

**Advanced Dynamics**  
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**Module No # 06**  
**Lecture No # 28**  
**Planar Kinetics of Rigid Bodies – II**

We will continue our discussions on planar kinetics of rigid bodies.

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**Overview**

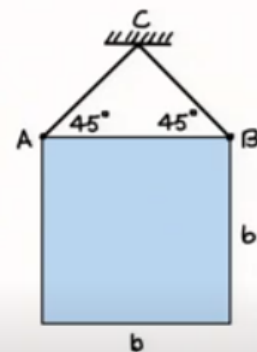
- Plane kinetics of rigid bodies
- Applications of equations of translational and rotational dynamics
- Problems

I am going to take up a few more problems and show you applications of the equations of motion of translational and rotational dynamics for plane kinetics of rigid bodies.

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**Problem 1:**

A uniform 12 kg square panel is suspended from point C by two wires at A and B. If the wire at B suddenly breaks, calculate the tension T in the wire at A immediately after the break occurs.



Source: Dynamics, Meriam and Kraige

The problem statement is shown in the slide above.

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Coordinate system and FBD

Equations of motion (using A)  $\vec{a}_A = a_t \hat{e}_t + \frac{v^2}{\rho} \hat{e}_n$

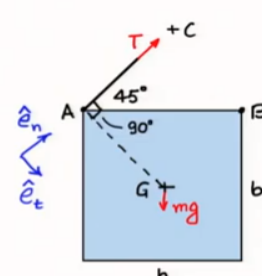
$$I_A \ddot{\alpha} + \underbrace{\vec{r}_{AG} \times m \vec{a}_A}_0 = \vec{M}_A = \frac{b}{\sqrt{2}} \hat{e}_t \times \frac{mg}{\sqrt{2}} (\hat{e}_t - \hat{e}_n)$$

$$\Rightarrow \frac{2}{3} m b^2 \alpha = - \frac{m g b}{2} \Rightarrow \alpha = - \frac{3}{4} \frac{g}{b}$$

Equations of motion (using G)

$$I_G \ddot{\alpha} = \vec{M}_G = - \frac{b}{\sqrt{2}} \hat{e}_t \times T \hat{e}_n$$

$$\Rightarrow \frac{1}{6} m b^2 \alpha = - \frac{b T}{\sqrt{2}} \Rightarrow T = \frac{1}{4\sqrt{2}} m g$$

$$\Rightarrow \underline{T = 20.8 \text{ N}}$$


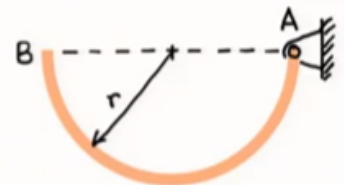
$I_G = \frac{1}{6} m b^2$   
 $I_A = \frac{2}{3} m b^2$

The detailed solution is presented in the slide above. It is important to note that the solution is obtained by writing out the rotational dynamics of the plate about 2 special points: (i) about A, and (ii) about G.

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## Problem 2:

A uniform semi-circular bar of mass  $m$  and radius  $r$  is hinged freely about a horizontal axis through A. If the bar is released from rest in the position shown, where AB is horizontal, determine the initial angular acceleration  $\alpha$  of the bar and the expression for the force exerted on the bar by the pin at A.



Source: Dynamics, Meriam and Kraige

The problem statement is shown in the slide above.

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Coordinate system and FBD

Equations of motion

$$\vec{a}_G = -\rho \dot{\theta}^2 \hat{e}_r + \rho \ddot{\theta} \hat{e}_\theta$$

$$-m\rho \dot{\theta}^2 = -A_r + mg \sin \theta$$

$$m\rho \ddot{\theta} = -A_\theta + mg \cos \theta$$

$$2mr^2 \ddot{\theta} = mg\rho \cos \theta \Rightarrow \ddot{\theta} = \frac{1}{2} \frac{g\rho}{r^2} \cos \theta \quad (\text{using A})$$

At  $t=0$ ,  $\sin \theta = \frac{2r}{\pi\rho}$ ,  $\cos \theta = \frac{r}{\rho}$ ,  $\dot{\theta} = 0$

$$\Rightarrow \ddot{\theta} = \alpha = \frac{g}{2r}$$

$$\Rightarrow A_r = \frac{2mgr}{\pi\rho} \quad A_\theta = \frac{mgr}{\rho} \left( -\frac{1}{2} \frac{\rho^2}{r^2} + 1 \right)$$

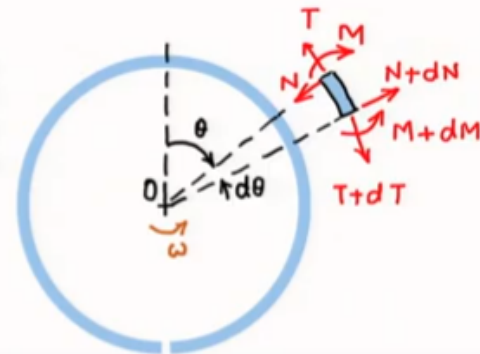
$\rho = r\sqrt{\frac{4}{\pi^2} + 1}$   
 $I_A = 2mr^2$

The detailed solution is provided in the slide above.

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## Problem 3:

A split ring of radius  $r$  and mass per unit length  $\rho$  is rotating with a constant speed  $\omega$  about a vertical axis (perpendicular to the screen) through the center  $O$ . Using the differential element shown, derive expressions of shear force  $N$ , ring tension  $T$  and bending moment  $M$  in terms of the angle  $\theta$ .



Source: Dynamics, Meriam and Kraige

The next problem statement is shown in the slide above.

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Coordinate system and FBD

Equations of motion  $\vec{a} = -r\omega^2 \hat{e}_r$

$$\rho r d\theta \vec{a} = \left[ (N+dN)\cos\frac{d\theta}{2} - N\cos\frac{d\theta}{2} - (T+dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} \right] \hat{e}_r$$

$$+ \left[ (T+dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} + (N+dN)\sin\frac{d\theta}{2} + N\sin\frac{d\theta}{2} \right] \hat{e}_\theta$$

$$\Rightarrow \begin{cases} -\rho r \omega^2 = \frac{dN}{d\theta} - T \\ 0 = \frac{dT}{d\theta} + N \end{cases} \quad \begin{cases} \cos\frac{d\theta}{2} \approx 1 \\ \sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \end{cases}$$

Rotational dynamics

$$0 = M + dM - M + (N+dN)r\frac{d\theta}{2} + Nr\frac{d\theta}{2}$$

$$\Rightarrow 0 = \frac{dM}{d\theta} + Nr$$

The solution starts by considering the free body diagram of the infinitesimal element as shown in the slide above. The equation of motion of the element is written in the plane polar coordinates as shown.

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Coordinate system and FBD

Equations of motion  $\vec{a} = -r\omega^2 \hat{e}_r$

$$\left. \begin{aligned} -pr\omega^2 &= \frac{dN}{d\theta} - T & -(1) \\ 0 &= \frac{dT}{d\theta} + N & -(2) \\ 0 &= \frac{dM}{d\theta} + Nr & -(3) \end{aligned} \right\} \begin{aligned} N(\pi) &= 0 \\ T(\pi) &= 0 \\ M(\pi) &= 0 \end{aligned}$$

From (1) and (2)  $\frac{d^2 N}{d\theta^2} + N = 0$

$$\Rightarrow N = C_1 \cos \theta + C_2 \sin \theta \Rightarrow N = C_2 \sin \theta \quad (N(\pi) = 0)$$

$$\Rightarrow T = C_2 \cos \theta + pr\omega^2 = \underline{pr\omega^2 [\cos \theta + 1]} \quad (T(\pi) = 0 \Rightarrow C_2 = pr\omega^2)$$

$$\Rightarrow \underline{M = pr^3 \omega^2 (\cos \theta + 1)} \quad (M(\pi) = 0)$$

$N = pr\omega^2 \sin \theta$

The obtained equations are then integrated using the boundary conditions as given in the slide above. That completes the solution.

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## Summary

- Plane kinetics of rigid bodies
- Applications of equations of translational and rotational dynamics
- Problems

The discussions are summarized above.