Advanced Dynamics
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Module No # 01
Lecture No # 03
Relative Motion – I

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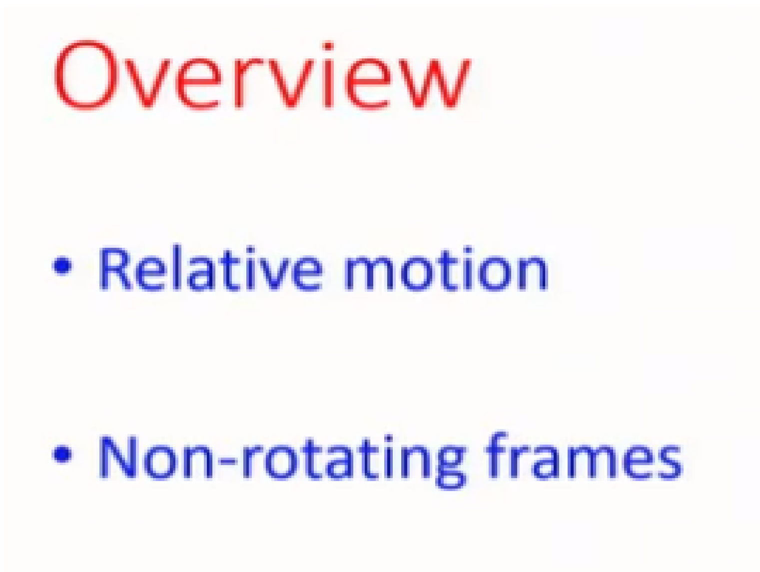

In this lecture we are going to look at relative motion. This is a very important concept in kinematics, and this actually complicates kinematics to a large extent. So it is a very important to understand how relative motion is described kinematically.

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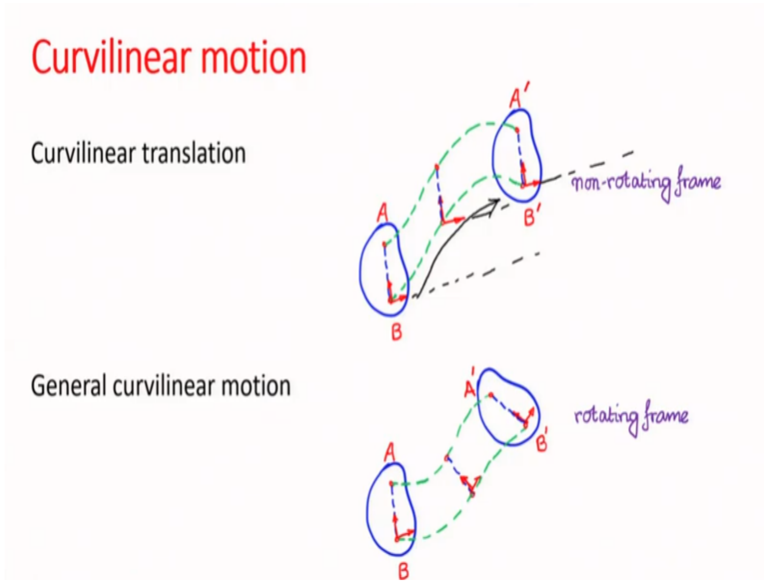

Before we go to this topic let me briefly tell you about general curvilinear motion and curvilinear translation; the difference between these 2 kinds of motion that we can have. Suppose there is a body with a fixed frame which is moving on this green path so it is part of the rigid body. So it is moving on this path but it is not changing its orientation and its direction. Such a motion is called curvilinear translation. On the other hand, if you have a general curvilinear motion with rotation, we have a general curvilinear motion.

In the curvilinear translation the moving frame is non-rotating whereas general curvilinear motion the frame can rotate as it moves.

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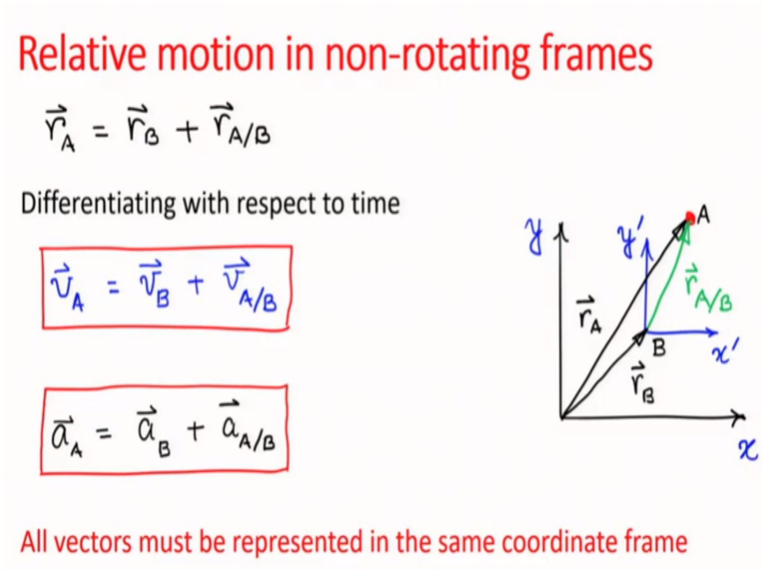

We start with relative motion in non-rotating frames imagine that we have this frame x-y which fixed in inertial space. Whereas this x'-y' is a non-rotating moving frame. I can describe a point A in the fixed frame xy, and I can also define in terms of this x prime y prime frame where the x prime y prime frame location is indicated by this vector rB. The vector $r_{A/B}$ is this vector which an observer sitting in frame B sees to point A.

We can write $r_A = r_B + r_{A/B}$. Now if you differentiate with respect to time then we have the velocity relation

$$\boxed{\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}}$$

And if you differentiate agin with respect to time, we obtain the acceleration relation

$$\boxed{\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}}$$

All these vectors of course must be representing in the same coordinate system for this vector relation to be valid. So either you can choose x-y frame or you can choose x'-y' frame.

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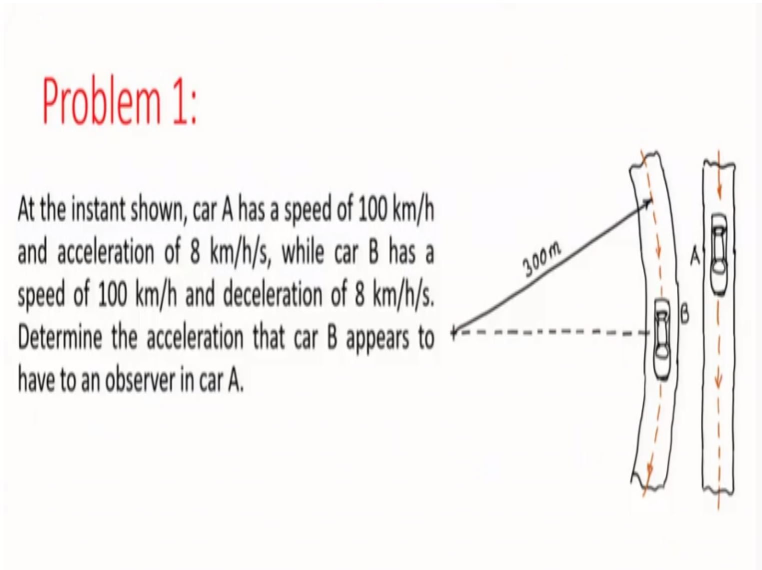

We consider the above problem. We have to determine the acceleration that car B appears to have to an observer in car A. So this is clearly a problem of finding the relative acceleration of another body which is B as measured by an observer sitting in a non-rotating moving frame which is A.

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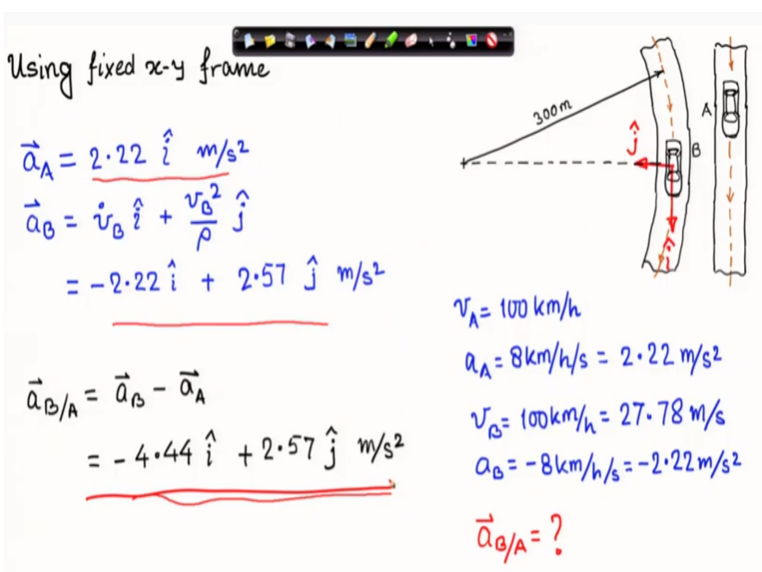

So here I have the coordinate system which I use I have fixed up i cap and j cap at the current location of B. But remember that the i cap and j cap coordinate system is not moving or rotating with B.

Now that I will use this frame to represent all vectors. Acceleration of B has acceleration because of 2 things: one is the tangential acceleration (tangent to the path), and the normal acceleration (normal to the path) because of path curvature.

$$\vec{a}_A = 2.22\,\hat{i} \ \ m/s^2$$

$$\vec{a}_B = \dot{v}_B\,\hat{i} + \frac{v_B^2}{\rho}\,\hat{j}$$

$$= -2.22\,\hat{i} + 2.57\,\hat{j} \ \ m/s^2$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

$$= -4.44\,\hat{i} + 2.57\,\hat{j} \ \ m/s^2$$

So this is the acceleration of car B relative to car A as seen by the observer in frame A or car A.

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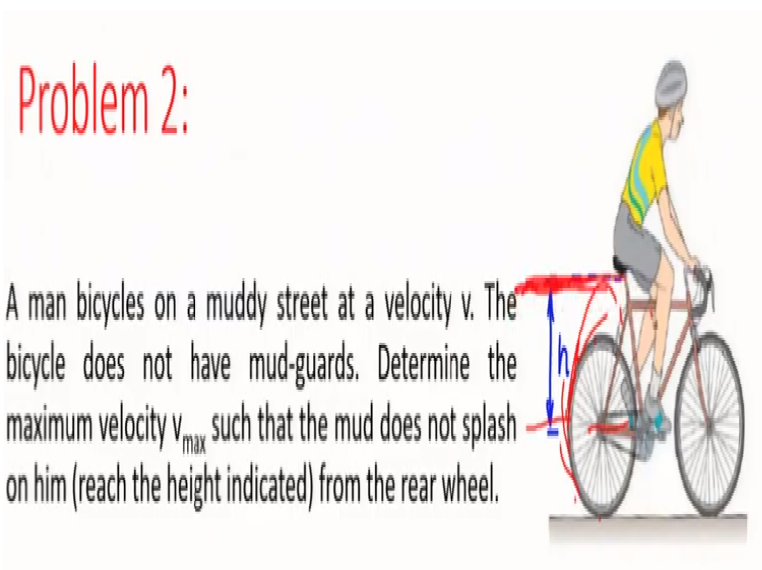

Next, we have a very practical problem above that we face if we do not have mud guard on our cycles. When you are bicycling on muddy road then this mud gets splashed. We have to find out what can be the maximum speed for no splashing. To simplify the problem, what I have done is I have asked what should be v_{max} so that h is the maximum height that is reached by any mud particles starting from anywhere from the rim of the rear wheel.

So I consider that from all points on the rear wheel mud will be splashed. At some point it will get loosened and it will move in the projectile path. What should be the maximum speed so that the maximum height that a mud particle can reach at that speed is h as measured from the center of the wheel to this seat.

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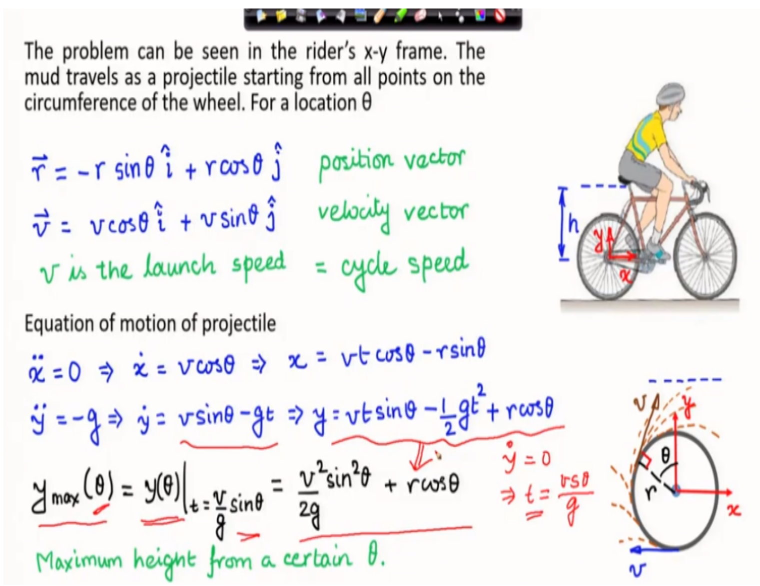

It is very convenient to look at this problem in the frame of the bicycle. The magnitude of velocity of launch of the mud particles is exactly equal to the velocity of the bicycle.

This is because, when I am on the bicycle, I will see all points on the wheel moving with the same velocity as the velocity of the bicycle. Now, we have

$\vec{r} = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j}$ position vector

$\vec{v} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$ velocity vector

v is the launch speed = cycle speed

For the projectile motion of the mud particles

$$\ddot{x} = 0 \Rightarrow \dot{x} = v\cos\theta \Rightarrow x = vt\cos\theta - r\sin\theta$$
$$\ddot{y} = -g \Rightarrow \dot{y} = v\sin\theta - gt \Rightarrow y = vt\sin\theta - \frac{1}{2}gt^2 + r\cos\theta$$

Now I want to find out what is the maximum height reached from a certain θ value on the rim. To do that I will find out the time it takes for the vertical velocity to reach 0

$$y_{max}(\theta) = y(\theta)\Big|_{t=\frac{v}{g}\sin\theta} = \frac{v^2\sin^2\theta}{2g} + r\cos\theta$$

Maximum height from a certain θ.

Now I want to find out which theta will actually give me the global maximum height.

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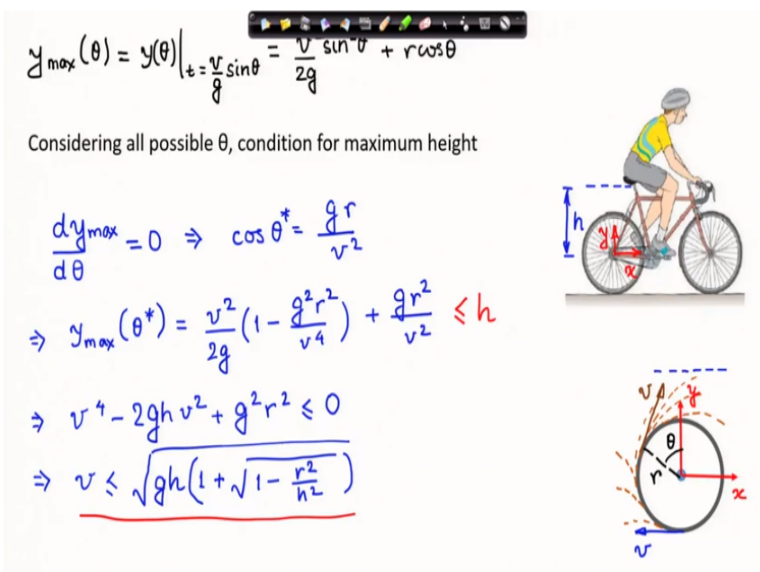

To find that, I will differentiate y_{max} as the function of θ with respect to θ and put it equal to 0. So this is this will give me that θ location from which the mud particle will reach the maximum height from among all mud particle starting from anywhere on the rim of the wheel. This maximum height should be less than h, and this condition will give me the maximum velocity of the bike. So, we have

$$\frac{dy_{max}}{d\theta} = 0 \Rightarrow \cos\theta^* = \frac{gr}{v^2}$$

$$\Rightarrow y_{max}(\theta^*) = \frac{v^2}{2g}\left(1 - \frac{g^2r^2}{v^4}\right) + \frac{gr^2}{v^2} \leq h$$

$$\Rightarrow v^4 - 2gh\,v^2 + g^2r^2 \leq 0$$

$$\Rightarrow v \leq \sqrt{gh\left(1 + \sqrt{1 - \frac{r^2}{h^2}}\right)}$$

Thus, we have found out what should be the maximum speed of cycling so that even without mud guards the mud will not splash on the rider.

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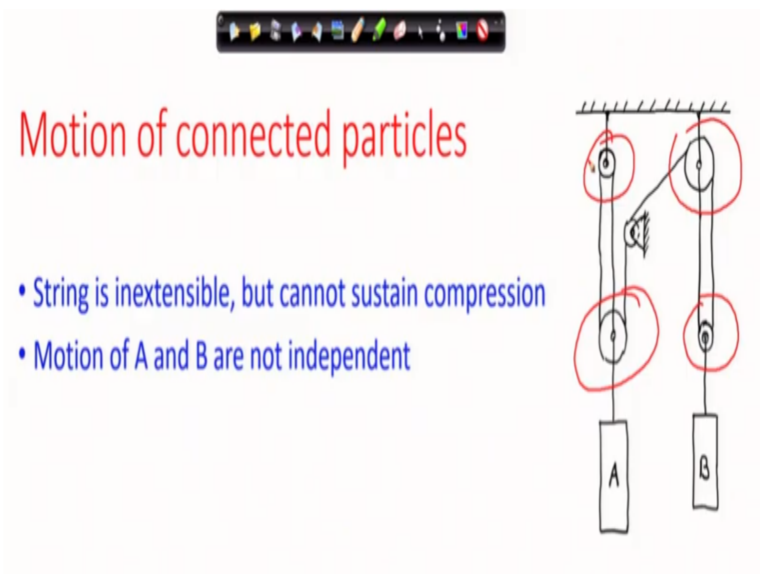

Next we are going to look at motion of connected particles. Here I have an example where 2 blocks A and B are connected by inextensible strings or chords. If these chords can be kept in tension then the motion of A and B are related. In such problems, the complicated parts are the compound pulleys and that is what we are going to analyze first.

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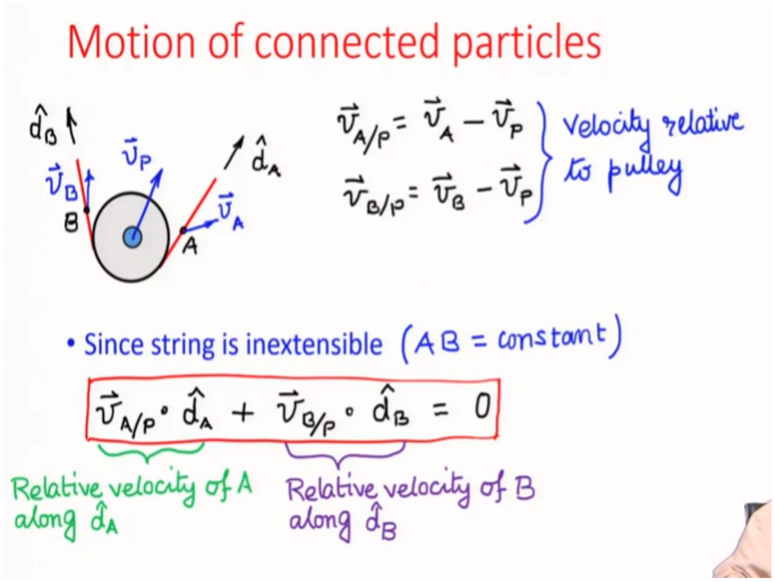

Above is a general schematic of a pulley. I have considered 2 points on the string, points A and B which are points fixed to the string. They can have velocity is v_A and v_B as I have shown and the center of the pulley can have velocity v_P. The velocity of point A is composed of a velocity of unwrapping the string that is perpendicular to the string, and the other velocity is along the string which means the string is moving on the pulley. Similarly for point B. The relative velocities of A and B with respect to the pulley can be written as

$$\left.\begin{aligned} \vec{v}_{A/P} &= \vec{v}_A - \vec{v}_P \\ \vec{v}_{B/P} &= \vec{v}_B - \vec{v}_P \end{aligned}\right\} \text{Velocity relative to pulley}$$

These relative velocities by themselves can be arbitrary. However, because the string is inextensible, the projection of these relative velocities along the string must add up to zero so that the length of the string is conserved. This is very similar to conservation of mass for which we have the continuity equation derived in a similar way.

We consider the two directions d_A and d_B as shown. Because both d_A and d_B are directed away from the pulley, the projection of the corresponding relative velocities along these 2 directions must sum up to 0. Hence

$$\boxed{\underbrace{\vec{v}_{A/P}\cdot\hat{d}_A}_{\text{Relative velocity of A along } \hat{d}_A} + \underbrace{\vec{v}_{B/P}\cdot\hat{d}_B}_{\text{Relative velocity of B along } \hat{d}_B} = 0}$$

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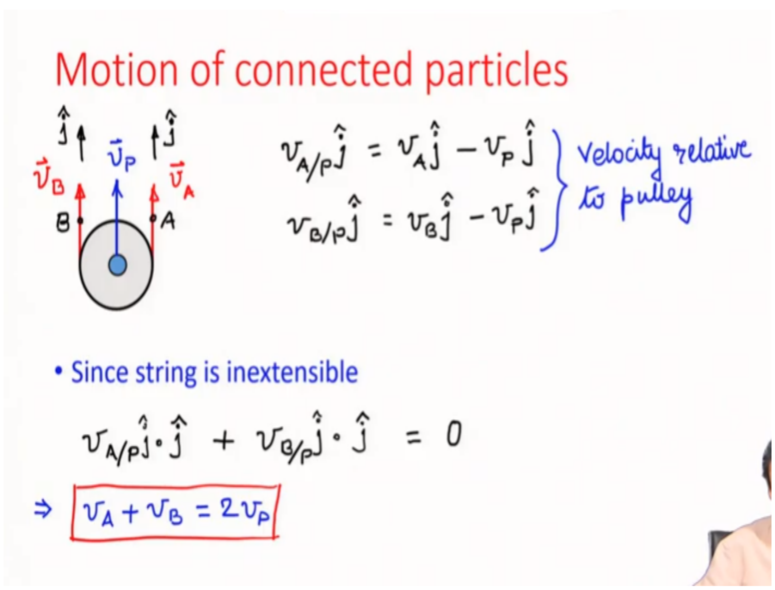
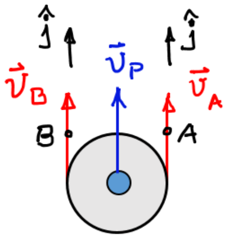


Now if you consider a special case where the direction are both arms of the string are vertical. This gives



$$\left.\begin{aligned} v_{A/P}\hat{j} &= v_A\hat{j} - v_P\hat{j} \\ v_{B/P}\hat{j} &= v_B\hat{j} - v_P\hat{j} \end{aligned}\right\} \text{Velocity relative to pulley}$$

- Since string is inextensible

$$v_{A/P}\hat{j}\cdot\hat{j} + v_{B/P}\hat{j}\cdot\hat{j} = 0$$

$$\Rightarrow \boxed{v_A + v_B = 2v_P}$$

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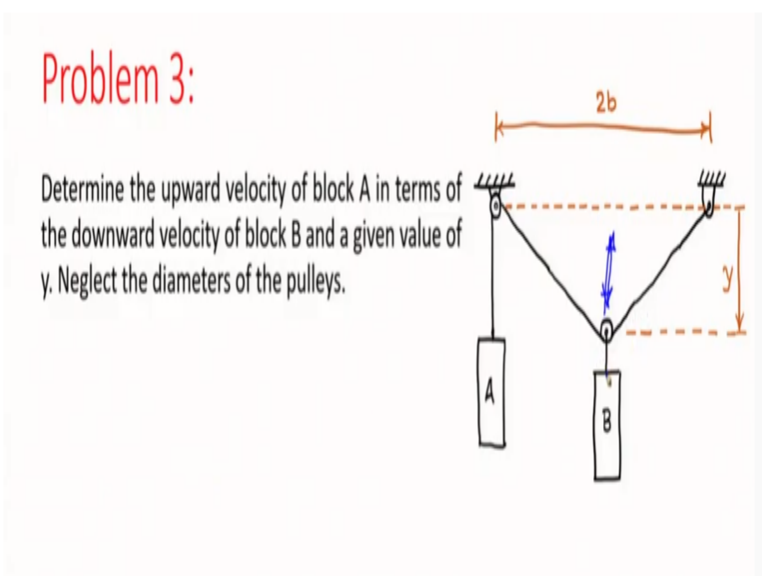

Now we look at the problem above and see how this can be solved using the concepts developed. I will also show you an alternative method using plane geometry. First notice one thing that the block B will always remain at the center of this span of the string. Therefore, the motion of block B will only be in the vertical direction.

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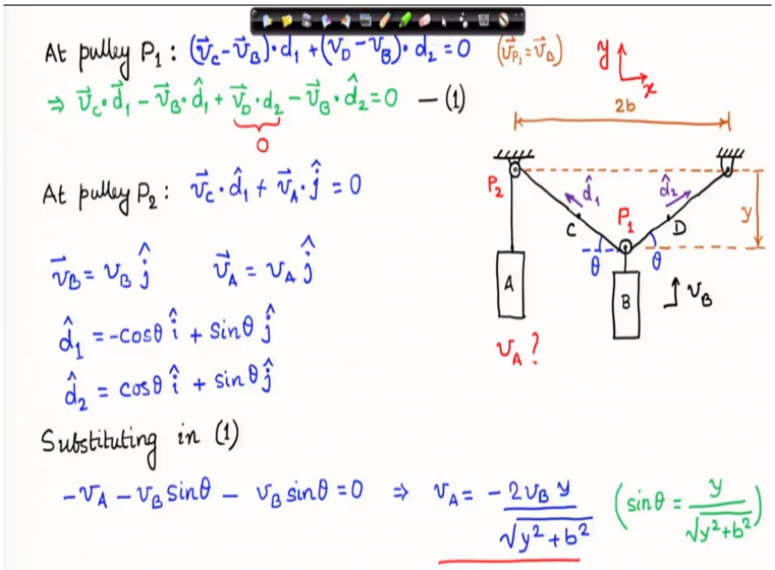

So here I have marked out v_B and the coordinate system that we use is the x-y coordinate system. Note that velocity of the pulley P1 will be same as motion of block B. I have marked these unit vector directions as we are used d_1 and d_2 in the direction of the string and I have considered these 2 points C and D on these two spans of the string. Using the inextensibility condition, we have

At pulley P_1 : $(\vec{v}_C-\vec{v}_B)\cdot\hat{d}_1 + (\vec{v}_D-\vec{v}_B)\cdot\hat{d}_2 = 0$ $\quad (\vec{v}_{P_1}=\vec{v}_B)$

$$\Rightarrow \vec{v}_C\cdot\vec{d}_1 - \vec{v}_B\cdot\hat{d}_1 + \underbrace{\vec{v}_D\cdot d_2}_{0} - \vec{v}_B\cdot\hat{d}_2 = 0 \quad \text{— (1)}$$

Also

At pulley P_2: $\vec{v}_C \cdot \hat{d}_1 + \vec{v}_A \cdot \hat{j} = 0$

$$\vec{v}_B = v_B \hat{j} \qquad \vec{v}_A = v_A \hat{j}$$

$$\hat{d}_1 = -\cos\theta \, \hat{i} + \sin\theta \, \hat{j}$$

$$\hat{d}_2 = \cos\theta \, \hat{i} + \sin\theta \, \hat{j}$$

Substituting in (1)

$$-v_A - v_B \sin\theta - v_B \sin\theta = 0$$

$$\Rightarrow \quad v_A = \frac{-2 v_B y}{\sqrt{y^2+b^2}} \qquad \left(\sin\theta = \frac{y}{\sqrt{y^2+b^2}}\right)$$

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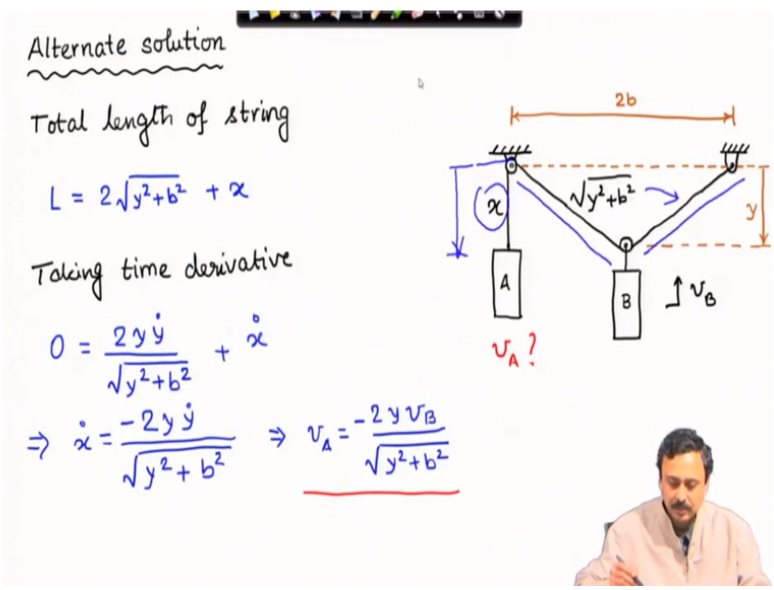

Now we discuss an alternate solution method for this problem. You find out the total length of the string and use the condition of inextensibility as follows

Alternate solution

Total length of string

$$L = 2\sqrt{y^2+b^2} + x$$

Taking time derivative

$$0 = \frac{2y\dot{y}}{\sqrt{y^2+b^2}} + \dot{x}$$

$$\Rightarrow \dot{x} = \frac{-2y\dot{y}}{\sqrt{y^2+b^2}} \Rightarrow v_A = \frac{-2yv_B}{\sqrt{y^2+b^2}}$$

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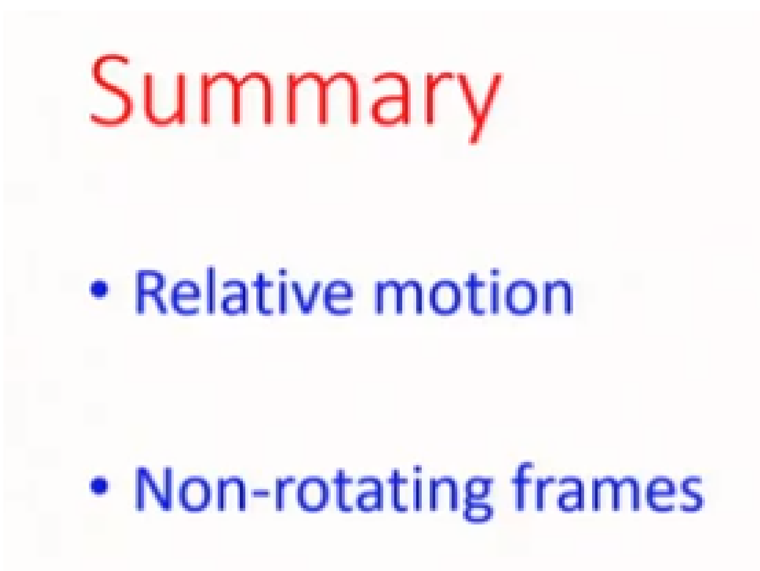

This is the summary of what we have discussed: relative motion in non-rotating frames.