Advanced Dynamics
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Module No # 06
Lecture No # 26
Kinetics of a system of Particles – Extension of Rigid Bodies

In this lecture, I am going to continue on kinetics of a system of particles, and towards the end of the lecture I am going to show how the equations governing the kinetics of a system of particles can be extended to that of rigid bodies.

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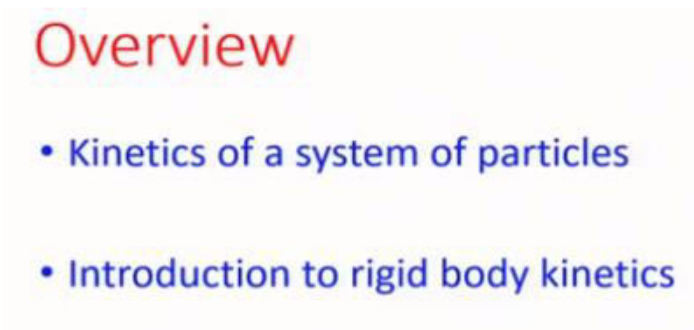

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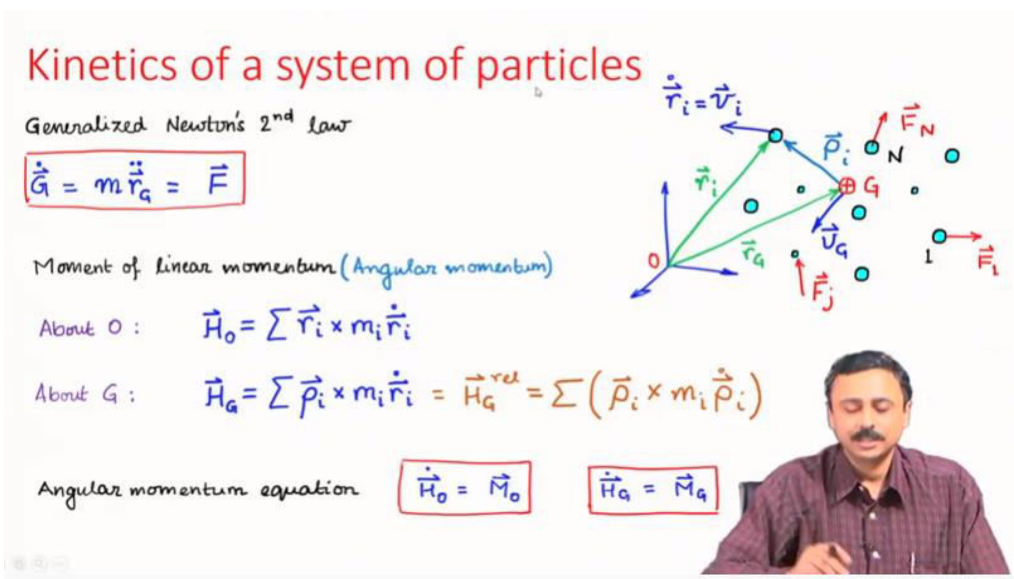

The above slide recapitulates the generalized Newton's second law for a system of particles. The definition of moment of linear momentum or what is also known as the angular momentum is also presented above.

The linear momentum equation, which is the generalized Newton's second law tells us something about how this set is moving in terms of the motion of the center of mass. On the other hand, the angular momentum equation tells us in some way how the orientation of the system is changing though orientation for a system of particle is ill defined in some sense. But as we move on we will show that, for a rigid body, this gives us a very concrete way of knowing the orientation of the body.

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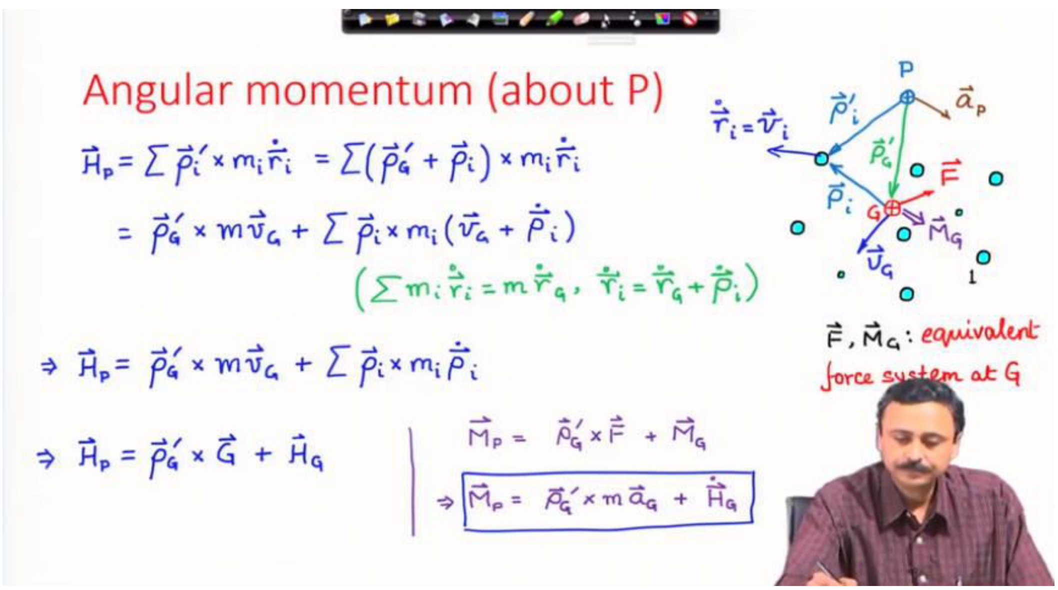

The calculation of angular momentum about an arbitrary point P as shown in the slide above.

$$\vec{H}_P^{rel} = \sum \vec{\rho}_i' \times m_i \dot{\vec{\rho}}_i' \qquad \dot{\vec{\rho}}_i' = (\dot{\vec{r}}_i - \dot{\vec{r}}_P)$$

Differentiating with respect to time

$$\dot{\vec{H}}_P^{rel} = \sum \vec{\rho}_i' \times m_i (\ddot{\vec{r}}_i - \ddot{\vec{r}}_P)$$

$$\Rightarrow \dot{\vec{H}}_P^{rel} = \sum \vec{\rho}_i' \times \vec{F}_i - \sum m_i \vec{\rho}_i' \times \ddot{\vec{r}}_P \qquad \left(\sum_i \vec{\rho}_i' \times \sum_j f_{ij} = 0\right)$$

$$\Rightarrow \dot{\vec{H}}_P^{rel} = \vec{M}_P - m\vec{\rho}_G' \times \ddot{\vec{r}}_P \qquad \left(\sum m_i \vec{\rho}_i' = m\vec{\rho}_G'\right)$$

$$\Rightarrow \boxed{\dot{\vec{H}}_P^{rel} + \vec{\rho}_G' \times m\ddot{\vec{r}}_P = \vec{M}_P}$$

Next, we look at the relative angular momentum about P, as shown in the slide above. The time derivative of the relative angular momentum leads to the corresponding angular momentum equation.

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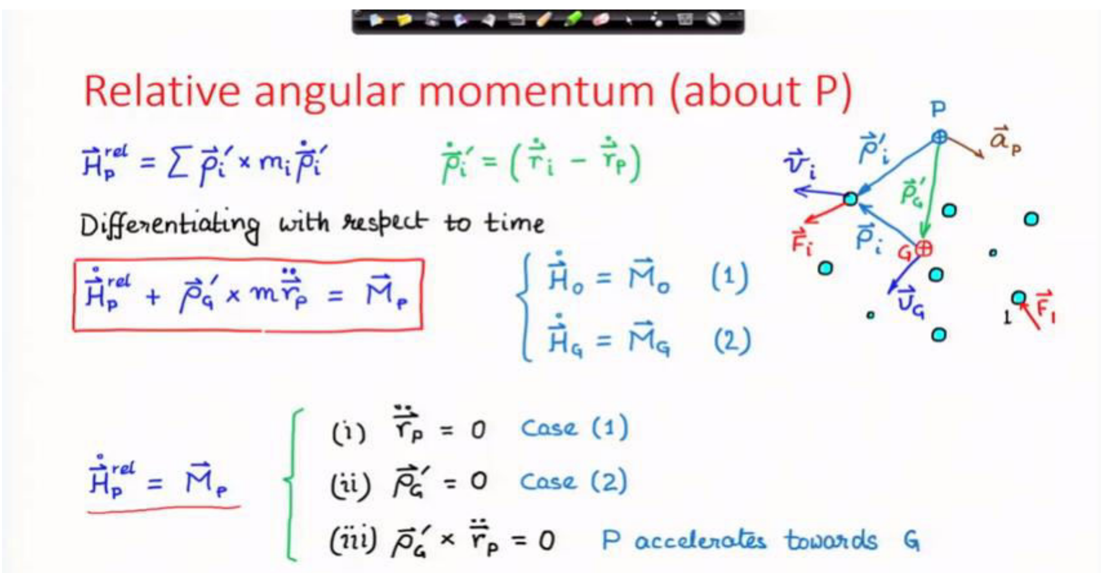

We obtain 3 special cases as shown above for which the angular momentum equation takes a particularly simple form. These cases are: (i) when P is a fixed point in inertial space, (ii) when P is the center of mass of the system, and (iii) when P is accelerating towards/away from the center of mass.

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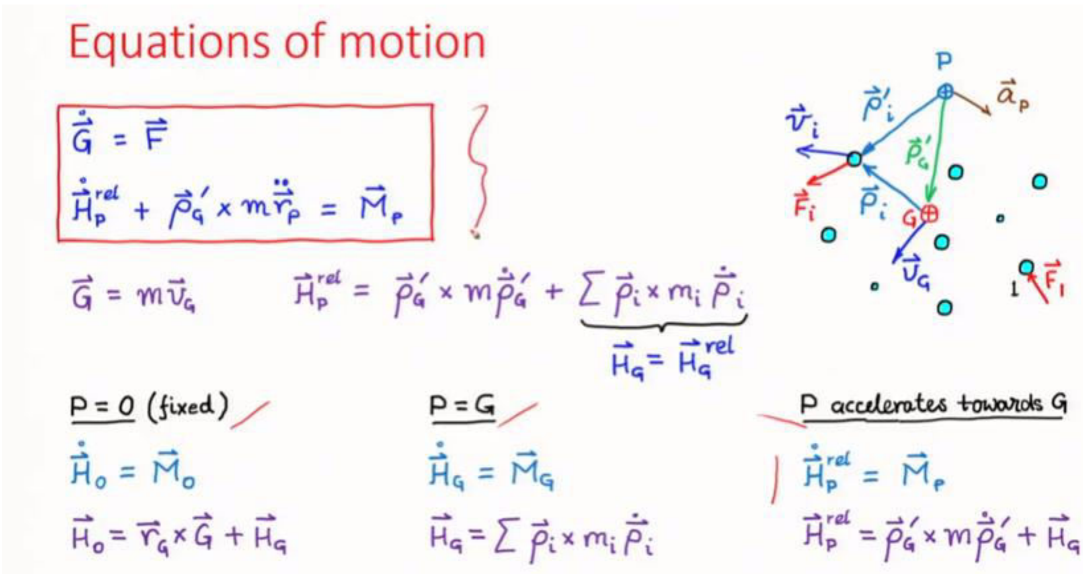

The complete set of equations governing the dynamics of a system of particles is summarized in the slide above.

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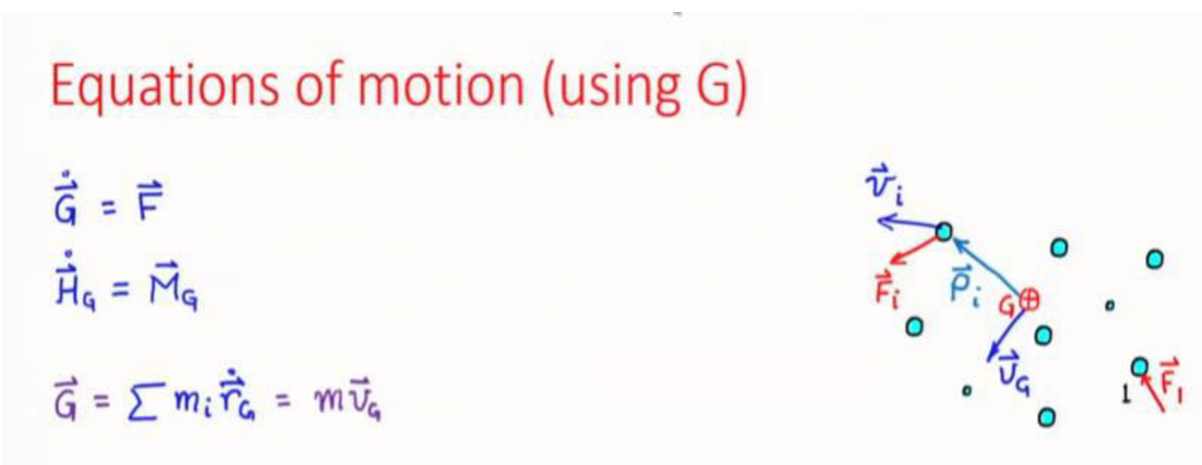

Now let us go back to equation of motion that we have derived about G now for a system of particles.

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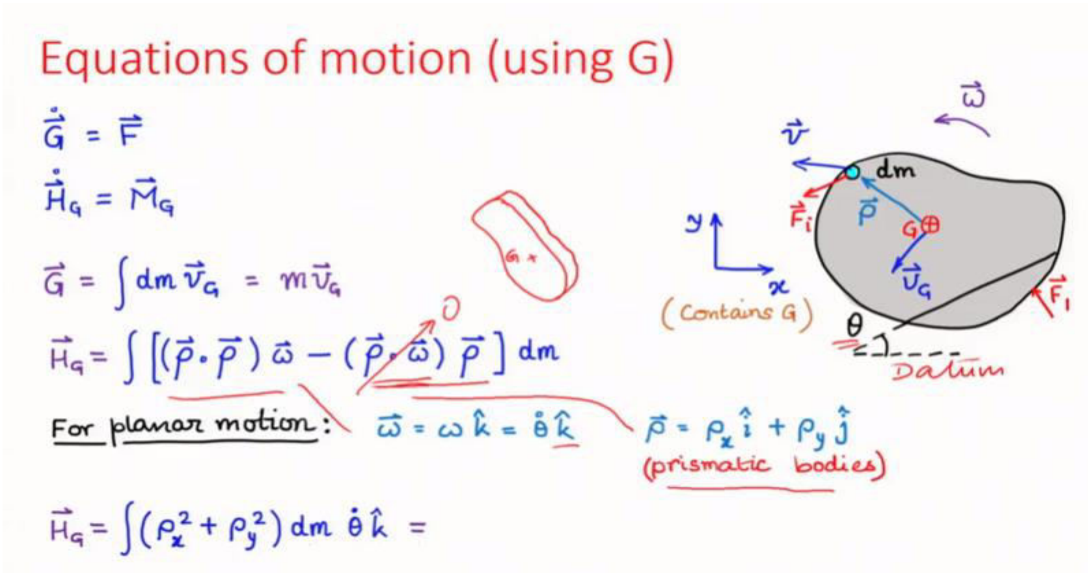

When we go to rigid bodies the summation of linear momenta just gets converted to an integral as shown above. We can similarly express the angular momentum about the center of mass as an integral as

$$\vec{H}_G = \int \vec{\rho} \times \dot{\vec{\rho}}\, dm = \int \vec{\rho} \times (\vec{\omega} \times \vec{\rho})\, dm$$

$$\Rightarrow \vec{H}_G = \int \left[(\vec{\rho}\cdot\vec{\rho})\,\vec{\omega} - (\vec{\rho}\cdot\vec{\omega})\,\vec{\rho}\,\right] dm \qquad \left(\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}\right)$$

If we consider the planar motion of an extruded body (lamina) we can simplify further to obtain

For planar motion: $\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}$ $\qquad \vec{\rho} = \rho_x\hat{i} + \rho_y\hat{j}$ (prismatic bodies)

$$\vec{H}_G = \int (\rho_x^2 + \rho_y^2)\, dm\, \dot{\theta}\hat{k} = I_G\,\vec{\omega}$$

I_G: Moment of Inertia about G

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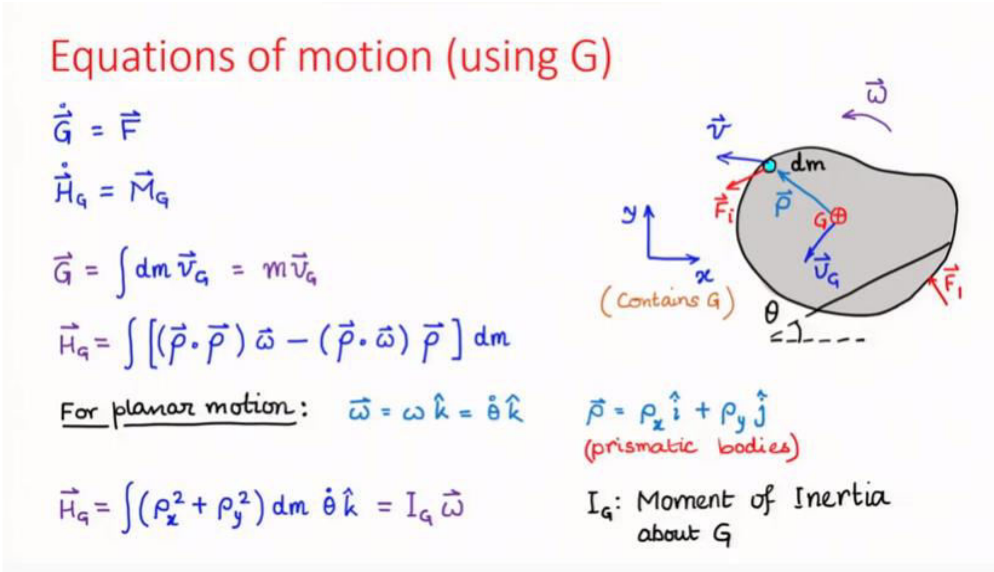

The angular momentum about the center of mass of a lamina is presented in the slide above.

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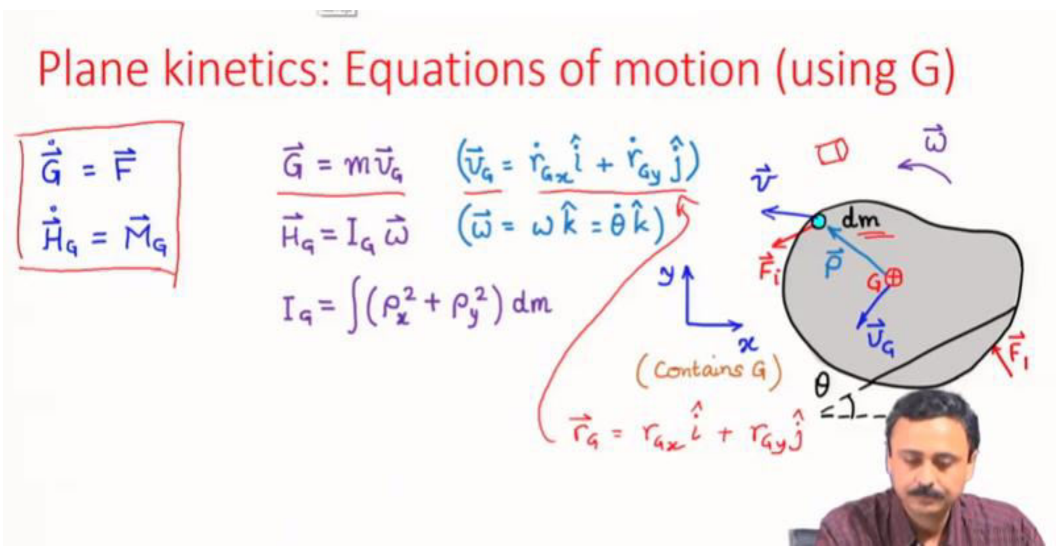

The complete equations of motion of a lamina in planar motion is presented above.

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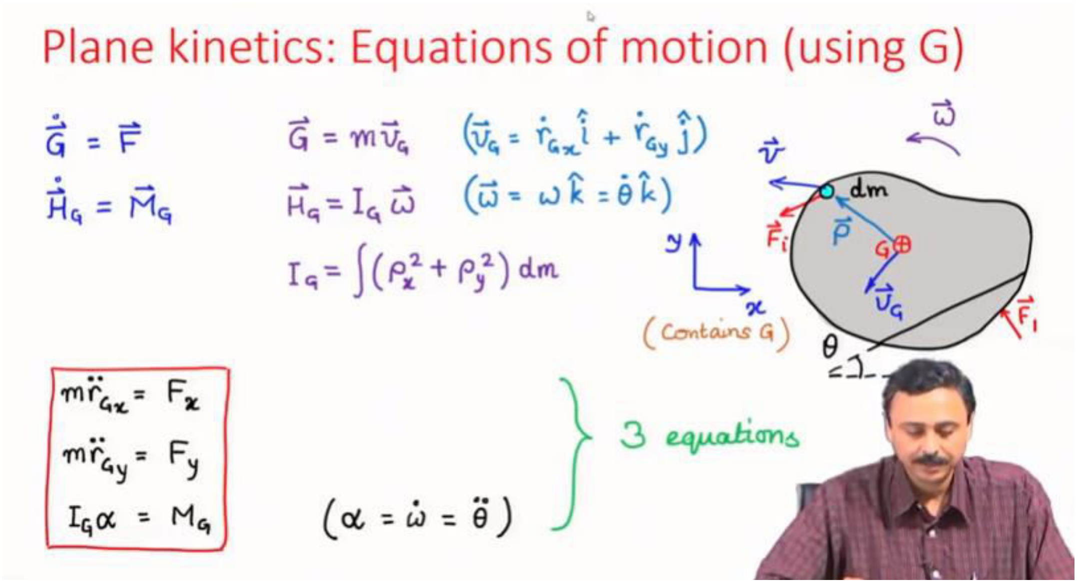

The 3 scalar equations obtained from the vector equations are presented in the slide above.

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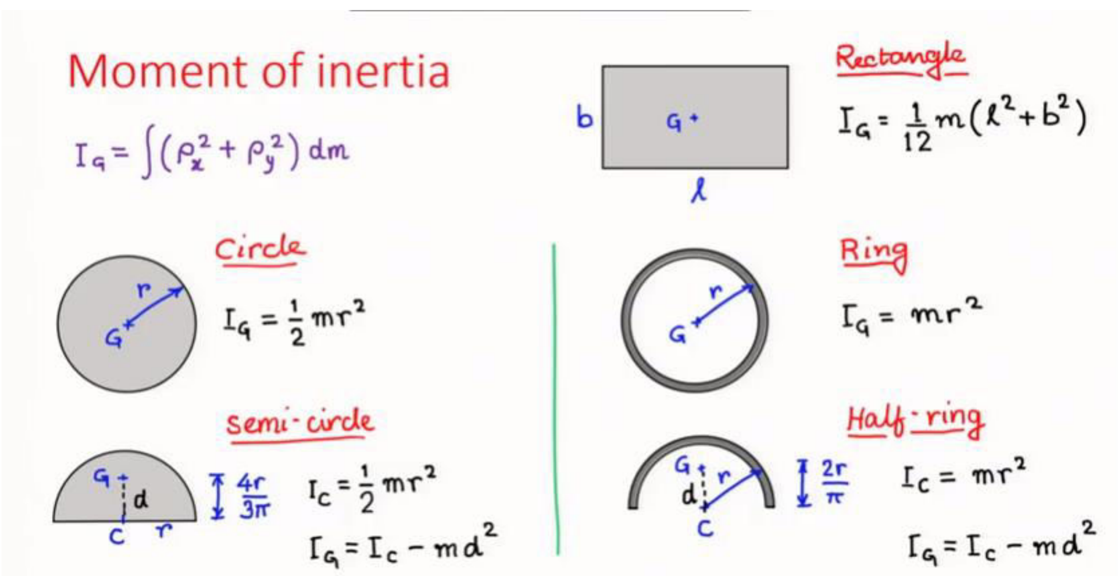

Moment of inertia of some typical bodies are shown in the slide above.

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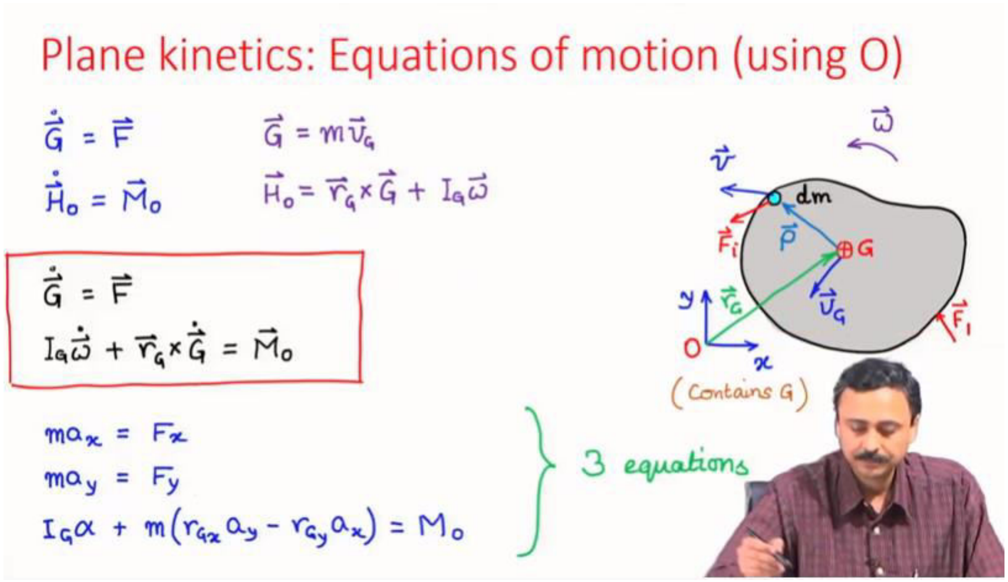

Now we look at the equations of motion of plane kinetics of a prismatic rigid body about a fixed point O. This is presented in the slide above.

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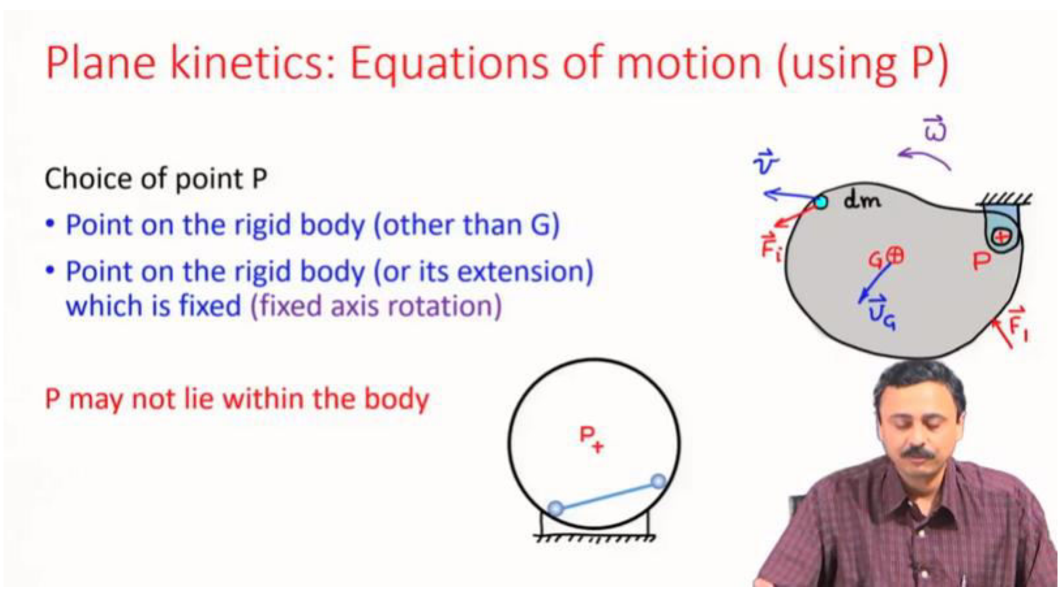

Now we consider a general point P. There can be 2 possible choices of P. The point P can be a point on the rigid body which is not G, or it can be a point on the rigid body or its extension which is fixed.

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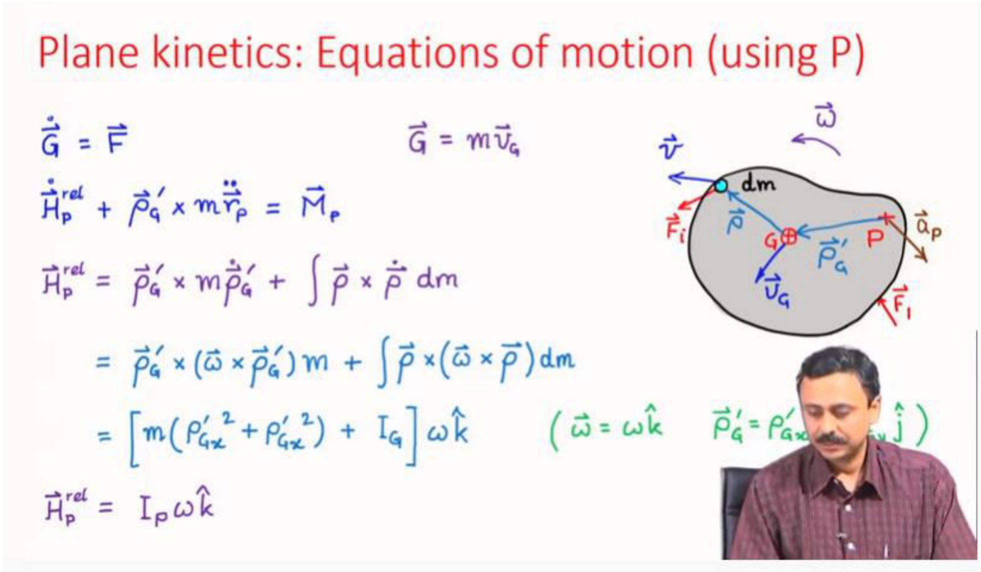

The above slide shows the calculation of the relative angular momentum of the body about the point P fixed to the body.

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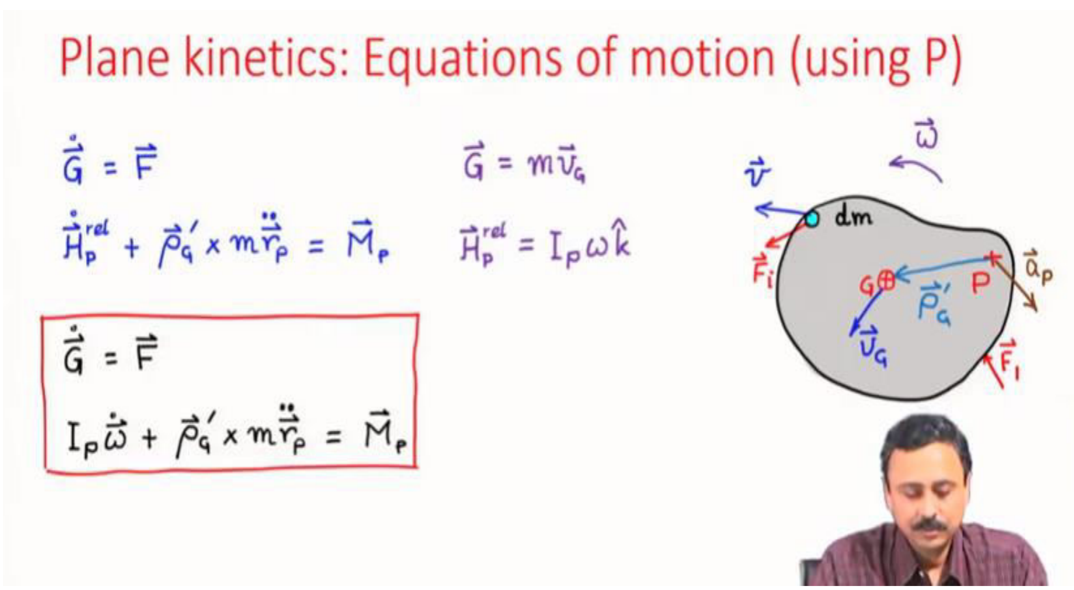

In the first case when P belonging to the rigid body is not fixed in inertial space, the equations of motion of the body are as shown above.

Next, we consider the case when P is a point of the rigid body which is fixed (hinged) in the inertial space. Then we must be careful in drawing the free body diagram of the body and consider the forces at the hinge. The generalized Newton's second law must include these forces on the right hand side.

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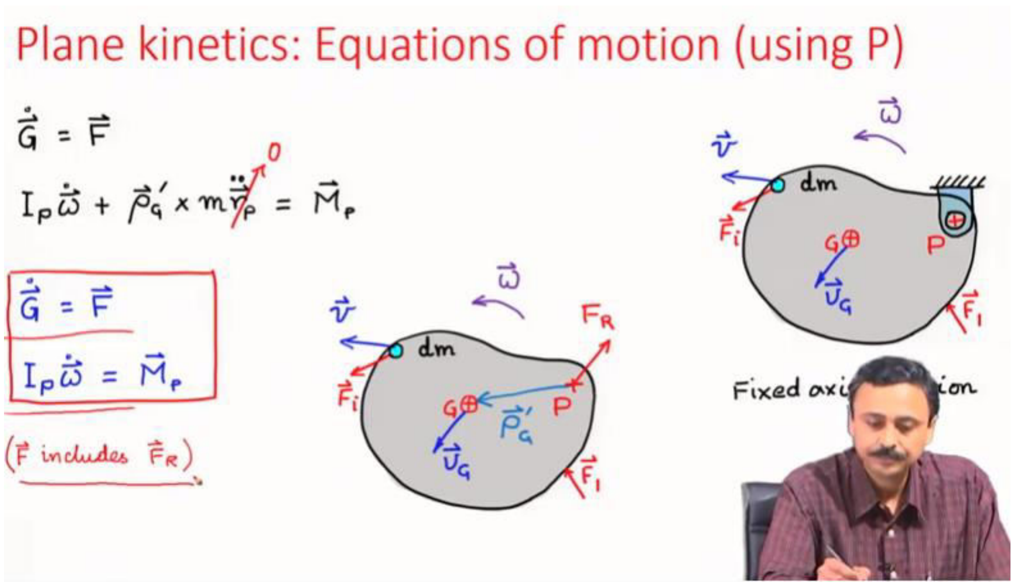

The equations of motion are shown in the slide above.

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The discussions are summarized in the slide above.