

**Advanced Dynamics**  
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**Module No # 06**  
**Lecture No # 31**  
**Planar Kinetics: Impulse Momentum Relations – I**

In this lecture we are going to look at the impulse momentum relations for planar kinetics of rigid bodies.

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**Overview**

- Impulse-momentum relations from equations of motion of plane kinetics of rigid bodies
- Conservation of momentum

**Impulse-momentum relation**

$$\begin{aligned}\dot{\vec{G}} &= \vec{F} & \vec{G} &= m\vec{v}_G \\ \dot{\vec{H}}_G &= \vec{M}_G & \vec{H}_G &= I_G \vec{\omega}\end{aligned}$$

Integrating over time

$$\Delta \vec{G} = \vec{G}_2 - \vec{G}_1 = \int_{t_1}^{t_2} \vec{F} dt$$

Linear impulse-momentum relation

$$\Delta \vec{H}_G = \vec{H}_2 - \vec{H}_1 = \int_{t_1}^{t_2} \vec{M}_G dt$$

Angular impulse-momentum relation

The above slide recapitulates the equations of motion of planar kinetics of a rigid body with the angular momentum equations written about the center of mass G. The linear and angular impulse momentum relations are also shown.

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### Plane kinetics: Equations of motion (using P)

$$\dot{\vec{G}} = \vec{F}$$

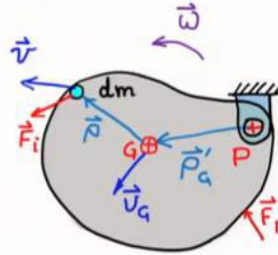
$$\vec{G} = m\vec{v}_G$$

$$\dot{\vec{H}}_P^{rel} + \vec{p}_G' \times m\ddot{\vec{r}}_P = \vec{M}_P$$

$$\dot{\vec{H}}_P^{rel} = \vec{p}_G' \times m\dot{\vec{p}}_G' + \int \vec{p} \times \dot{\vec{p}} dm$$

$$= \vec{p}_G' \times (\vec{\omega} \times \vec{p}_G') m + \int \vec{p} \times (\vec{\omega} \times \vec{p}) dm$$

$$= [m(\rho_{Gx}^2 + \rho_{Gy}^2) + I_G] \omega \hat{k} \quad (\vec{\omega} = \omega \hat{k} \quad \vec{p}_G' = \rho_{Gx} \hat{i} + \rho_{Gy} \hat{j})$$

$$\dot{\vec{H}}_P^{rel} = I_P \omega \hat{k}$$


The angular dynamics equation of motion about a point P which is fixed in inertial space is shown in the above slide. Since P is a fixed point, acceleration of P is 0, and we have a simplification. The expression of the relative angular momentum of the body is derived above.

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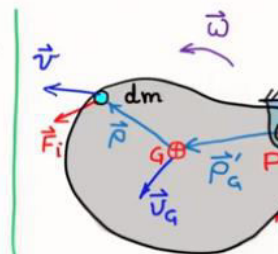

### Impulse-momentum relation

$$\dot{\vec{H}}_P^{rel} = \vec{M}_P \quad \vec{H}_P^{rel} = \vec{p}_G' \times m\vec{v}_G + I_G \vec{\omega} = I_P \omega \hat{k}$$

Integrating over time

$$\Delta \vec{H}_P = \vec{H}_{P2} - \vec{H}_{P1} = \int_{t_1}^{t_2} \vec{M}_P dt$$

Angular impulse-momentum relation

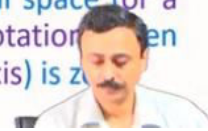



The angular impulse – momentum relation about P is presented in the above slide.

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## Conservation of momentum

- If the net force on a rigid body is zero, then the linear momentum of its center of mass is conserved
- If the net moment about the center of mass of a rigid body is zero, the angular momentum about the center of mass is conserved
- If the net moment is zero about a fixed point in inertial space (or a fixed axis about which the body undergoes fixed-axis rotation), then the angular momentum about the fixed point (or fixed axis) is conserved

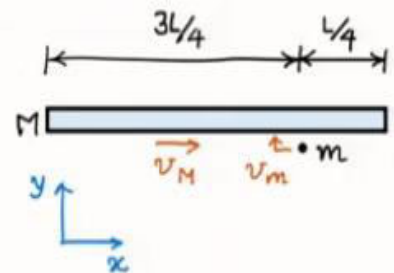


The conditions under which linear momentum and angular momentum of a body are conserved is presented above.

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## Problem 1:

A uniform slender bar of mass  $M$  and length  $L$  is translating on a smooth horizontal  $x$ - $y$  plane with velocity  $v_M$  when a particle of mass  $m$  traveling with velocity  $v_m$  perpendicular to the bar strikes the bar at the location shown and becomes embedded in it. Determine the final linear and angular velocities of the bar with its embedded particle.



Source: Dynamics, Meriam and Kraige

We consider the above problem.

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Conservation of linear momentum

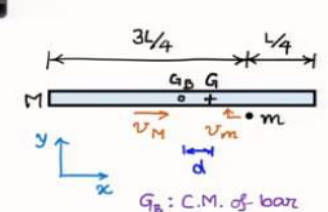
$$M v_M \hat{i} + m v_m \hat{j} = (M+m) \vec{v}_G$$

$$\Rightarrow \vec{v}_G = \frac{M v_M}{M+m} \hat{i} + \frac{m v_m}{M+m} \hat{j}$$

Conservation of angular momentum

$$\left(\frac{L}{4} - d\right) \hat{i} \times m v_m \hat{j} = I_G \omega \hat{k}$$

$$\Rightarrow \frac{M L^2}{12} \left[ \frac{7m+4M}{4(m+M)} \right] \omega = \frac{M L m v_m}{4(m+M)}$$

$$\Rightarrow \omega = \frac{12 m v_m}{L(7m+4M)}$$


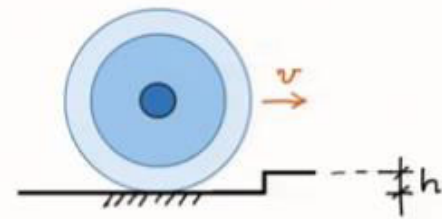
$G_B$ : C.M. of bar  
 $G$ : C.M. of combined body  
 $d = \frac{mL}{4(m+M)}$   
 Moment of inertia  
 $I_G = \frac{M L^2}{12} + M d^2 + m \left(\frac{L}{4} - d\right)^2$   
 $= \frac{M L^2}{12} \left[ \frac{7m+4M}{4(m+M)} \right]$

The solution steps are detailed in the above slide.

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## Problem 2:

Determine the minimum velocity  $v$  which the wheel must have to just roll over the obstruction. The centroidal radius of gyration of the wheel is  $k$ , and it is assumed that the wheel does not slip or leave contact.



Source: Dynamics, Meriam and Kraige

We consider the above problem.

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Angular momentum conservation about A after impact

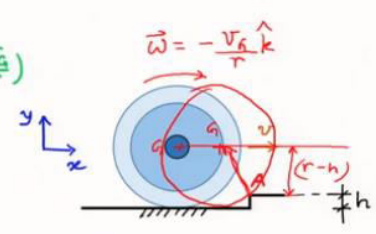
$$\Delta \vec{H}_A = 0 \Rightarrow \vec{H}_{A1} = \vec{H}_{A2}$$

$$\vec{H}_{A1} = I_G \omega \hat{k} + \vec{AG} \times m \vec{v}_G$$

$$= \left[ -\frac{mk^2}{r} \frac{v_G}{r} - m v_G (r-h) \right] \hat{k} \quad (\omega = -\frac{v_G}{r})$$

$$\vec{H}_{A2} = I_A \omega' \hat{k} = (mk^2 + mr^2) \omega' \hat{k}$$

$$\Rightarrow -\left[ k^2 + r(r-h) \right] \frac{v_G}{r} = (k^2 + r^2) \omega'$$

$$\Rightarrow \omega' = -\frac{k^2 + r(r-h)}{k^2 + r^2} \frac{v_G}{r}$$


Roll-over point A

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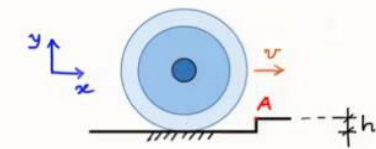
Energy conservation till highest configuration

$$T_1 + V_1 = T_2 + V_2 \quad \text{for minimum } v_G$$

$$\Rightarrow \frac{1}{2} I_A \omega'^2 = mgh$$

$$\Rightarrow \frac{1}{2} m(k^2 + r^2) \frac{[k^2 + r(r-h)]^2}{(k^2 + r^2)^2} \frac{v_G^2}{r^2} = mgh$$

$$\Rightarrow \frac{1}{2} \frac{[k^2 + r(r-h)]^2}{(k^2 + r^2) r^2} v_G^2 = gh$$

$$\Rightarrow v_G = \frac{\sqrt{2gh(k^2 + r^2)r^2}}{[k^2 + r(r-h)]}$$


Roll-over point A

The 2 slides above provides the detailed solution of the problem.

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## Summary

- Impulse-momentum relations from equations of motion of plane kinetics of rigid bodies
- Conservation of momentum

The summary of the discussions is provided above.