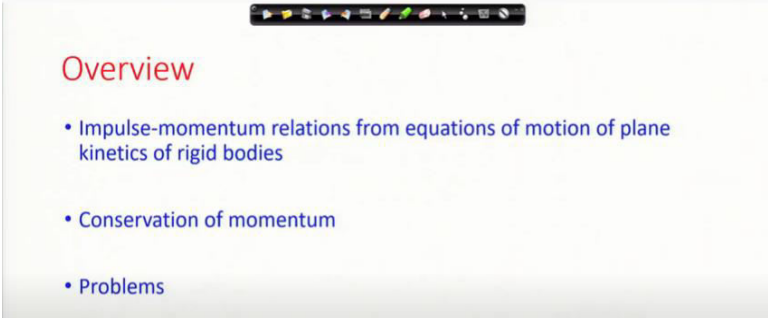


**Advanced Dynamics**  
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**Indian Institute of Technology - Kharagpur**

**Module No # 07**  
**Lecture No # 32**  
**Planar Kinetics: Impulse – Momentum Relations – II**

We will continue our discussions on impulse momentum relations for planar kinetics of rigid bodies.

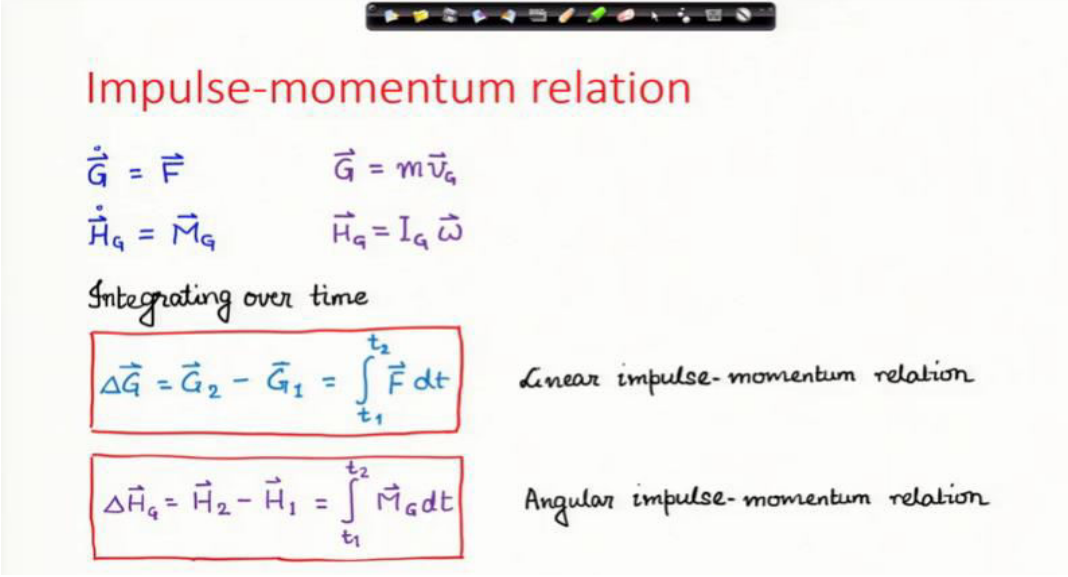
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**Overview**

- Impulse-momentum relations from equations of motion of plane kinetics of rigid bodies
- Conservation of momentum
- Problems

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**Impulse-momentum relation**

$$\dot{\vec{G}} = \vec{F} \quad \vec{G} = m \vec{v}_G$$
$$\dot{\vec{H}}_G = \vec{M}_G \quad \vec{H}_G = I_G \vec{\omega}$$

Integrating over time

$$\Delta \vec{G} = \vec{G}_2 - \vec{G}_1 = \int_{t_1}^{t_2} \vec{F} dt$$

Linear impulse-momentum relation

$$\Delta \vec{H}_G = \vec{H}_2 - \vec{H}_1 = \int_{t_1}^{t_2} \vec{M}_G dt$$

Angular impulse-momentum relation

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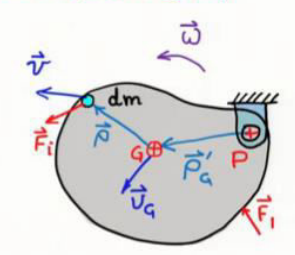
### Plane kinetics: Equations of motion (using P)

$$\vec{\dot{G}} = \vec{F}$$

$$\vec{\dot{H}}_P^{rel} + \vec{r}_{G'P} \times m\vec{\ddot{r}}_P = \vec{M}_P$$

$$\vec{G} = m\vec{v}_G$$


$$\vec{H}_P^{rel} = I_P \omega \hat{k}$$



Integrating over time

$$\Delta \vec{H}_P = \vec{H}_{P2} - \vec{H}_{P1} = \int_{t_1}^{t_2} \vec{M}_P dt$$

Angular impulse-momentum relation

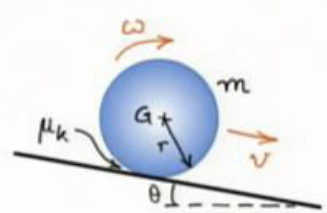


The above 2 slides recapitulate our discussions in the previous lecture.

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### Problem 1:

A homogeneous sphere of mass  $m$  and radius  $r$  is projected along an incline of angle  $\theta$  with an initial speed  $v_0$  and no angular velocity ( $\omega_0=0$ ). If the coefficient of kinetic friction is  $\mu_k$ , determine the time duration  $t$  of the period of slipping. Also, determine the velocity  $v$  of the center of mass  $G$  and the angular velocity  $\omega$  at the end of the slipping period.



Source: Dynamics, Meriam and Kraige

We consider the above problem.

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Angular impulse-momentum relation

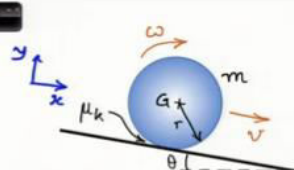
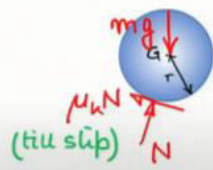
$$\Delta \vec{H}_G = \vec{H}_2 - \vec{H}_1 = \int_{t_1}^{t_2} \vec{M}_G dt$$

$$I_G \omega_2 - 0 = \int_0^t (-r \mu_k N) dt$$

$$\Rightarrow \frac{2}{5} m r^2 \omega_2 = -(r \mu_k m g \cos \theta) t$$

$$\Rightarrow \frac{2}{5} r \omega_2 = -\mu_k g \cos \theta t$$

When slip ceases  $v_2 = -r \omega_2$

$$\Rightarrow \frac{2}{5} v_2 = (\mu_k \cos \theta) g t$$



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Linear impulse-momentum relation

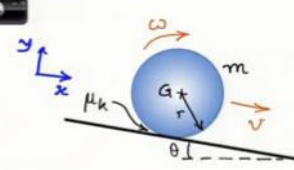
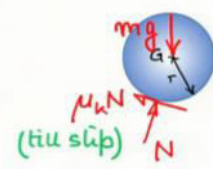
$$\Delta \vec{G} = \vec{G}_2 - \vec{G}_1 = \int_{t_1}^{t_2} \vec{F} dt$$

$$m v_2 - m v_0 = \int_0^t (-\mu_k N + m g \sin \theta) dt$$

$$\Rightarrow v_2 = v_0 + (-\mu_k \cos \theta + \sin \theta) g t$$

From previous calculation  $\frac{2}{5} v_2 = (\mu_k \cos \theta) g t$

$$\Rightarrow v_0 + (-\mu_k \cos \theta + \sin \theta) g t = \frac{5}{2} (\mu_k \cos \theta) g t$$

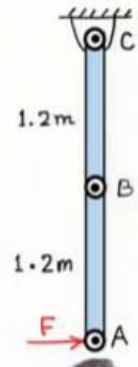
$$\Rightarrow t = \frac{2 v_0}{g (7 \mu_k \cos \theta - 2 \sin \theta)} \quad v_2 = \frac{5 v_0 \mu_k \cos \theta}{7 \mu_k \cos \theta - 2 \sin \theta} \quad \omega_2 = \frac{-5 v_0 \mu_k \cos \theta}{r (7 \mu_k \cos \theta - 2 \sin \theta)}$$



The solution is detailed in the 2 slides above.

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## Problem 2:

Two identical slender bars, each of mass 4 kg, are hinged at B and pivoted at C as shown. If a horizontal impulse of 14 Ns is applied to end A of the lower bar during an interval of 0.1 s (during which the bars are still essentially in their vertical positions), compute the angular velocity  $\omega_2$  of the upper bar immediately after the impulse.



We consider the above problem.

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Angular impulse-momentum relation about C

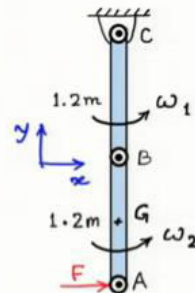
$$\Delta \vec{H}_C = \vec{H}_{C2} - \vec{H}_{C1} = \int \vec{M}_C dt$$

$$\Rightarrow I_C \omega_1 \hat{k} + (I_A \omega_2 \hat{k} + \vec{CG} \times m \vec{v}_G) = \vec{CA} \times \int \vec{F} dt$$

$$\begin{aligned} \vec{v}_G &= \vec{v}_B + \omega_2 \hat{k} \times \vec{BG} = \omega_1 \hat{k} \times (-1\hat{j}) + \omega_2 \hat{k} \times (-\frac{1}{2}\hat{j}) \\ &= (\omega_1 + \omega_2 \frac{1}{2}) \hat{i} \end{aligned}$$

$$\Rightarrow \frac{ml^2}{3} \omega_1 + \frac{ml^2}{12} \omega_2 + \frac{3}{2} ml^2 (\omega_1 + \frac{\omega_2}{2}) = 2l(14)$$

$$\Rightarrow 11\omega_1 + 5\omega_2 = 35$$



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Angular impulse-momentum relations

$$I_C \omega_1 \hat{k} = \int t \theta_x dt \hat{k} \quad - (1)$$

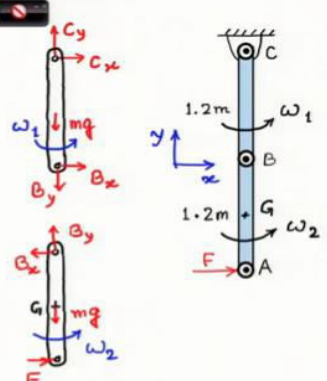
$$I_G \omega_2 \hat{k} = \int \frac{l}{2} \theta_x dt \hat{k} + \int \frac{l}{2} F dt \hat{k} \quad - (2)$$

Evaluating  $2 \times (2) - (1)$

$$2 \times \frac{ml^2}{12} \omega_2 - \frac{ml^2}{3} \omega_1 = 2 \times \frac{l}{2} (14)$$

$$\Rightarrow \omega_2 - 2\omega_1 = \frac{35}{2}$$

From previous calculation  $11\omega_1 + 5\omega_2 = 35$

$$\Rightarrow 21\omega_1 = -\frac{105}{2} \Rightarrow \underline{\omega_1 = -2.5 \text{ rad/s}}$$


The diagram shows a vertical rod of length 2.4m pivoted at the top (C). The center of mass G is at 1.2m from the pivot. A horizontal force F is applied at the bottom (A). The rod has angular velocities ω1 and ω2 at different points. The diagram also shows the rod's position at a later time with angular velocities ω1 and ω2.

The detailed solution is given in the the 2 slides above.

(Refer Slide Time: 19:12)

### Overview

- Impulse-momentum relations from equations of motion of plane kinetics of rigid bodies
- Conservation of momentum
- Problems

The summary is shown above.