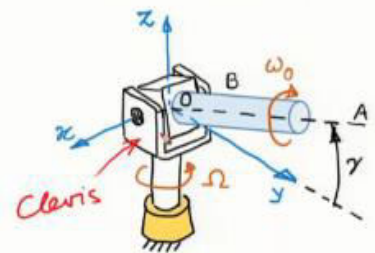
s: spin  
n: nutation  
p: precession

The above slide recapitulates the discussions of spatial kinematics of rotation of an axisymmetric body about a fixed point.

(Refer Slide Time: 03:40)

## Problem 1:

A vertical shaft and attached clevis rotate about the z axis at a constant rate  $\Omega = 4 \text{ rad/s}$ . Simultaneously, the shaft B revolves about its axis OA at a constant rate  $\omega_0 = 3 \text{ rad/s}$ , and the angle  $\gamma$  is decreasing at a constant rate of  $\pi/4 \text{ rad/s}$ . Determine the angular velocity  $\omega$  and magnitude of the angular acceleration  $\alpha$  of shaft B when  $\gamma = 30^\circ$ . The x-y-z axes are attached to the clevis and rotate with it.



Source: Dynamics, Meriam and Kraige

Consider the above problem. The detailed solution is presented in the following slide.

(Refer Slide Time: 5:26)

Coordinate frame x-y-z

$$\vec{\Omega} = \vec{\Omega}_f = 4 \hat{k} \text{ rad/s} \quad \vec{\omega}_\gamma = \dot{\gamma} \hat{i} = -\frac{\pi}{4} \hat{i} \text{ rad/s}$$

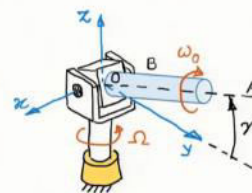
$$\vec{\omega}_0 = -3(\cos\gamma \hat{j} + \sin\gamma \hat{k}) \text{ rad/s}$$

$$\begin{aligned} \vec{\omega}_B &= \vec{\Omega}_f + \vec{\omega}_\gamma + \vec{\omega}_0 = 4\hat{k} - \frac{\pi}{4}\hat{i} - 3\left(\frac{\sqrt{3}}{2}\hat{j} + \frac{1}{2}\hat{k}\right) \text{ rad/s} \\ &= \underline{-0.785 \hat{i} - 2.6 \hat{j} + 2.5 \hat{k} \text{ rad/s}} \end{aligned}$$

$$\vec{\alpha}_B = \frac{\partial \vec{\omega}_B}{\partial t} + \vec{\Omega}_f \times \vec{\omega}_0 \quad \left(\frac{\partial \omega_B}{\partial t} = \frac{\partial \omega_0}{\partial t}\right)$$

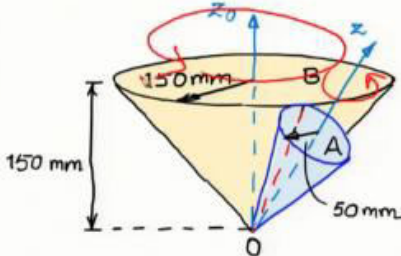
$$\begin{aligned} &= -3(-\dot{\gamma} \sin 30^\circ \hat{j} + \dot{\gamma} \cos 30^\circ \hat{k}) + 4\hat{k} \times (-0.785 \hat{i} - 2.6 \hat{j} + 2.5 \hat{k}) \\ &= \underline{10.4 \hat{i} - 4.32 \hat{j} + 2.04 \hat{k} \text{ rad/s}^2} \end{aligned}$$

$$|\vec{\alpha}_B| = 11.44 \text{ rad/s}^2$$



**Problem 2:**

A right-circular cone A rolls on the fixed right-circular cone B at a constant rate and makes one complete trip around B every 4 seconds. Compute the magnitude of the angular acceleration  $\alpha$  of the cone A during its motion.



Source: Dynamics, Meriam and Kraige

**(Refer Slide Time: 15:36)**

Coordinate frame x-y-z

$$\vec{\Omega} = \Omega (-\sin 58.63^\circ \hat{j} + \cos 58.63^\circ \hat{k})$$

$$\vec{p} = p \hat{k}$$

Net angular velocity

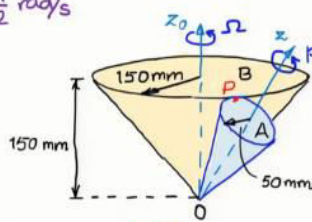

$$\vec{\omega} = -\frac{\pi}{2} \sin 58.63^\circ \hat{j} + (p + \frac{\pi}{2} \cos 58.63^\circ) \hat{k}$$


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$$\vec{v}_P = \vec{v}_O + \vec{\omega} \times \vec{OP} = 0 \quad (\text{no slip condition}) \Rightarrow \vec{\omega} = \omega \hat{OP}$$

$$\vec{OP} = OP (-\sin 13.63^\circ \hat{j} + \cos 13.63^\circ \hat{k})$$

$$\Rightarrow -\frac{\pi}{2} \sin 58.63^\circ \cos 13.63^\circ + (p + \frac{\pi}{2} \cos 58.63^\circ) \sin 13.63^\circ = 0$$

$$\Rightarrow p = 4.724 \text{ rad/s}$$



$\sin \phi = \frac{1}{2}$

(Refer Slide Time: 24:07)

Coordinate frame  $x-y-z$

$$\vec{\Omega} = \Omega (-\sin 58.63^\circ \hat{j} + \cos 58.63^\circ \hat{k}) \quad \Omega = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad/s}$$

$$\vec{p} = p \hat{k}$$

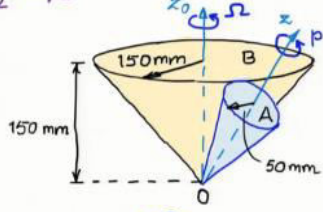
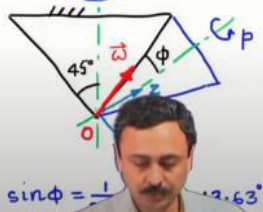
Net angular velocity

$$\vec{\omega} = -\frac{\pi}{2} \sin 58.63^\circ \hat{j} + (p + \frac{\pi}{2} \cos 58.63^\circ) \hat{k}$$

$$p = 4.724 \text{ rad/s}$$

Angular acceleration

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{\partial \vec{\omega}}{\partial t} + \vec{\Omega} \times \vec{\omega} \quad \vec{\Omega}_f = \vec{\Omega}$$

$$\Rightarrow \vec{\alpha} = \vec{\Omega}_f \times \vec{p} = -p \Omega \sin 58.63^\circ \hat{i} = -6.33 \hat{i} \text{ rad/s}^2$$



$\sin \phi = \frac{1}{2}$

(Refer Slide Time: 25:34)

Overview

- Kinematics in 3D
- Prototype problem: rotation about a fixed point
- Problems

The summary is shown above.