

Advanced Dynamics
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Module No # 05
Lecture No # 25
Kinetics of a System of Particles – III

We will continue discussing the kinetics of a system of particles.

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Overview

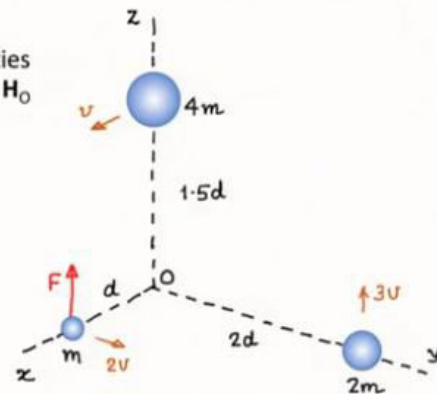
- Kinetics of a system of particles
- Problems

In this lecture we will solve problems based on what we discussed in the past 2 lectures.

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Problem 1:

The system of three particles has the indicated masses, velocities and external force as shown. Determine \mathbf{r}_G , $d\mathbf{r}_G/dt$, $d^2\mathbf{r}_G/dt^2$, \mathbf{T} , \mathbf{H}_O and $d\mathbf{H}_O/dt$ for this system.



Source: Dynamics, Meriam and Kraige

The above slide presents the first problem.

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$$\vec{r}_G = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{7m} [4m(1.5d)\hat{k} + 2m(2d)\hat{j} + md\hat{i}]$$

$$= \frac{d}{7} [4\hat{i} + 6\hat{j} + \hat{k}]$$

$$\dot{\vec{r}}_G = \frac{\sum m_i \dot{\vec{r}}_i}{\sum m_i} = \frac{1}{7m} [4m(v\hat{i}) + 2m(3v\hat{k}) + m(2v\hat{j})]$$

$$= \frac{v}{7} [4\hat{i} + 2\hat{j} + 6\hat{k}]$$

$$m\ddot{\vec{r}}_G = \vec{F} \Rightarrow \ddot{\vec{r}}_G = \frac{1}{m} F\hat{k}$$

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$$T = \sum \frac{1}{2} m_i v_i^2 = \frac{1}{2} (4m) v^2 + \frac{1}{2} (2m) (3v)^2 + \frac{1}{2} m (2v)^2$$

$$= 13mv^2$$

Decomposition of Kinetic energy

$$T = \underbrace{\frac{1}{2} m v_G^2}_{T_T} + \underbrace{\sum \frac{1}{2} m_i \dot{\vec{r}}_i \cdot \dot{\vec{r}}_i}_{T_R} = 13mv^2$$

$$T_T = \frac{1}{2} m \dot{\vec{r}}_G \cdot \dot{\vec{r}}_G = \frac{1}{2} (7m) \frac{v^2}{49} [56] = 4mv^2$$

$$(\dot{\vec{r}}_G = \frac{v}{7} [4\hat{i} + 2\hat{j} + 6\hat{k}])$$

$$\Rightarrow T_R = 9mv^2 = \sum \frac{1}{2} m_i \dot{\vec{r}}_i \cdot \dot{\vec{r}}_i \quad (\dot{\vec{r}}_i = \vec{v}_i - \dot{\vec{r}}_G)$$

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$$\begin{aligned}\vec{H}_0 &= \sum \vec{r}_i \times m_i \vec{v}_i \\ &= 1.5d\hat{k} \times 4m v\hat{i} + 2d\hat{j} \times 2m(3v\hat{k}) + d\hat{i} \times m(2v\hat{j}) \\ &= mvd[12\hat{i} + 6\hat{j} + 2\hat{k}]\end{aligned}$$

$$\dot{\vec{H}}_0 = \vec{M}_0 = d\hat{i} \times F\hat{k} = -dF\hat{j}$$

Decomposition of angular momentum

$$\vec{H}_0 = \vec{r}_G \times m\dot{\vec{r}}_G + \underbrace{\sum \vec{p}_i \times m_i \dot{\vec{p}}_i}_{\vec{H}_G} = mvd[12\hat{i} + 6\hat{j} + 2\hat{k}]$$

$$\vec{r}_G \times m\dot{\vec{r}}_G = \frac{d}{7}[\hat{i} + 4\hat{j} + 6\hat{k}] \times (7m) \frac{v}{7}[4\hat{i} + 2\hat{j} + 6\hat{k}] = \frac{mvd}{7}[12\hat{i} + 18\hat{j} - 14\hat{k}]$$

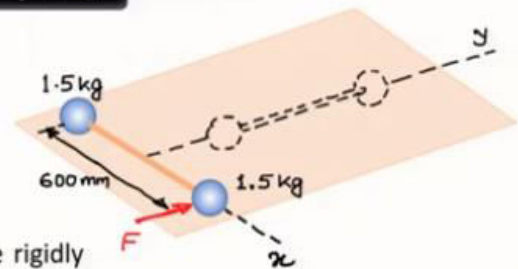
$$\Rightarrow \vec{H}_G = \frac{mvd}{7}[72\hat{i} + 24\hat{j} + 28\hat{k}]$$

The detailed solution is presented in the 3 slide above.

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Problem 2:

Two spheres (treated as particles) of mass 1.5 kg each are rigidly connected to a 600 mm rigid rod of negligible mass and initially at rest on a smooth horizontal surface oriented along the x-axis as shown. A force F along the y-axis direction is suddenly applied that imparts an impulse of 10 Ns during a negligibly short period of time. As the spheres pass the y-axis aligned dashed configuration, calculate the velocity of each sphere.



Source: Dynamics, Meriam and Kraige

The second problem is presented in the slide above.

We use the impulse momentum relation for the center of mass of the system. The change of linear momentum of the center of mass is equal to the impulse of force that acts on the system. Using this idea, we obtain

Impulse - momentum form

$$\Delta \vec{G} = \vec{G}_2 - \vec{G}_1 = \int_{t_1}^{t_2} \vec{F} dt$$

$$\Rightarrow m v_x \hat{i} + m v_y \hat{j} - 0 = 10 \hat{j} \text{ Ns}$$

$$\Rightarrow v_x = 0, \quad v_y = \frac{10}{3} \text{ m/s}$$

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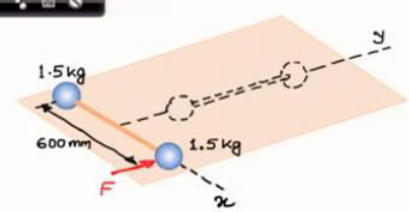
Angular impulse-momentum form

$$\Delta \vec{H}_G = \vec{H}_{G2} - \vec{H}_{G1} = \int_{t_1}^{t_2} \vec{M}_G dt$$

$$\begin{aligned} \vec{H}_G &= \sum \vec{r}_i \times m_i \vec{v}_i \\ &= \sum \vec{r}_i \times m_i (\vec{\omega} \times \vec{r}_i) \\ &= 0.3 \hat{i} \times (1.5) (\omega \hat{k} \times 0.3 \hat{i}) \\ &\quad + (-0.3 \hat{i}) \times (1.5) [\omega \hat{k} \times (-0.3 \hat{i})] \\ &= 0.27 \omega \hat{k} \text{ kgm}^2/\text{s} \end{aligned}$$

$$\Rightarrow 0.27 \omega \hat{k} - 0 = 0.3 \hat{i} \times 10 \hat{j} \text{ Nms}$$

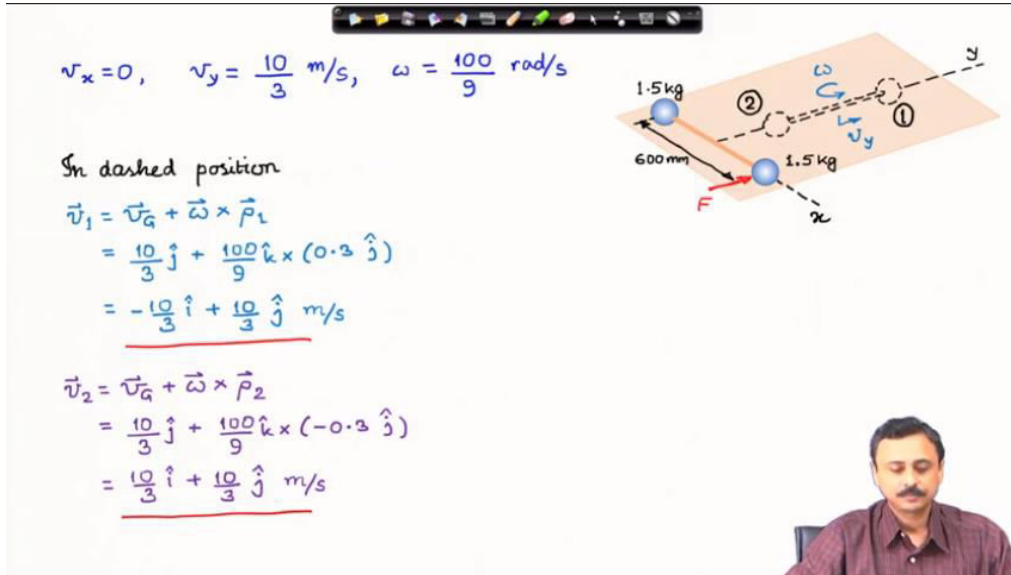
$$\Rightarrow 0.27 \omega = 3 \text{ Nms}$$

$$\Rightarrow \omega = \frac{100}{9} \text{ rad/s}$$


Now we look at the angular impulse momentum relation as shown in the slide above: the change of angular momentum about the center of mass is equal to the moment the impulse of the moment about the center of mass. Using this, we get the angular velocity of this system. Now the angular velocity of the system tells us how the system will change its orientation as time

progresses. Thus, the linear impulse momentum equation tells us how the center of mass will move, and the angular impulse momentum equation tells us how the rigid frame is going to change its orientation.

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$v_x = 0, \quad v_y = \frac{10}{3} \text{ m/s}, \quad \omega = \frac{100}{9} \text{ rad/s}$

In dashed position

$$\vec{v}_1 = \vec{v}_G + \vec{\omega} \times \vec{r}_1$$

$$= \frac{10}{3} \hat{j} + \frac{100}{9} \hat{k} \times (0.3 \hat{j})$$

$$= \underline{\underline{-\frac{10}{3} \hat{i} + \frac{10}{3} \hat{j} \text{ m/s}}}$$

$$\vec{v}_2 = \vec{v}_G + \vec{\omega} \times \vec{r}_2$$

$$= \frac{10}{3} \hat{j} + \frac{100}{9} \hat{k} \times (-0.3 \hat{j})$$

$$= \underline{\underline{\frac{10}{3} \hat{i} + \frac{10}{3} \hat{j} \text{ m/s}}}$$

Finally, using these two information, we calculate the velocities of the two balls as presented in the slide above.

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Summary

- Kinetics of a system of particles
- Problems

The discussions are summarized in the slide above.