**Advanced Dynamics**
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**Module No # 06**
**Lecture No # 29**
**Planar Kinetics: Work Energy Relations – I**

In this lecture I am going to discuss the work energy relation for planar kinetics.

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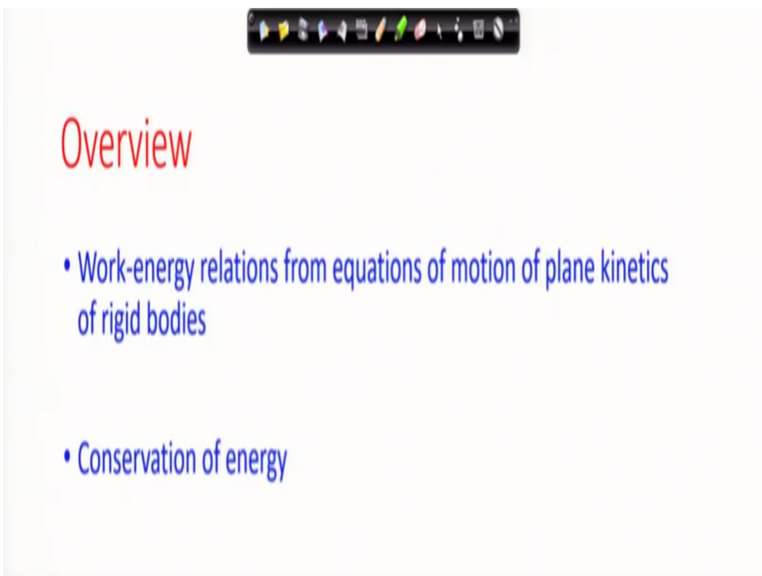


To give you an overview of what we are going to discuss we are going to derive the work energy relations from equations of motion of plane kinetics. Look at the conservation of energy and then look at some problems.

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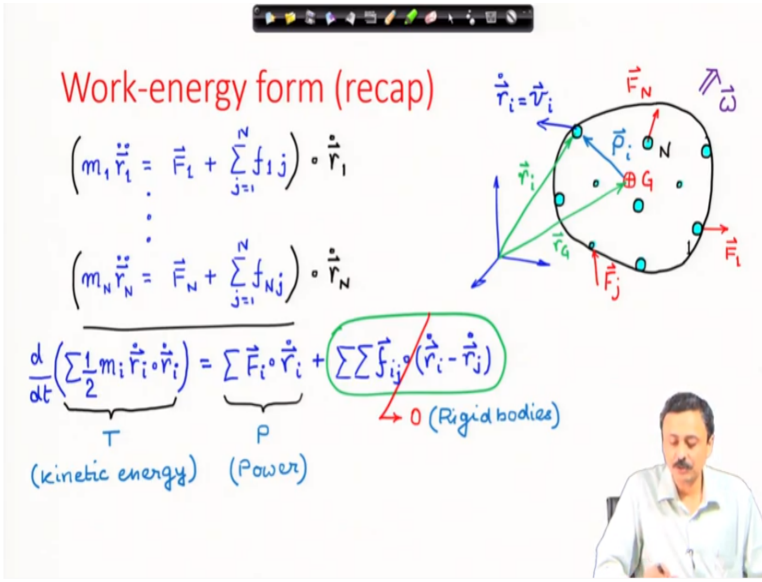


Just to recapitulate what we have discussed for a system of particles because this is what we are going to extend to rigid bodies. Here we had written out the Newton's second law for all the particles we had n particles suppose we have n particles. We write down the Newton's second law for all these particles. And we had taken dot product of each of these equations with the corresponding velocity of that particle.

So first equation gets a dot product with r 1 dot and so on the nth particle equation is dot product with the velocity of the nth particle. And then we had summed up and we found that the rate of change of kinetic energy is equal to the power and for rigid bodies this was taken as a special case. For rigid bodies this contribution of internal forces is 0 because the internal force direction and the relative velocity direction for rigid bodies they are orthogonal.

Because remember the relative velocity r i dot - r j dot is omega cross r i - r j therefore r i dot this relative velocity is perpendicular to the line joining the 2 points and f i j is along the line joining the 2 points. Therefore is a dot product of 2 perpendicular orthogonal things and that is 0.

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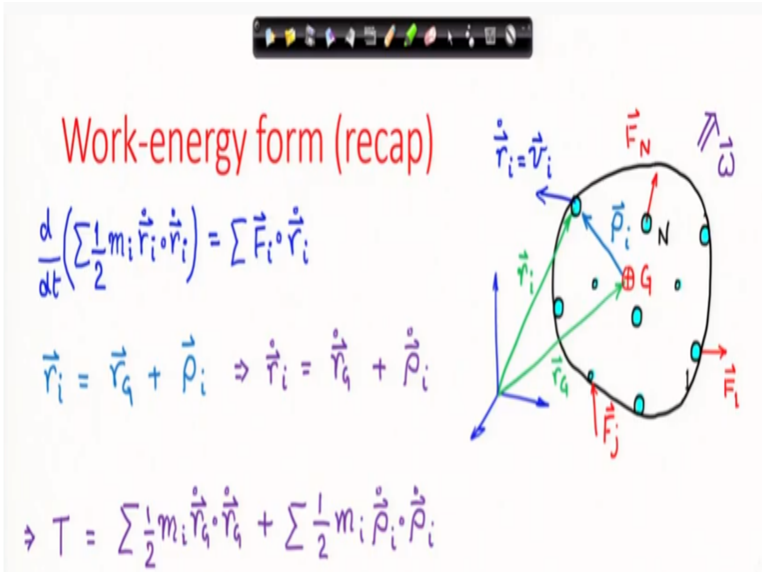


And then we had looked at the decomposition of the kinetic energy expression. And we have found that the total kinetic energy of this system is equal to the kinetic energy of translation of center of mass with all the mass concentrated at the center of mass as if center of mass is a point particle. And we are writing the kinetic energy of the point particle this is one term in that expression of total kinetic energy.

The second term comes from motion of the particles about the center of mass. And for rigid bodies this rho i dot will be related in terms of the angular velocity.

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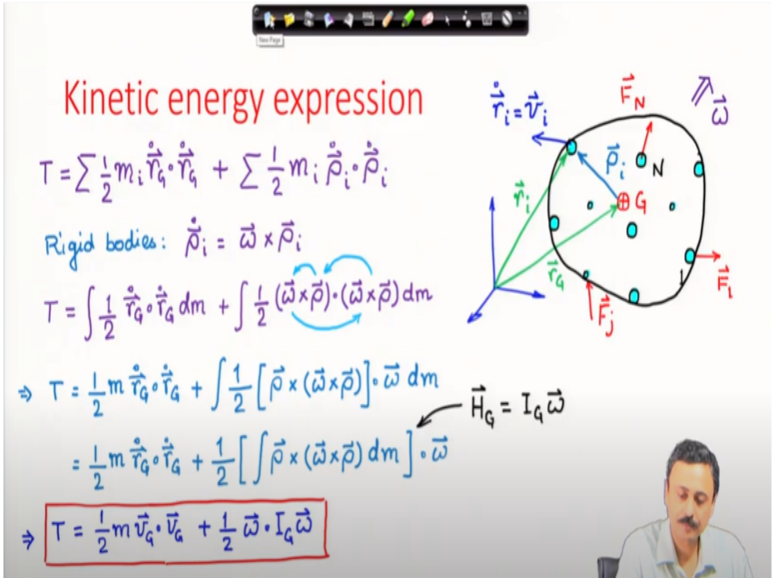


So this was the expression of kinetic energy of translation of center of mass and kinetic energy of motion about the center of mass kinetic energy because of motion of the particles about the center of mass. Remember that rho i dot is the relative velocity of the ith particle with respect to the center of mass. Therefore the second term in the kinetic energy expression gives us the kinetic energy because of motion of the particles about the center of mass.

For rigid bodies this can be written very simply like this omega cross rho i this follows from our discussions on kinematics. Because the center of mass and the point particle ith point particle both are parts of a rigid body therefore the relative velocity is omega cross the relative vector which is rho i. For rigid bodies that summation goes over to an integral and our measure for this integration is the small mass of that infinitesimal particle that we are considering d m.

As you can see the first integral is because of the motion of center of mass it is a velocity of center of mass square 1 half of d m times velocity of center of mass square integrated over the whole body. The second term is 1 half omega cross rho dot omega cross rho d m now this is because of the rotation of the body about the center of mass. This sometimes called the rotational kinetic energy and we will see this detailed expression.

Now here I can look at this term as a scalar triple product. I can look at this expression as a scalar triple product what I do is? I see that this is a cross b dot c this is scalar triple product. And this can be written by rotational permutation you can write it as I can take this c in place of B therefore this becomes equal to B cross C dot A. So this becomes B cross C dot A therefore this term this vector goes here and the vector a goes here right. This is like cyclic permutation that is what I use here C vector is this whole thing.

Therefore if I use that I can write it as so first term its integration of d m is nothing but the total mass of the system. The second integral can be written because this omega goes here. So this comes here and this omega cross rho goes to the second place so this is omega cross rho and rho goes to the first place so here sits rho. And that is integrated over the mass total body. This expression you have seen before and the first term is straight forward is the translational kinetic energy because of the translation of the center of mass.

The second one this bracketed quantity the bracketed quantity happens to be I G times omega this we have discussed before this happens to be I G time's omega. The moment of inertia about the center of mass of the rigid body times the omega vector. Now since we are discussing planar kinetics I G is a scalar quantity but planar kinetics for prismatic bodies. Therefore the expression of total kinetic energy of this prismatic rigid body you can think is 1 half m v G square + 1 half omega dot I G time's omega.

Here as an expression I G can also be a tensile the moment of inertia tensile which we will discuss a little later but for prismatic bodies this will turn out to be a scalar. Therefore we have these 2 terms the kinetic energy of translation and kinetic energy of rotation that constitutes the net kinetic energy of the body.

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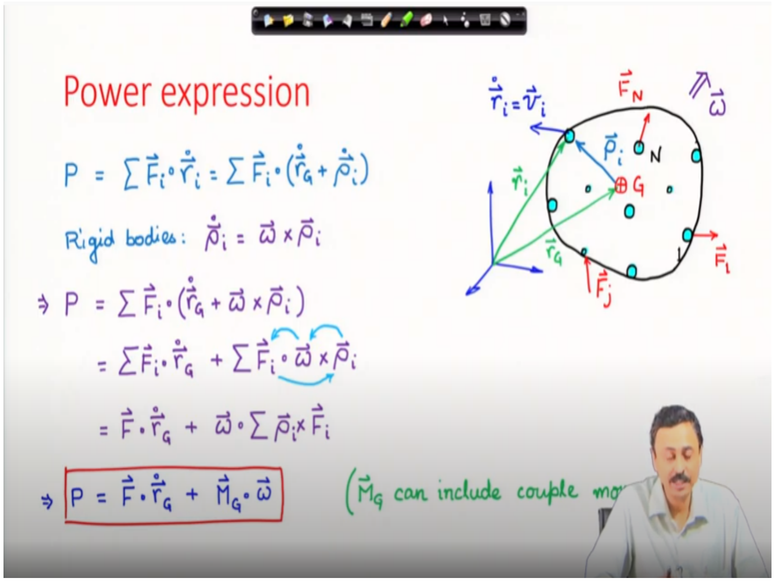


Now the power expression we had written like this force time's velocity force dot velocity vector. And when I write r i dot in terms of the velocity of center of mass and the relative velocity. For the rigid bodies as you know the relative velocity is omega cross the relative vector rho i. And if you substitute these things in the power expression as I have done here the first term when I open this brackets the first term it will be only summation of F i because r G has nothing to do with the summation.

Therefore I will get the net force dot velocity of the center of mass plus there will be a summation here which here I am going to again do some processing. I am going to use this is again a in a scalar triple product the first term is simple its net force dot velocity of the center of mass. Whereas the second summation I can use the scalar triple product and I can rotate this in this form write it like omega dot summation of rho i cross F what is rho i cross F. F i is the force on the ith particle recall that F i is the force on the ith particle.

Therefore rho i cross f this is nothing but the moment because of the force on the ith particle moment about the center of mass. Therefore the net power is equal to net force dot velocity of center of mass plus net moment about the center of mass dot omega. And this moment can include couple moments as well so that is the power expression.

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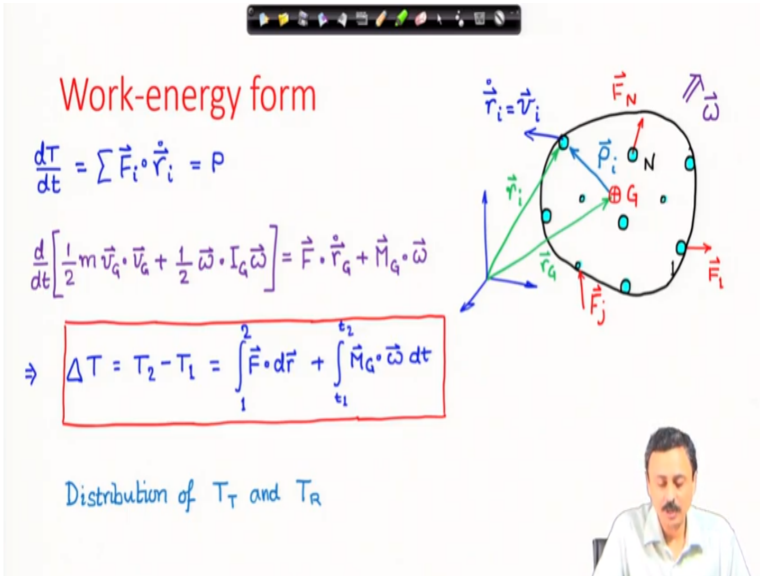


And our work energy relation turns out to be rate of change of kinetic energy is equal to the power. And if when I do this substitutions I get this as my final expression therefore if I integrate over time on the left I will get the change in kinetic energy. Now this kinetic energy change in kinetic energy means total kinetic energy rotation a translation and rotation. So t indicates the total kinetic energy of translation and rotation this total sum the change in the sum is equal to F dot d r which is work done on the system by the forces net force.

And this is of course this is on the center of mass. And the second integral mg dot omega d t I have not written here omega as some theta dot k cap. If I do then I can write as M G d theta but in general for a general rigid body I cannot write omega as a time derivative of a vector quantity omega is not time derivative of a vector quantity. A general omega is not time derivative of a vector quantity but for planar rotation when I write omega as theta dot k cap for planar motion I write omega as theta dot k cap.

In this case I can do this integral so omega d t in this case omega d t I can write as d theta k cap this is only for planar rotation not in general I cannot do this. Now the kinetic energy is of course composed of the translational part and the rotational part. And we are going to look at this distribution of or how the translational kinetic energy changes and how rotational kinetic energy changes a little later?

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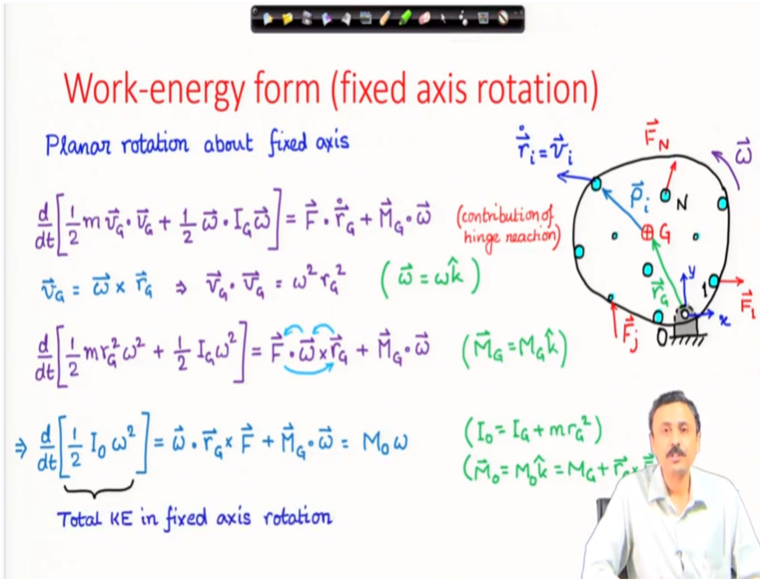


If you consider planar rotation about a fixed axis then there are some simplifications possible that I was mentioning. Now velocity of center of mass can be written as omega cross r G and you note what is r G here? This is r G vector from O to the center of mass velocity of center of mass is omega cross r G and therefore velocity of the center of mass square is omega square r G square and omega has this form.

And then I substitute these expressions here and I use once again that scalar triple product rotation because of scalar triple product I can do this rotation and on the left hand side I have d, d t of the bracketed quantity you will notice what is the bracketed quantity? The bracketed quantity is 1 half m r G square + I G time's omega square. This is nothing but the parallel axis theorem to shift or to find out the moment of inertia of the body about O therefore this is I O.

Therefore half moment of inertia about O time's omega square is the net kinetic energy of the body. And on the right hand side by using this rotation I can write it omega dot r G cross f + M G dot omega. And this I can write as i if I take this omega dot outside common then in I have r G cross F + M G which I have written out already. And this dot omega and omega is omega k cap you should note one thing.

In this expression and also here the net force at the center of mass includes the hinge reaction forces the net force at the center of mass includes the hinge reaction forces. Similarly the moment at the center of mass because of all forces will include the moment because of the hinge reaction forces. Therefore this I have mentioned contribution of the hinge reaction forces but when I finally do this about the center of mass I get M O net moment about the hinge time's omega.

And here those forces the hinge reaction forces will not contribute. Therefore it is very easy for rotation with a fixed axis fixed axis rotation problem. It is always good to apply at the hinge. Therefore this is the total kinetic energy in fixed axis rotation.

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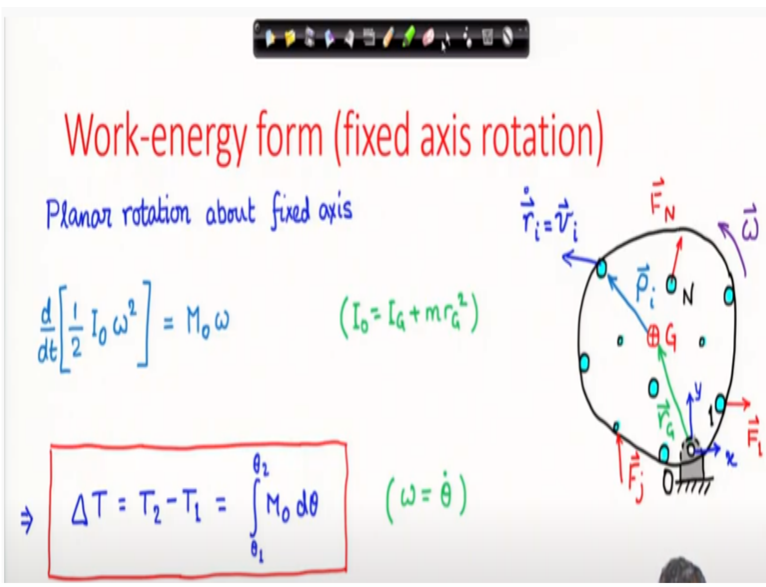


So this is our expression now here. I can write omega as theta dot k cap this which I had written previously. The omega vector was theta dot k cap because this is linear motion and in that I can write it like this. Therefore this d t omega vector d t this is what I will have here and that turns out to be theta dot d t k cap. And everything is in k cap direction actually. Everything here is a scalar finally it has reduced to a scalar.

Because I have taken a dot product because this moment is also in the k cap direction therefore I had the dot product and finally everything reduced to a scalar quantity. Now this is the net change in the kinetic energy which can be written as the kinetic energy calculated about the hinge point. So change in kinetic energy of the body is equal to the net moment about the hinge point this is the work done because of the moment about the hinge point.

The integral M O d theta when theta goes from theta 1 to theta 2 is the net work done by the moment. And therefore the change in kinetic energy of this body is equal to the work done by the all the moments acting there can be couple moments there can be forces whose moment you can calculate about O. And you have to calculate the work done by these moments and that must be equal to the change in the kinetic energy of the body.

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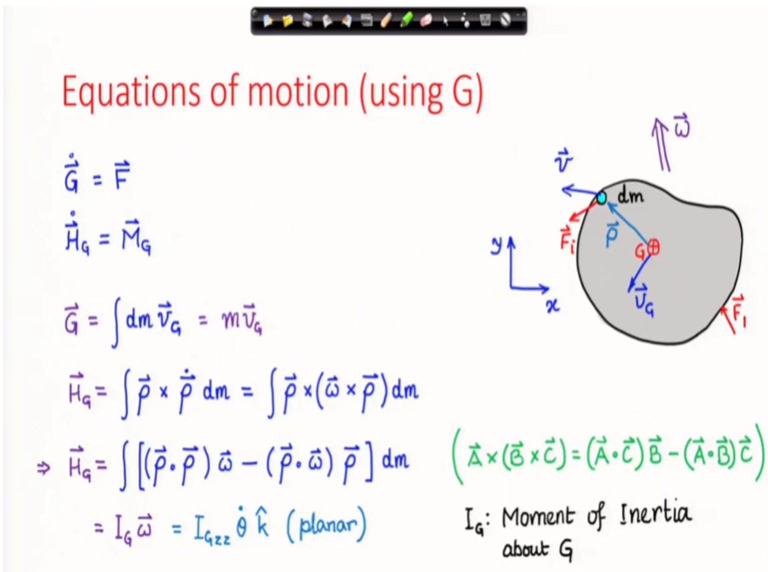


Let us look at the equation of motion of this rigid body about with the rotational dynamics equation or the angular momentum equation about G. So we have the translational dynamics equation which is the extended Newton's second law for the center of mass and the angular momentum equation about the center of mass. And you know that the linear momentum of the center of mass is defined this way the total mass times velocity of center of mass.

And the angular momentum we are defined it this way about G its arm cross the relative velocity d m for a rigid body and for a rigid body rho dot the relative velocity is omega cross rho. And we had used this vector identity to finally obtain the angular momentum in a simple form as I G in the moment of inertia about G time's omega I G is the moment of inertia about G. And if you consider this expression of omega representation of omega as some theta dot k cap.

Then I have the moment of inertia about the z axis that is the precise definition times of theta dot k cap for the planar situation is equal to H G the angular momentum about G.

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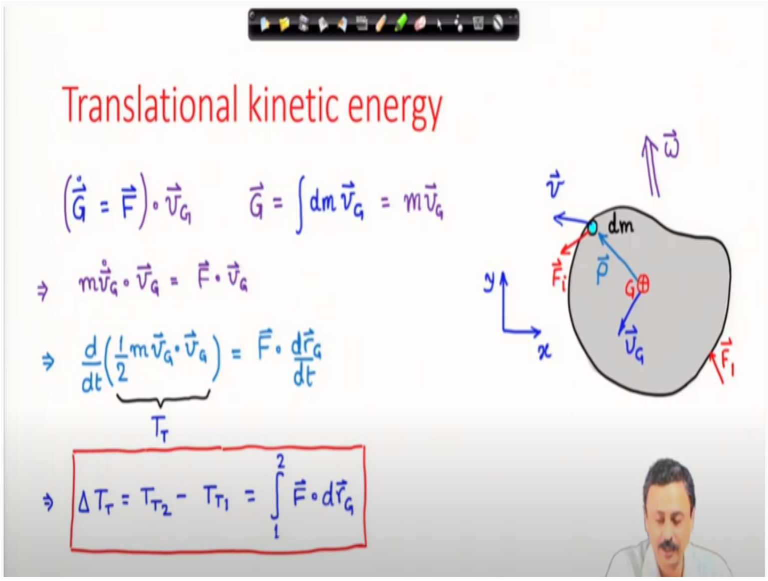


Now we have the translational dynamics if I take dot product of this translational dynamics with the velocity of center of mass then what do you have? On my left hand side I have mass times acceleration of center of mass dot velocity of center of mass is equal to on the right hand side force dot velocity of center of mass. The left hand side can be written as d, d t of the translational kinetic energy now you see that we are only if I use only the translational equation of motion of the extended Newton second law.

And do that same processing as we have done before consider this as a particle and dot product with the velocity of that particle. Then I obtain d, d t of the translational kinetic energy and that is equal to on the right hand side it is the power the net force on the body dot the velocity of the center of mass. Therefore I can integrate and I can write that the change in the translational kinetic energy is equal to the work done by all forces at the center of mass.

The F dot d r G is the work done by all forces and that I integrate as the system moves from state 1 to state 2 so that changes the translational kinetic energy of the system.

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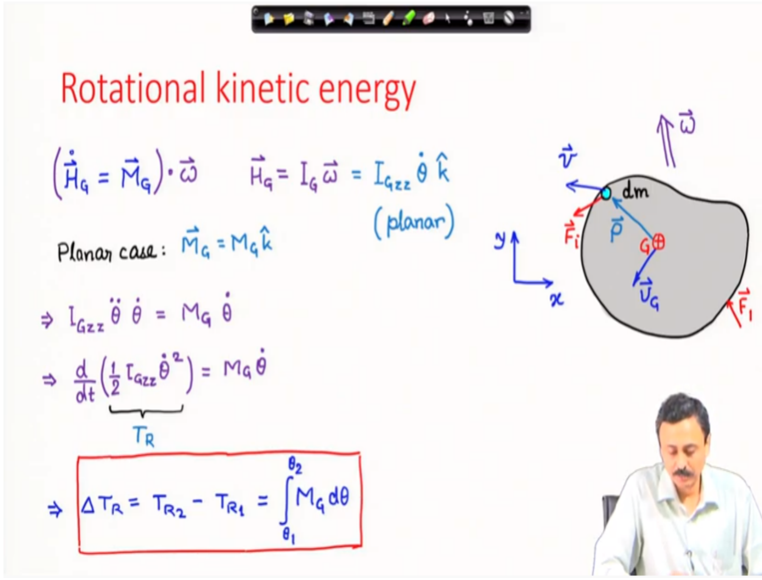


What changes the rotational kinetic energy? For that we have to look at the angular momentum equation in the planar case we are treating the thing for the planar case I take a dot product of the angular momentum equation with omega. For the planar case the moment about the center of mass moment of all forces about center of mass will be in the k cap direction therefore its representation will be M G k cap.

I have taken dot product with omega and I have written actually omega as theta dot omega vector as theta dot k cap. And the angular momentum is I G z z theta dot k cap and its time derivative is therefore I G z z theta double dot on the left hand side therefore I have this theta double dot k cap dot theta dot k cap that gives me this product. And on the right hand side I have M G vector dot omega which turns out to be this with these expressions.

Therefore the left hand side I can write d, d t of half I G z z theta dot square which you will recognize is the rotational kinetic energy the bracketed term is the rotational kinetic energy. And on the right hand side it is the power fed because of the moments about the center of mass. because of all the moments about the center of mass. Therefore rate of change of the rotational kinetic energy is equal to moment times the angular velocity.

Hence the change in the rotational kinetic energy is equal to the work done by all forces and maybe couple moments in changing the orientation of the body it goes from state 1 to state 2 and that changes the rotational kinetic energy.

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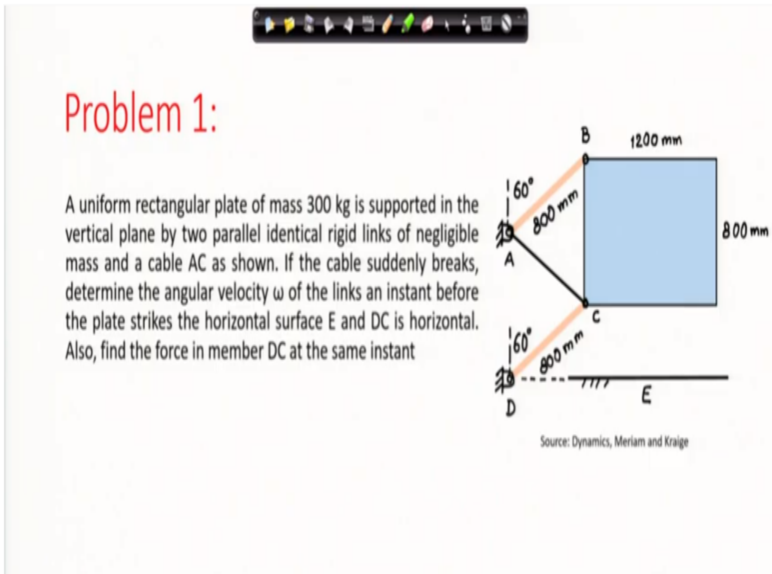


Let us look at this problem this is taken from the book of Meriam and Kraige dynamics a uniform rectangular plate of mass 300 kilogram is supported in the vertical plane by 2 parallel identical links of negligible mass and a cable A C as shown. The cable A C which goes diagonally from A to C is to prevent the mechanism from falling down. If the cable suddenly breaks determine the angular velocity omega of the links and instant before the plate strikes the horizontal surface E and D C is horizontal.

So the link D C becomes horizontal just before it strikes the ground also find the force in member D C at the same instant. This plate is of course a prismatic body and we are considering the motion in the plane which contains the center of mass of the plate.

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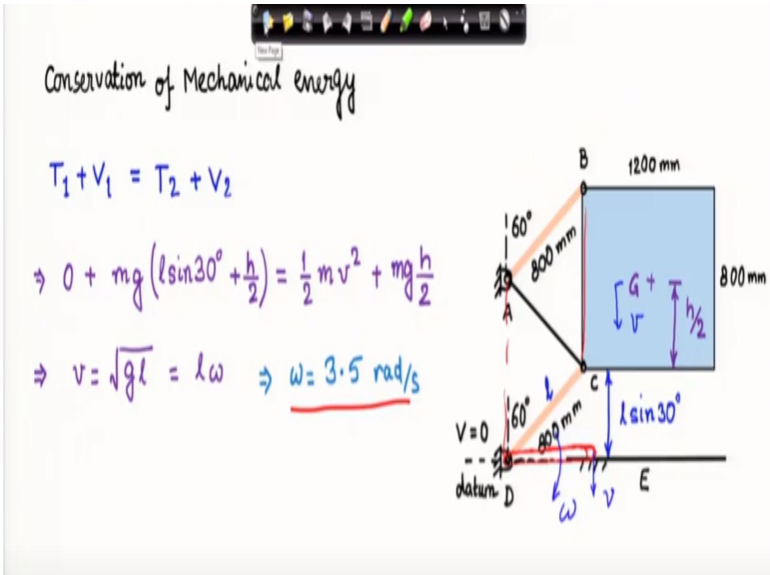


The forces that are acting on the plate external forces are because of the links of course and the weight. Weight is a potential force because of the gravitational force and we consider uniform gravitational field and there is a potential corresponding to it. We look at conservation of mechanical energy of the system the links are rigid and massless. Therefore they do not contribute to the mechanical energy but the plate is massive and therefore we have to deal with the total mechanical energy of the plate.

And that is conserved because we have this gravitational field which is a potential field. Therefore the total mechanical energy at the initial state when it is tied like this by the string and after the string is broken and the plate is about to hit the ground. We consider the total mechanical energy at that state. So just after the string is broken it is starts from rest at this configuration and the second state is just before it hits the ground.

The initial kinetic energy is of course 0 because it starts from rest at t = 0 + it is at rest at t = 0 the string breaks and the initial the velocity is 0 at that instant. The initial potential energy which is v 1 is measured from the datum. The datum I have shown here, the ground is the datum. And you can easily see that m g h height of the center of mass from the datum is l sin 30 degree + h by 2 therefore this is the initial potential energy just after the string breaks.

And that must be equal to the total mechanical energy when it just before it hits the ground it has a velocity therefore the kinetic energy of the plate is half m v square now notice that this plate is in curvilinear translation there is no rotation of the plate. Because the mechanism is such it is a parallelogram mechanism. The mechanism is such that it will not allow rotation this is a parallelogram mechanism therefore this edge and this edge they will remain parallel and that is why there is no rotation of the plate.

Therefore it is only the translational kinetic energy half 2 m velocity of center of mass square plus the final potential energy it is m g h by 2 that is the height from the datum of the center of mass. If you do the simplification and carry out this calculation you will find velocities under root g l. And that must also be equal to length of this member times omega this is purely from kinematics. This member now has become horizontal this is in the final state this member has become horizontal.

And the velocity that we have at this point which we indicate by v which is same as the velocity of the center of mass why? Because the body is not in rotation if there is only translation in a translating body every point moves with the same velocity. Therefore this point C has the same velocity as g which is v and this v must be this length time's omega. And if you calculate omega from here by plugging in the values that are provided you will get omega as 3.5 radians per second that is about determination of omega.

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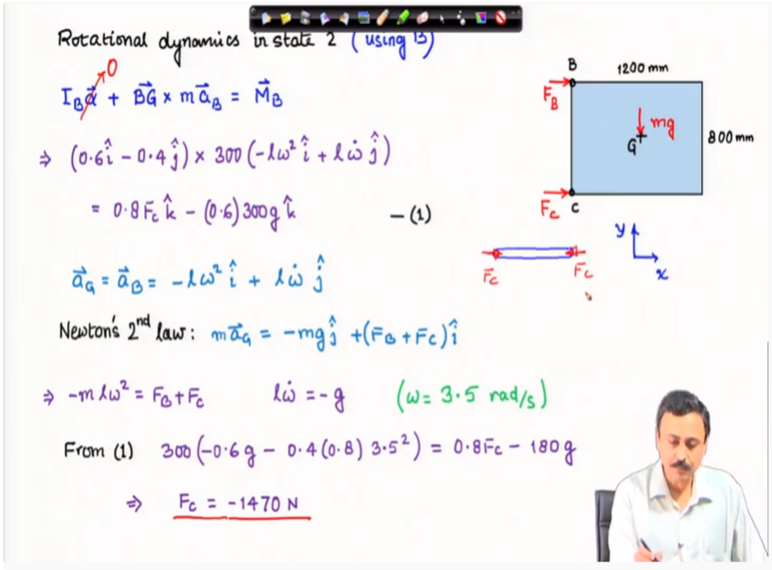


Next we move to determine the forces in that member that connects at C the member D C for that I am going to write the rotational dynamics of this plate in the second configuration or state 2 and I will use the point B. Why I use the point B? Because the angular momentum equation about B is the moment of inertia of the plate about B times angular acceleration + B G cross mass of the plate times acceleration of B that is equal to the net moment about B.

Since we want to find out force in member at point C in member D C we want to find out force in member C. I am considering point B because about that point F C and the weight these are the only 2 forces that contribute to the moment about B. Weight is known the only thing then the unknown would be F C. Also note one more thing since the links are massless therefore they form what are known as 2 force members in mechanics these are called 2 force members.

If this is the that link then the force on that member C D is like this this is a 2 force member therefore the force on the plate at point C will be horizontal because we are dealing with state 2 where the link D C has become horizontal. Now angular acceleration of course is 0 angular velocity of the plate is also 0 therefore I B times alpha that drops out the rest of the terms I can write I can represent B G vector in the coordinate frame that I have chosen.

Because the dimensions of the body are specified cross mass of the plate 300 kilogram times acceleration of point B. Note that B is moving on a circular path with a as the center. Therefore this is moving on this circular path therefore the acceleration of point B will have 2 components 1 is tangential to the path the other is normal to the path. Normal to the path is in the negative x direction therefore and that is the centripetal acceleration.

-l omega square i cap that is the centripetal acceleration term plus this is the tangential acceleration l omega dot j cap. This is angular velocity and angular acceleration of the link. If I write down the right hand side of this angular momentum equation is the moment of all forces about B. 2 forces will contribute 1 is the force at c which is horizontal in this state and the other is the weight and we know the moment arms.

Because the dimensions are specified therefore I have this angular momentum equation note in this angular momentum equation. We still have some unknowns omega dot the angular acceleration of the connecting link that is not known. Therefore what we do is we write down the linear momentum equation Newton's second law for the center of mass of the plate. Therefore we need the acceleration of the center of mass a G and acceleration of center of mass is same as acceleration of point B.

Once again because this body is not rotating the angular velocity is 0 angular acceleration is 0. Therefore we have acceleration of G same as acceleration of B and which is actually same as acceleration of C. If I now write down Newton's second law for the plate for the center of mass of the plate mass times acceleration of center of mass of the plate is equal to the net force acting on the plate we have the weight and the 2 forces in the x direction.

If I put in the expression of acceleration of G which is same as acceleration of B into this equation on the left hand side and look at the terms, the i cap equation. The, i cap equation gives me - m l omega square is the sum of the 2 forces hinge forces this is the, i cap equation and the j cap equation gives me l omega dot is equal to –g. Now I have omega dot from here and omega is already known from our previous calculation.

Therefore I know the left hand side of the first equation as well i cap equation. If I substitute in equation 1 which is the angular momentum equation I can solve for F C turns out to be -1470

Newton. Now this negative sign indicates that the way I have taken the force F C in the positive x direction that is incorrect it is actually in the opposite direction therefore F C is. So according to this figure by Newton's third law I should have F C in this direction and F on the other end of the bar in the opposite direction both are F C the 2 force member.

But because now after calculation we find that the sign is negative which means the bar will be under tension. This figure what I have drawn now shows that it seems that the bar is in compression. But if you reverse the direction of fc the bar is under tensile. The actual thing is the bar will be under tension what is the purpose of this equation? We can find out F B as well from here. Once I have solved for F C I know omega and I can solve for F B so that completes this problem.

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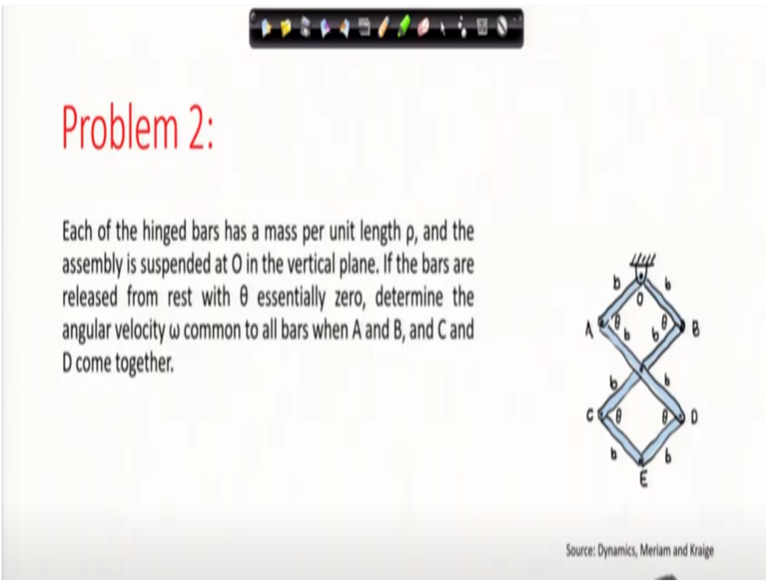


Let us move to the next problem again from the book of Meriam and Kraige each of the hinged bars has a mass per unit length row and the assembly suspended at O in the vertical plane. If the bars are released from rest with theta essentially 0 determine the angular velocity omega common to all bars when A and B and C and B come together. When this mechanism is fully stretched you will find that A and B will come together and similarly C and D will also come together and that will happen when theta is equal to pi and it starts from theta equal to 0.

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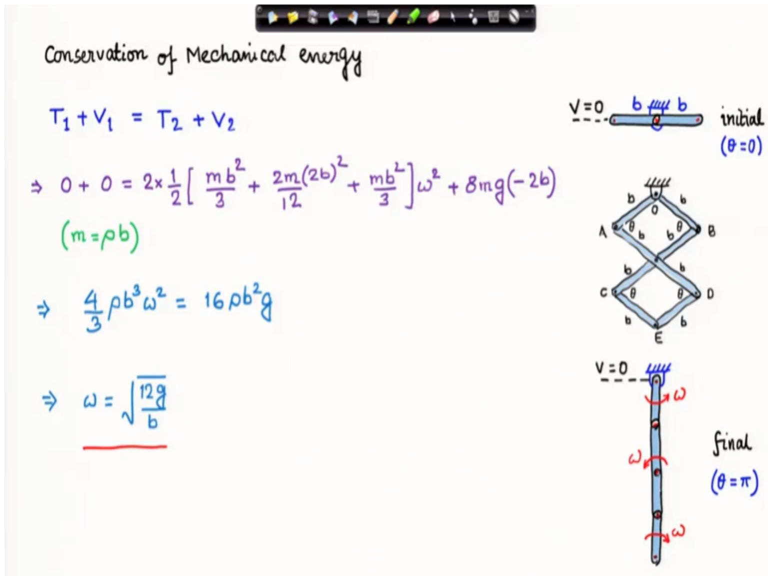


I have drawn the 3 states the starting state the intermediate state and the final state of the mechanism as it moves. This is in a uniform gravitational field there is no other active forces therefore the total mechanical energy of the system will be conserved. In state 1 which is the folded configuration initial and, state 2 is the final configuration which is the stressed configuration we have the total mechanical energy.

Here in state 3, I have shown the direction of angular velocities of some of the links. These links will rotate like this in opposite direction this is going to rotate like this. I have shown some of these omegas in the third state. Note that the potential energy datum has been taken in the initial coinciding with the ceiling at the hinge at the ceiling. The total mechanical energy at the initial state is 0 because there is no kinetic energy and potential energy is also 0 because it coincides with that datum.

In the second state we have kinetic energy and some potential energy let us look at the kinetic energy expression first. As I mentioned these 2 bars are moving inwards like this they are hinged at O. I can write the kinetic energy of this of each bar as half i about o times omega square at the final state I can write down the kinetic energy as half i about O the moment of inertia about O times omega square. And i naught is 1 by 3 m times length square one third m b square and there are 2 such bars.

What I have done is? I have written i o here pulled out 1 half here and omega square here. Now let us look at the bar A D or B C they are identical bars the kinetic energy of these bars in the final state they are actually moving or rotating about this point. This point has 0, velocity at this instant because under fully stretched condition this hinge will come to rest and therefore this bar these 2 bars are rotating about this fixed point.

Therefore I can write the kinetic energy of the bars A D or B C as 1 by 2 i about g times omega square omega remains the same for all bars and i g is one by twelve the mass of this bar is 2 times the mass of the smaller bars because the length is twice. And the length of this bar is 2 b and that is the moment of inertia of the longer bar A D or B C about its center of mass. And hence we have this as i g 1 half is pulled out omega square is pulled out.

Similarly this point e at the bottom the bottom most point has also come to rest. Therefore I can write for the 2 bottom bars. I can write the kinetic energy of half i e omega square and i e is one third m length square. Potential energy this is completely stretched the center of mass of the whole system is at this level.

And from the datum it is at a distance of 2 b in the negative that is why - 2 b and the total mass of the system is 8 m and g is the acceleration due to gravity. Once everything is in place I can make simplifications and finally I Find that common angular velocity as under root 12 g by b the length of the smaller bars this completes this problem.

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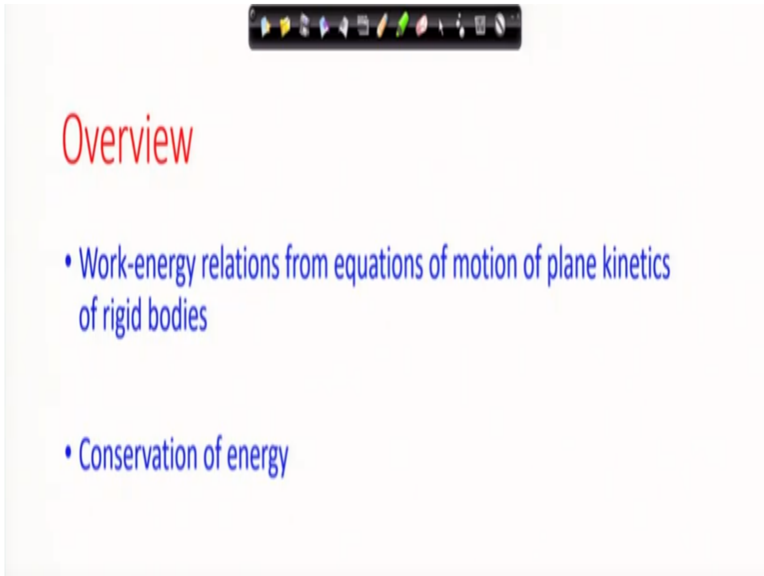


To summarize we have discussed; the work energy relation and used them in some problems to understand their application. And we have looked at conservation of energy solved problems based on conservation of energy with that I will close this lecture.