

Advanced Dynamics
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Module No # 06
Lecture No # 30
Planar Kinetics: Work – Energy Relations – II

We will continue with work energy relation for planar kinetics of rigid bodies.

(Refer Slide Time: 00:22)

Overview

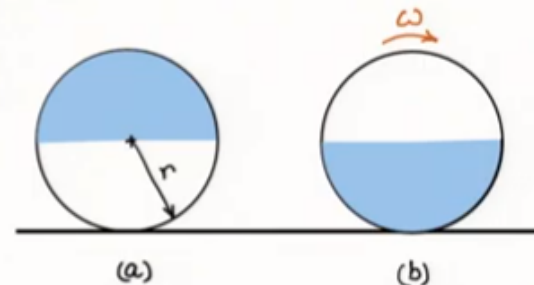
- Work-energy relations from equations of motion of plane kinetics of rigid bodies
- Conservation of energy

Here we are going to look at applications of the work energy relations through some problems.

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Problem 1:

A semi-circular disc of mass $m=2$ kg is mounted in a light hoop of radius $r=150$ mm and released from rest in position (a). If the hoop rolls without slipping, determine the angular velocity ω of the hoop and the normal force N under the hoop as it passes position (b) after rotating through 180 deg.



Source: Dynamics, Meriam and Kraige

The statement of the problem is shown in the slide above.

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Conservation of Mechanical energy

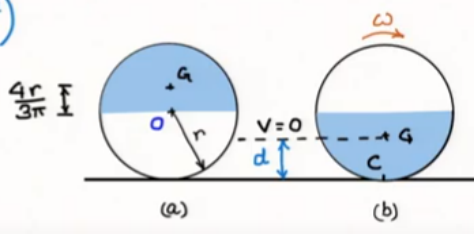
$$T_1 + V_1 = T_2 + V_2$$

$$\Rightarrow 0 + mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2} I_c \omega^2 \quad (I_c = I_G + md^2)$$

$$I_o = \frac{1}{2} mr^2 \quad I_G = I_o - m\left(\frac{16r^2}{9\pi}\right)$$

$$I_c = I_G + m\left(r - \frac{4r}{3\pi}\right)^2 = \frac{3}{2} mr^2 - \frac{8}{3\pi} mr^2$$

$$\Rightarrow \frac{1}{2} \left(\frac{3}{2} - \frac{8}{3\pi}\right) mr^2 \omega^2 = \frac{8}{3\pi} mgr$$

$$\Rightarrow \omega = \left[\frac{32}{9\pi - 16} \frac{g}{r} \right]^{1/2}$$


First, we determine the angular velocity in state (b) as shown in the above slide.

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Coordinate system and FBD

Newton's 2nd law

$$m \vec{a}_G = \vec{F} = f \hat{i} + (N - mg) \hat{j} \quad (1)$$

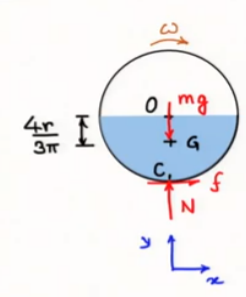
$$\vec{a}_G = \vec{a}_o + \vec{\alpha} \times \vec{OG} + \vec{\omega} \times \vec{\omega} \times \vec{OG}$$

$$= \alpha \hat{k} \times r \hat{j} + \alpha \hat{k} \times \left(-\frac{4r}{3\pi} \hat{j}\right) + \omega^2 \frac{4r}{3\pi} \hat{j}$$

$$= \alpha r \left(\frac{4}{3\pi} - 1\right) \hat{i} + \omega^2 \frac{4r}{3\pi} \hat{j}$$

From (1)

$$\hat{j}: m \omega^2 \frac{4r}{3\pi} = N - mg$$

$$\Rightarrow N = mg \left[1 + \frac{128}{3\pi(9\pi - 16)} \right]$$


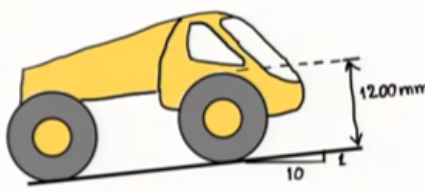
$$\left(\omega = \left[\frac{32}{9\pi - 16} \frac{g}{r} \right]^{1/2} \right)$$

The detailed steps for determination of the normal reaction at C is presented in the above slide.

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Problem 2:

A small experimental vehicle has a total mass m of 500 kg including the wheels and the driver. Each of the four wheels has a mass of 40 kg and centroidal radius of gyration of 400 mm. Total frictional resistance R to motion is found to be 400 N by towing the vehicle with the engine disengaged. Determine the power output of the engine for a speed of 72 km/h up the 10% incline with (a) zero acceleration, and (b) with an acceleration of 3 m/s^2 .



Source: Dynamics, Meriam and Kraige

The next problem statement is presented in the above slide.

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Work-energy relation in differential form

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dV}{dt} = P \quad P = P_R + P_E$$

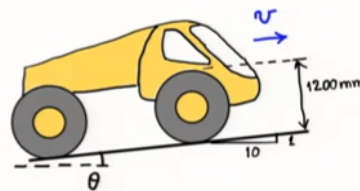
(a) Constant velocity ($\frac{dT}{dt} = 0$)

$$V = mg y \Rightarrow \frac{dV}{dt} = mg v \sin \theta$$

$$P = -400v + P_E$$

From work-energy relation

$$500(9.81)(20) \frac{1}{\sqrt{101}} = -400(20) + P_E$$

$$\Rightarrow \underline{P_E = 17.76 \text{ kW}}$$


$v = 72 \text{ km/h} = 20 \text{ m/s}$

For the constant velocity case, the calculation of engine power is presented in the slide above.

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Work-energy relation in differential form

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dV}{dt} = P \quad P = P_R + P_E$$

(b) acceleration $a = 3 \text{ m/s}^2$

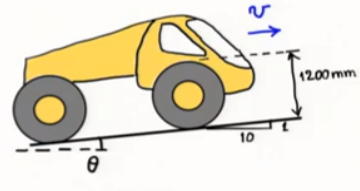
$$T = \frac{1}{2} M v^2 + 4 \left(\frac{1}{2} m_w k^2 \frac{v^2}{r_w^2} \right)$$

$$\frac{dT}{dt} = M v a + 4 m_w \frac{k^2}{r_w^2} v a \quad \frac{dV}{dt} = mg v \sin \theta$$

$$P = -400v + P_E$$

From work-energy relation

$$500(20)3 + 160 \frac{0.4^2}{0.6^2} (20)3 + 500(9.81) \frac{20}{\sqrt{101}} = -400(20) + P_E$$

$$\Rightarrow \underline{P_E = 52.03 \text{ kW}}$$


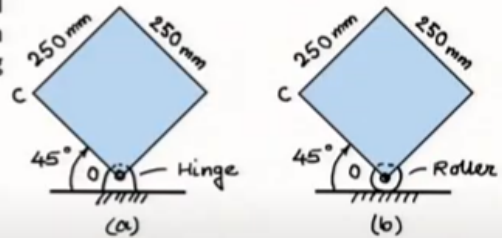
$v = 72 \text{ km/h} = 20 \text{ m/s}$

The solution of part (b) of the problem with accelerated motion, the calculation of engine power is shown above.

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Problem 3:

Each of the identical solid square blocks is allowed to fall by rotating counter-clockwise from the rest positions shown. The support at O is case (a) is a hinge, while in case (b) is a small frictionless roller. Determine the angular velocity ω of each block as the edge OC becomes horizontal just before striking the ground.



It may be noted in this problem that case (a) represents a fixed axis rotation of the block, while case (b) corresponds to a general planar motion of the block.

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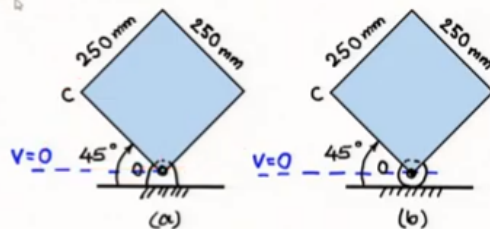
(a) Energy conservation: $I_1 + V_1 = I_2 + V_2$

$$0 + mg \frac{a}{\sqrt{2}} = \frac{1}{2} I_O \omega^2 + mg \frac{a}{2}$$

$$\left(I_O = \frac{m}{12} (2a^2) + m \frac{a^2}{2} = \frac{2}{3} ma^2 \right)$$

$$\Rightarrow \omega^2 = 0.621 \frac{g}{a}$$

$$\Rightarrow \underline{\omega = 4.94 \text{ rad/s}}$$



The above slide presents the solution for case (a).

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(b) Energy conservation: $T_1 + V_1 = T_2 + V_2$

$$0 + mg \frac{a}{\sqrt{2}} = \frac{1}{2} m v_a^2 + \frac{1}{2} I_G \omega^2 + mg \frac{a}{2}$$

Newton's 2nd law

$$m \vec{a}_a = (-mg + N) \hat{j} \Rightarrow \dot{v}_{ax} = 0 \Rightarrow v_{ax} = 0$$

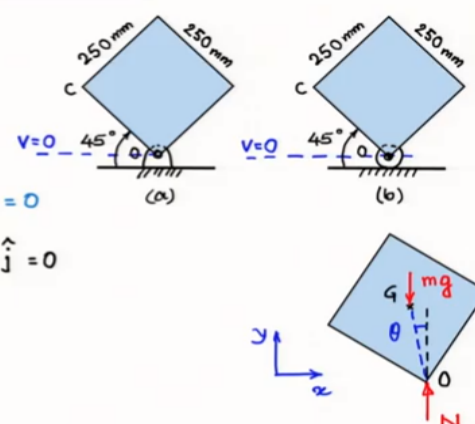
No vertical velocity of O Constraint: $\vec{v}_O \cdot \hat{j} = 0$

$$(\vec{v}_O = \vec{v}_a + \vec{\omega} \times \vec{aO}) \cdot \hat{j} = 0$$

$$\Rightarrow 0 = v_{ay} + \omega \hat{k} \times \frac{a}{\sqrt{2}} (\sin \theta \hat{i} - \cos \theta \hat{j}) \cdot \hat{j}$$

$$\Rightarrow v_{ay} = -\frac{a\omega}{\sqrt{2}} \sin \theta = -\frac{a\omega}{2} \quad (\theta = 45^\circ \text{ in state 2})$$

Using work-energy relation: $\frac{1}{2} m \left(\frac{a^2}{4} + \frac{a^2}{6} \right) \omega^2 = mg(0.207) \Rightarrow \omega = 6.25 \text{ rad/s}$



The solution for case (b) is a little involved since an additional constraint on the motion of G needs to be determined. This has been shown in the slide above.

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Summary

- Work-energy relations from equations of motion of plane kinetics of rigid bodies
- Conservation of energy

The discussions are summarized in the above slide.