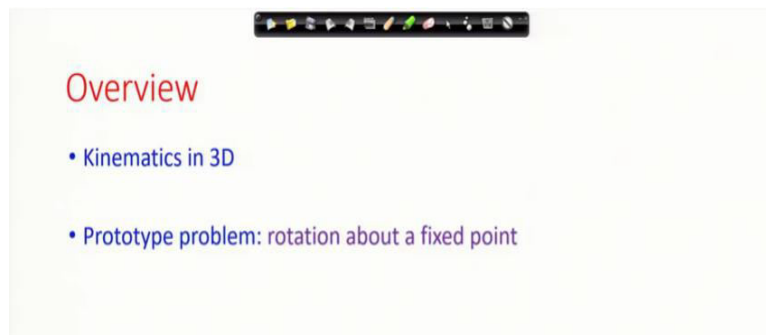


Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Module No # 07
Lecture No # 33
Spatial Kinematics of Rigid Bodies – I

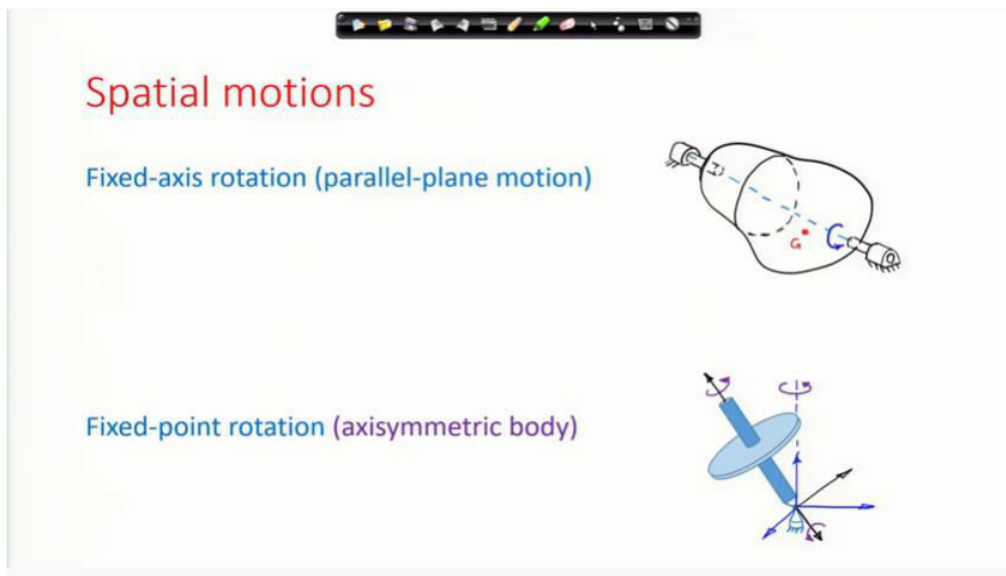
In this lecture we are going to look at spatial kinematics of rigid bodies.

(Refer Slide Time: 00:18)



We are going to look at kinematics in 3 dimensions we are going to look at a prototype problem which will essentially setup the background for kinematics of rotation about a fixed point.

(Refer Slide Time: 00:40)



Spatial motions that we will consider will essentially be of 2 types, as shown in the slide above. When there exists a fixed rotation axis, it is called a fixed axis rotation or also known as parallel

plane motion. The other is fixed point rotation of an axisymmetric body about a fixed point. Later, we will consider completely free motion of a rigid body.

(Refer Slide Time: 01:06)

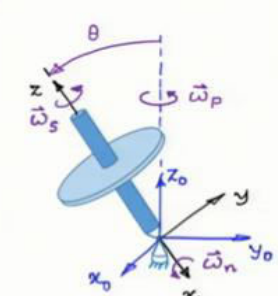
Spatial kinematics

Prototype system

- $x_0-y_0-z_0$: Inertial (non-rotating) frame (origin O)
- z : spin axis of body ($\vec{\omega}_s$)
- $\hat{x} = \hat{z}_0 \times \hat{z}$ (always in x_0-y_0 plane)
- $x-y-z$: Rotating frame

Angular velocity of $x-y-z$ frame: $\vec{\Omega}_f = \vec{\omega}_p + \vec{\omega}_n$

Total angular velocity of the body: $\vec{\omega} = \underbrace{\vec{\omega}_p + \vec{\omega}_n}_{\text{frame rotation}} + \underbrace{\vec{\omega}_s}_{\text{spin}}$



s: spin
n: nutation
p: precession

We are going to start with this prototype problem of rotation of a symmetric body about a fixed point. The parallel plane motion will be discussed a little later. In the above slide, I have shown in the figure an axisymmetric body rotating with a point fixed at the origin of the inertial $x_0-y_0-z_0$ frame shown. The setting up of the $x-y-z$ frame, and the angular velocity of the body is expressed in terms of the spin, precession and nutation angular velocities.

(Refer Slide Time: 06:48)

Spatial kinematics

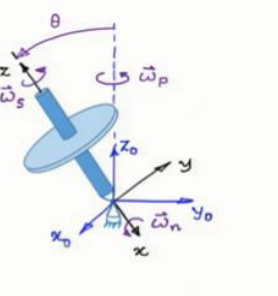
Prototype system

Angular velocity of $x-y-z$ frame: $\vec{\Omega}_f = \vec{\omega}_p + \vec{\omega}_n$

Total angular velocity of the body: $\vec{\omega} = \vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s$

Angular acceleration: $\frac{d\vec{\omega}}{dt} = \frac{\partial}{\partial t} + \vec{\Omega}_f \times$

• Body: $\vec{\alpha} = \underbrace{\frac{d\vec{\omega}}{dt}}_{\text{local angular acceleration}} = \underbrace{\frac{\partial \vec{\omega}_p}{\partial t} + \frac{\partial \vec{\omega}_n}{\partial t} + \frac{\partial \vec{\omega}_s}{\partial t}}_{\text{angular acceleration due to frame rotation}} + \underbrace{\vec{\Omega}_f \times (\vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s)}_{\text{angular acceleration due to frame rotation}} = (\vec{\omega}_p + \vec{\omega}_n) \times \vec{\omega}_s$



s: spin
n: nutation
p: precession

The above slide presents the calculation of the angular acceleration of the axisymmetric body.

(Refer Slide Time: 09:26)

Spatial kinematics

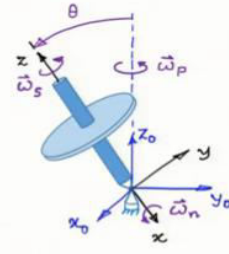
Prototype system

Angular velocity of x-y-z frame: $\vec{\Omega}_f = \vec{\omega}_p + \vec{\omega}_n$

Total angular velocity of the body: $\vec{\omega} = \vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s$

Angular acceleration: $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{\Omega}_f \times$

- Body: $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{\partial \vec{\omega}_p}{\partial t} + \frac{\partial \vec{\omega}_n}{\partial t} + \frac{\partial \vec{\omega}_s}{\partial t} + (\vec{\omega}_p + \vec{\omega}_n) \times \vec{\omega}_s$
- Frame: $\vec{\alpha}_f = \frac{d\vec{\Omega}_f}{dt} = \frac{\partial \vec{\omega}_p}{\partial t} + \frac{\partial \vec{\omega}_n}{\partial t}$



Next we look at the angular acceleration of frame, as shown in the above slide.

(Refer Slide Time: 10:26)

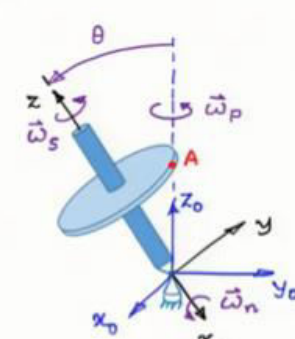
Spatial kinematics

$\vec{v}_A = \vec{v}_O + \vec{v}_{rel} + \vec{\Omega}_f \times \vec{OA} = \vec{\omega}_s \times \vec{OA} + (\vec{\omega}_p + \vec{\omega}_n) \times \vec{OA}$

$\Rightarrow \vec{v}_A = (\vec{\omega}_s + \vec{\omega}_p + \vec{\omega}_n) \times \vec{OA}$

$\vec{a}_A = \vec{a}_O + \vec{a}_{rel} + \vec{\alpha}_f \times \vec{OA} + \vec{\Omega}_f \times \vec{\Omega}_f \times \vec{OA} + 2\vec{\Omega}_f \times \vec{v}_{rel}$

If $\omega_s = \text{constant}$ $\vec{a}_{rel} = \vec{\omega}_s \times \vec{\omega}_s \times \vec{OA}$

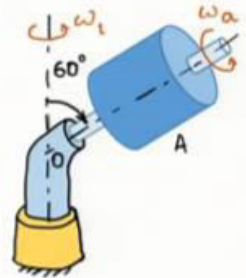


Consider now a point A as shown in red on the disk in the above figure. We can express the velocity and acceleration of A as presented above.

(Refer Slide Time: 13:26)

Problem 1:

A spool A rotates about its axis with an angular velocity of $\omega_a = 20 \text{ rad/s}$. Simultaneously, the assembly rotates about the vertical axis with an angular velocity $\omega_1 = 10 \text{ rad/s}$. Determine the magnitude ω of the total angular velocity, and α the angular acceleration of the spool.



Source: Dynamics, Meri and Kraige

We consider the above problem.

(Refer Slide Time: 14:19)

Coordinate system

$$\vec{\omega}_a = 20 \hat{k} \text{ rad/s}$$

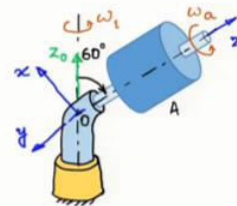
$$\vec{\omega}_1 = 10(\sin 60^\circ \hat{i} + \cos 60^\circ \hat{k}) \text{ rad/s} = \vec{\omega}_f$$

$$\begin{aligned} \vec{\omega}_{\text{spool}} &= \vec{\omega}_1 + \vec{\omega}_a \\ &= 5\sqrt{3} \hat{i} + 25 \hat{k} \text{ rad/s} \end{aligned}$$

$$\Rightarrow \underline{\omega_{\text{spool}} = 26.5 \text{ rad/s}}$$

$$\begin{aligned} \vec{\alpha}_{\text{spool}} &= \frac{\partial \vec{\omega}_{\text{spool}}}{\partial t} + \vec{\omega}_f \times \vec{\omega}_{\text{spool}} \\ &= \vec{\omega}_f \times (\vec{\omega}_1 + \vec{\omega}_a) = \vec{\omega}_f \times \vec{\omega}_a \end{aligned}$$

$$\Rightarrow \underline{\alpha_{\text{spool}} = -100\sqrt{3} \hat{j} \text{ rad/s}^2}$$

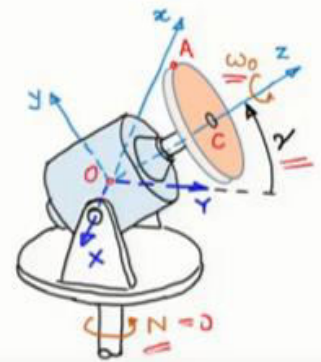


The detailed solution is presented above.

(Refer Slide Time: 18:42)

Problem 2:

If the motor shown pivots about the x-axis at a constant rate $d\gamma/dt = 3\pi$ rad/s with $N = 0$, determine the angular acceleration α of the disc as the position $\gamma = 30^\circ$ is passed. The constant speed of the motor is 120 rev/min. Also find the velocity and acceleration of point A, which is on the top of the disc for this position. Take $OC = 25$ cm, and $CA = 12.5$ cm.



Source: Dynamics, Meriam and Kraige

The next problem is shown above.

(Refer Slide Time: 20:29)

Using $x-y-z$ frame

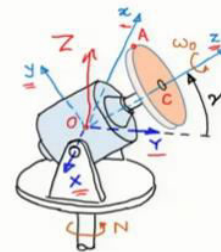
$$\vec{\omega}_0 = 4\pi \hat{k} \text{ rad/s} \quad \vec{\Omega}_f = -3\pi \hat{i} \text{ rad/s}$$

$$\begin{aligned} \text{Angular velocity of disc: } \vec{\omega} &= \vec{\Omega}_f + \vec{\omega}_0 \\ &= (-3\pi \hat{i} + 4\pi \hat{k}) \text{ rad/s} \end{aligned}$$

$$\begin{aligned} \text{Angular acceleration of disc: } \vec{\alpha} &= \frac{\partial \vec{\omega}}{\partial t} + \vec{\Omega}_f \times \vec{\omega} \\ &= \vec{\Omega}_f \times \vec{\omega}_0 \\ &= 12\pi^2 \hat{j} \text{ rad/s}^2 \end{aligned}$$

Transformation to $X-Y-Z$ frame:

$$\hat{i} = -\hat{I} \quad \hat{j} = -\sin\gamma \hat{J} + \cos\gamma \hat{K} \quad \hat{k} = \cos\gamma \hat{J} + \sin\gamma \hat{K}$$



(Refer Slide Time: 26:41)

Using x-y-z frame

$$\vec{\omega}_0 = 4\pi \hat{k} \text{ rad/s} \quad \vec{\Omega}_f = -3\pi \hat{i} \text{ rad/s}$$

Velocity of A: $\vec{v}_A = \vec{v}_O + \vec{v}_{rel} + \vec{\Omega}_f \times \vec{OA}$

$$= \vec{\omega}_0 \times \vec{OA} + \vec{\Omega}_f \times \vec{OA}$$

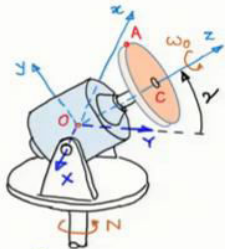

$$(\vec{OA} = 0.25 \hat{k} + 0.125 \hat{j} \text{ m})$$

$$\Rightarrow \vec{v}_A = -0.5\pi \hat{i} + 0.75\pi \hat{j} - 0.375\pi \hat{k} \text{ m/s}$$

Acceleration of A: $\vec{a}_A = \vec{a}_O + \vec{a}_{rel} + \vec{\alpha}_f \times \vec{OA} + \vec{\Omega}_f \times \vec{\Omega}_f \times \vec{OA} + 2\vec{\Omega}_f \times \vec{v}_{rel}$

$$\vec{\alpha}_f = \frac{\partial \vec{\Omega}_f}{\partial t} + \vec{\Omega}_f \times \vec{\Omega}_f = 0 \quad \vec{a}_{rel} = \vec{\omega}_0 \times \vec{\omega}_0 \times \vec{OA} = -2.4\pi^2 \hat{j} \text{ m/s}^2$$

$$\Rightarrow \vec{a}_A = -2.4\pi^2 \hat{j} + 9\pi^2(-0.25 \hat{k} - 0.125 \hat{j}) + 2(-3\pi \hat{i}) \times (-0.6\pi \hat{i})$$

$$= -3.525\pi^2 \hat{j} - 2.25\pi^2 \hat{k} \text{ m/s}^2$$



The detailed solution is presented in the 2 slides above.

(Refer Slide Time: 30:06)

Overview

- Kinematics in 3D
- Prototype problem: rotation about a fixed point

The discussions are summarized as shown above.