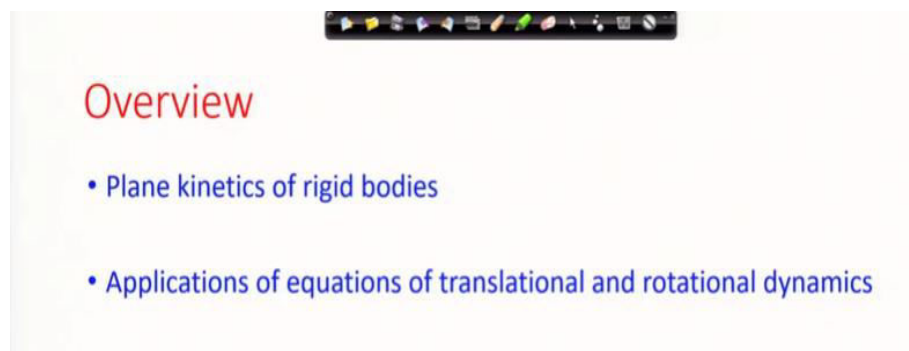


Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Module No # 06
Lecture No # 27
Planar Kinetics of Rigid Bodies – I

In this lecture I am going to discuss about planar kinetics of rigid bodies and we are going to look at problems.

(Refer Slide Time: 00:22)

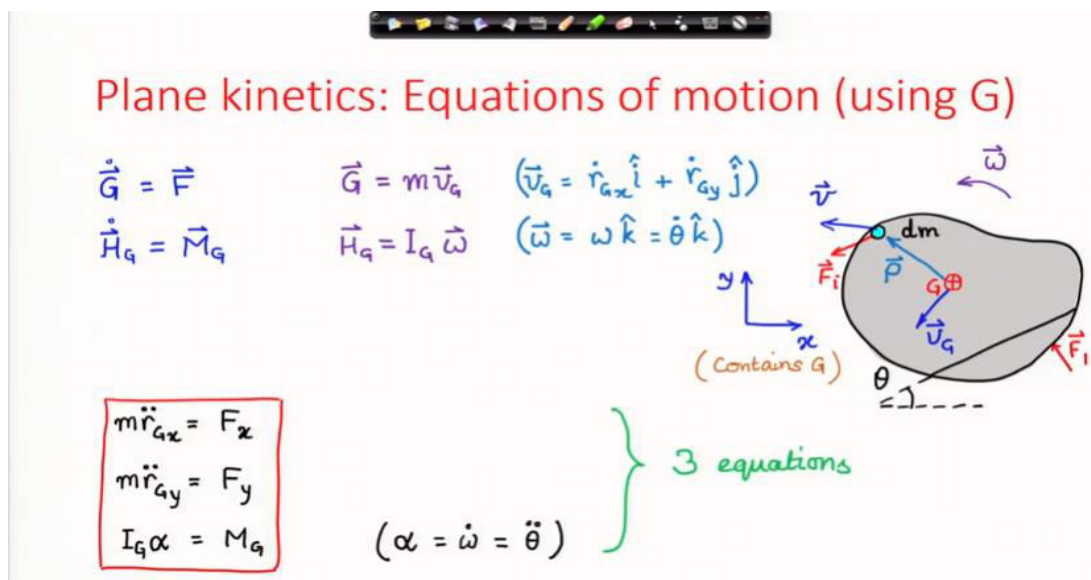


Overview

- Plane kinetics of rigid bodies
- Applications of equations of translational and rotational dynamics

I will show you through these problems some applications of these equations of translational and rotational dynamics.

(Refer Slide Time: 00:30)



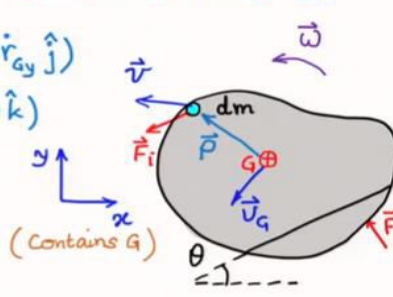
Plane kinetics: Equations of motion (using G)

$$\begin{aligned} \vec{G} &= \vec{F} & \vec{G} &= m \vec{v}_G & (\vec{v}_G &= \dot{r}_{Gx} \hat{i} + \dot{r}_{Gy} \hat{j}) \\ \dot{\vec{H}}_G &= \vec{M}_G & \vec{H}_G &= I_G \vec{\omega} & (\vec{\omega} &= \omega \hat{k} = \dot{\theta} \hat{k}) \end{aligned}$$

$$\begin{aligned} m \ddot{r}_{Gx} &= F_x \\ m \ddot{r}_{Gy} &= F_y \\ I_G \alpha &= M_G \end{aligned}$$

($\alpha = \dot{\omega} = \ddot{\theta}$)

} 3 equations



(Contains G)

(Refer Slide Time: 02:30)

Plane kinetics: Equations of motion (using O)

$$\dot{\vec{G}} = \vec{F}$$

$$\dot{\vec{H}}_O = \vec{M}_O$$

$$\vec{G} = m\vec{v}_G$$

$$\vec{H}_O = \vec{r}_G \times \vec{G} + I_G \vec{\omega}$$

(Contains G)

$$\left. \begin{aligned} ma_x &= F_x \\ ma_y &= F_y \\ I_G \alpha + m(r_{Gx} a_y - r_{Gy} a_x) &= M_O \end{aligned} \right\} 3 \text{ equations}$$

(Refer Slide Time: 04:28)

Plane kinetics: Equations of motion (using P)

$$\dot{\vec{G}} = \vec{F}$$

$$\dot{\vec{H}}_P^{rel} + \vec{\rho}'_G \times m\ddot{\vec{r}}_P = \vec{M}_P$$

$$\vec{G} = m\vec{v}_G$$

$$\dot{\vec{H}}_P^{rel} = I_P \vec{\omega} \hat{k}$$

$$\dot{\vec{G}} = \vec{F}$$

$$I_P \dot{\vec{\omega}} + \vec{\rho}'_G \times m\ddot{\vec{r}}_P = \vec{M}_P$$

(Refer Slide Time: 05:07)

Plane kinetics: Equations of motion (using P)

$$\dot{\vec{G}} = \vec{F}$$

$$I_P \dot{\omega} + \vec{r}_{G'} \times m \ddot{\vec{r}}_P = \vec{M}_P$$

$$\dot{\vec{G}} = \vec{F}$$

$$I_P \dot{\omega} = \vec{M}_P$$

(\vec{F} includes \vec{F}_R)

Fixed axis rotation.

The 4 slides above recapitulate the discussions on planar kinetics of prismatic bodies (lamina).

(Refer Slide Time: 05:44)

Problem 1:

A passenger car of an overhead monorail system is driven by one of its two small wheels A or B. Determine which driving wheel will give greater acceleration without slip, and compute the maximum acceleration if the effective coefficient of friction is limited to 0.25 between the rail and the wheels. Neglect the small mass of the wheels.

Source: Dynamics, Meriam and Kraige

The problem statement is shown in the slide above.

(Refer Slide Time: 06:55)

Coordinate system and FBD

(i) Powered wheel B

Equations of motion

$$ma_x = f_B$$

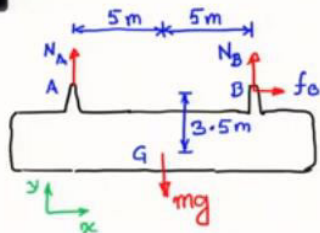
$$0 = N_A + N_B - mg$$

$$I_A \vec{\alpha} + \vec{r}_{AG} \times m \vec{a}_A = \vec{M}_A \quad (\text{using A})$$

$$\Rightarrow (5\hat{i} - 3.5\hat{j}) \times ma_x \hat{i} = -5mg\hat{k} + (10)N_B\hat{k}$$

$$\Rightarrow N_B = \frac{1}{10} (3.5ma_x + 5mg)$$

$$\text{Also, } f_B = ma_x$$

$$\left. \begin{array}{l} N_B = \frac{1}{10} (3.5ma_x + 5mg) \\ f_B = ma_x \end{array} \right\} \frac{|f_B|}{N_B} \leq 0.25 \Rightarrow a_x \leq 1.34 \text{ m/s}^2$$


The coordinate system and the free body diagram are shown in the slide above. To begin with, we consider that wheel B is powered. Then the calculation to determine the maximum acceleration is presented.

(Refer Slide Time: 10:58)

Coordinate system and FBD

(ii) Powered wheel A

Equations of motion

$$ma_x = f_A$$

$$0 = N_A + N_B - mg$$

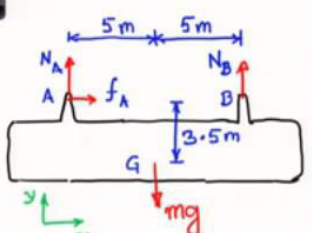
$$I_B \vec{\alpha} + \vec{r}_{BG} \times m \vec{a}_B = \vec{M}_B \quad (\text{using B})$$

$$\Rightarrow (-5\hat{i} - 3.5\hat{j}) \times ma_x \hat{i} = 5mg\hat{k} - (10)N_A\hat{k}$$

$$\Rightarrow N_A = \frac{1}{10} (-3.5ma_x + 5mg)$$

$$\text{Also, } f_A = ma_x$$

$$\left. \begin{array}{l} N_A = \frac{1}{10} (-3.5ma_x + 5mg) \\ f_A = ma_x \end{array} \right\} \frac{|f_A|}{N_A} \leq 0.25 \Rightarrow a_x \leq 1.13 \text{ m/s}^2$$

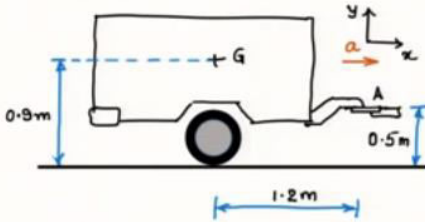
$$a_x^B = 1.34 \text{ m/s}^2 \Rightarrow \underline{a_x^B > a_x^A}$$


The calculation of maximum acceleration when wheel A is powered is shown above. It is found that we can achieve higher acceleration when wheel B is powered.

(Refer Slide Time: 13:06)

Problem 2:

The loaded trailer, having a mass of 900 kg with G as the center of mass, is attached at A to a rear-bumper hitch. If the car and trailer reach a velocity of 60 km/h on a level road in a distance of 30 m starting from rest with constant acceleration, compute the vertical component of the force supported by the hitch at A during this motion. Neglect any resistance at the relatively light wheels.



Source: Dynamics, Meriam and Kraige

The statement of the next problem is shown in the slide above.

(Refer Slide Time: 14:27)

Coordinate system and FBD

Equations of motion

$$ma = A_x$$

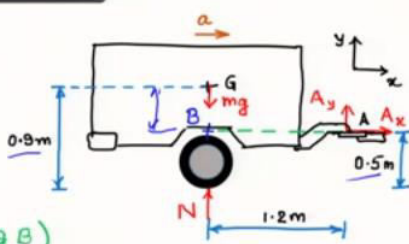
$$0 = A_y - mg + N$$

$$I_B \ddot{\theta} + \vec{r}_{BG} \times m \ddot{\alpha}_B = \vec{M}_B = 1.2 \hat{i} \times A_y \hat{j} \quad (\text{using B})$$

$$\Rightarrow 0.4 \hat{j} \times 900 (4.63 \hat{i}) = 1.2 A_y \hat{k}$$

$$\Rightarrow \underline{A_y = -1388.9 \text{ N}}$$

$\ddot{\alpha}_G = a \hat{i}$



$u = 0$
 $v = 60 \text{ km/h} = \frac{50}{3} \text{ m/s}$
 $a = \frac{1}{2(30)} \left(\frac{50}{3} \right)^2 = 4.63 \text{ m/s}^2$

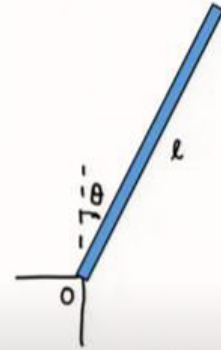
$A_y = ?$

The detailed solution is presented in the slide above.

(Refer Slide Time: 18:33)

Problem 3:

A uniform slender bar of mass m and length l , released from rest in the vertical position, pivots on its short flat end about the corner at O as shown. (a) If the bar is observed to slip when $\theta = 30^\circ$, find the coefficient of static friction μ_s between the bar and the corner. (b) If the end of the bar is notched (so that it cannot slip), find the angle θ at which contact between the bar and the corner ceases.



Source: Dynamics, Merriam & Kraige

The next problem statement is shown above.

(Refer Slide Time: 20:31)

Coordinate system and FBD

Equations of motion

$$\vec{a}_G = -\frac{l}{2}\ddot{\theta}\hat{e}_r + \frac{l}{2}\ddot{\theta}\hat{e}_\theta$$

$$-m\frac{l}{2}\dot{\theta}^2 = F_r - mg\cos\theta \quad (1)$$

$$m\frac{l}{2}\ddot{\theta} = -F_\theta + mg\sin\theta$$

$$\frac{1}{3}ml^2\ddot{\theta} = mg\frac{l}{2}\sin\theta \quad (\text{using } 0)$$

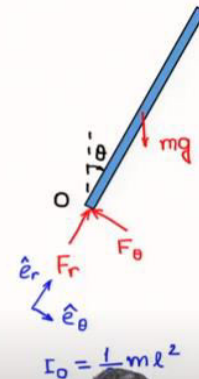
$$\Rightarrow \ddot{\theta} = \frac{3}{2}\frac{g}{l}\sin\theta$$

$$\Rightarrow \frac{d}{d\theta}\left(\frac{1}{2}\dot{\theta}^2\right) = \frac{3}{2}\frac{g}{l}\sin\theta$$

$$\Rightarrow \dot{\theta}^2 = \frac{3g}{l}(1 - \cos\theta)$$

$$\text{Substituting in (1)} \quad F_r = \frac{1}{2}mg(5\cos\theta - 3)$$

$$\left. \begin{array}{l} m\frac{l}{2}\ddot{\theta} = -F_\theta + mg\sin\theta \\ \frac{1}{3}ml^2\ddot{\theta} = mg\frac{l}{2}\sin\theta \end{array} \right\} F_\theta = \frac{1}{4}mg\sin\theta$$



(Refer Slide Time: 24:55)

Contact forces

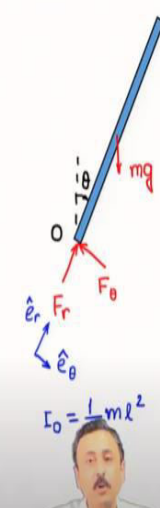
$$F_\theta = \frac{1}{4} mg \sin \theta$$

$$F_r = \frac{1}{2} mg (5 \cos \theta - 3)$$

At the point of slipping $\frac{F_\theta}{F_r} = \mu_s$

$$\Rightarrow \mu_s = \frac{\sin \theta}{2(5 \cos \theta - 3)} \bigg|_{\theta=30^\circ} = 0.188$$

Contact ceases when $F_r = 0$

$$\Rightarrow \cos \theta = \frac{3}{5} \Rightarrow \theta = 53.1^\circ$$


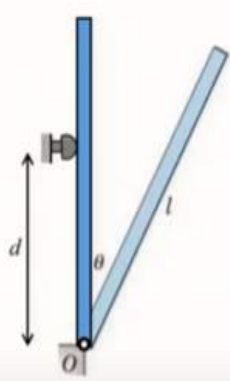
$I_O = \frac{1}{2} ml^2$

The 2 slides above present the detailed solution.

(Refer Slide Time: 26:13)

Problem 4:

A door stop is placed at a suitable location to prevent the door from hitting the wall. Determine d so that when the door hits the stop at a certain angular velocity, the reaction at the hinge O is minimized.



A problem related to the positioning of a door stop is shown in the slide above.

(Refer Slide Time: 27:21)

Coordinate system and FBD

Equations of motion

$$\vec{a}_G = -\frac{l}{2}\ddot{\theta}^2 \hat{e}_r + \frac{l}{2}\ddot{\theta} \hat{e}_\theta$$

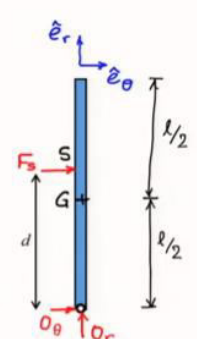

$$-m\frac{l}{2}\ddot{\theta}^2 = 0_r$$

$$m\frac{l}{2}\ddot{\theta} = 0_\theta + F_s$$

$$I_s \ddot{\alpha} + \vec{r}_{SG} \times m \vec{a}_S = \vec{M}_S = -d \hat{e}_r \times 0_\theta \hat{e}_\theta \quad (\text{using } S)$$

$$\vec{a}_S = -d\ddot{\theta}^2 \hat{e}_r + d\ddot{\theta} \hat{e}_\theta$$

$$\Rightarrow I_s \ddot{\theta} + \left[-\left(d - \frac{l}{2}\right) \hat{e}_r\right] \times m(-d\ddot{\theta}^2 \hat{e}_r + d\ddot{\theta} \hat{e}_\theta) = -d 0_\theta \hat{e}_z$$

$$\Rightarrow \left[I_s - m\left(d - \frac{l}{2}\right)d\right] \ddot{\theta} = -d 0_\theta$$



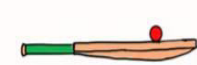
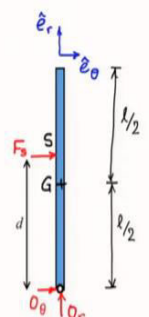

(Refer Slide Time: 31:15)

$$\left[I_s - m\left(d - \frac{l}{2}\right)d\right] \ddot{\theta} = -d 0_\theta$$

Minimum hinge reaction $\Rightarrow 0_\theta = 0$

$$\Rightarrow I_s - m\left(d - \frac{l}{2}\right)d = 0$$

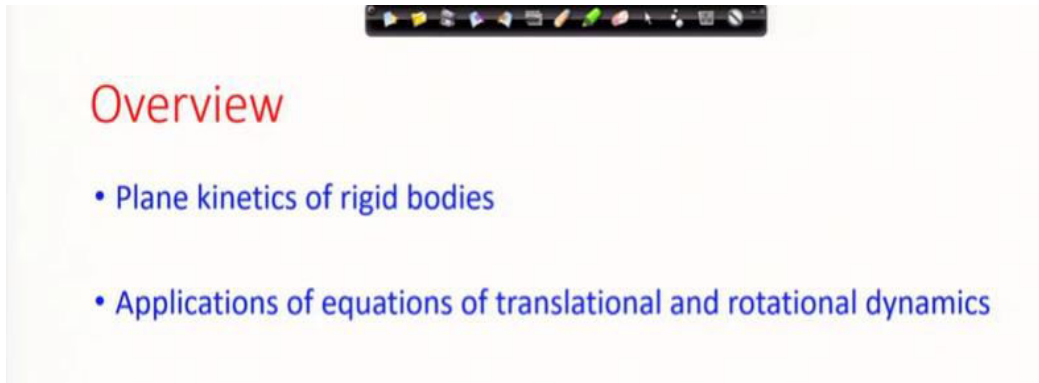
$$\Rightarrow \frac{ml^2}{12} + m\left(d - \frac{l}{2}\right)^2 - m\left(d - \frac{l}{2}\right)d = 0$$

$$\Rightarrow \underline{d = \frac{2}{3}l} \quad (S: \text{Center of Percussion})$$




The detailed solution steps are presented in the 2 slides above.

This problem has important implications in various other applications as well. For example in a cricket bat if a batsman can get the ball at the center of percussion of the bat (sweet spot) with respect to his/her hand (considered as a hinge), then the batsman will not feel any impulsive reaction of the ball. The same thing also applies for the tennis racquets.

(Refer Slide Time: 34:23)



Overview

- Plane kinetics of rigid bodies
- Applications of equations of translational and rotational dynamics

The discussions are summarized in the slide above.