Advanced Dynamics
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Module No # 05
Lecture No # 24
Kinetics of a System of Particles – II

We will continue our discussions on kinetics of system of particles.

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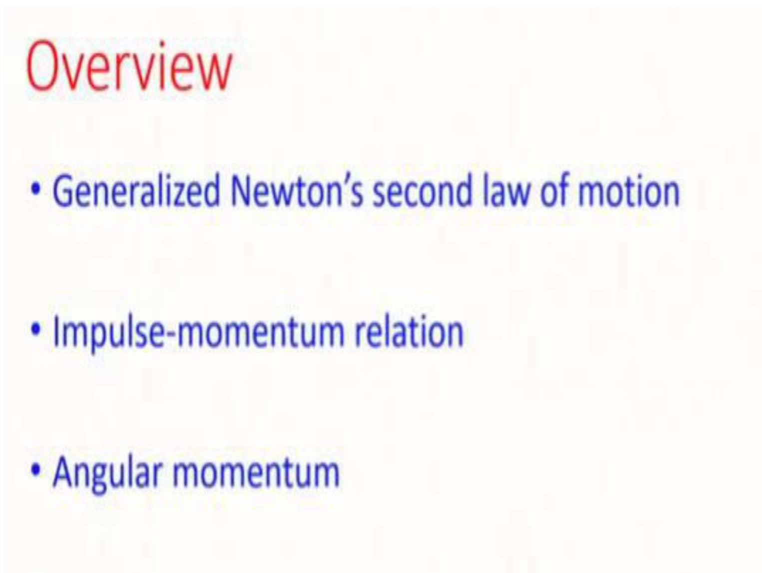

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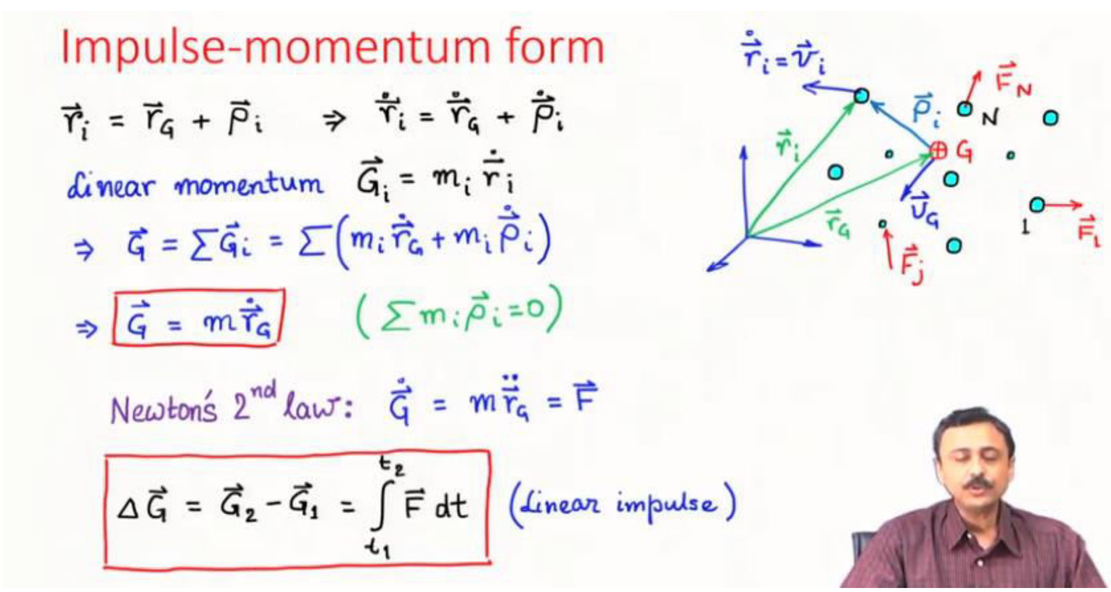

The impulse-momentum form on the generalized Newton's second law for a system of particles is presented in the slide above. It should be noted that linear momentum of a system of particles is given by G = m dr_G/dt, where m is the total mass of the system of particles and dr_G/dt is the velocity of the center of mass of the system.

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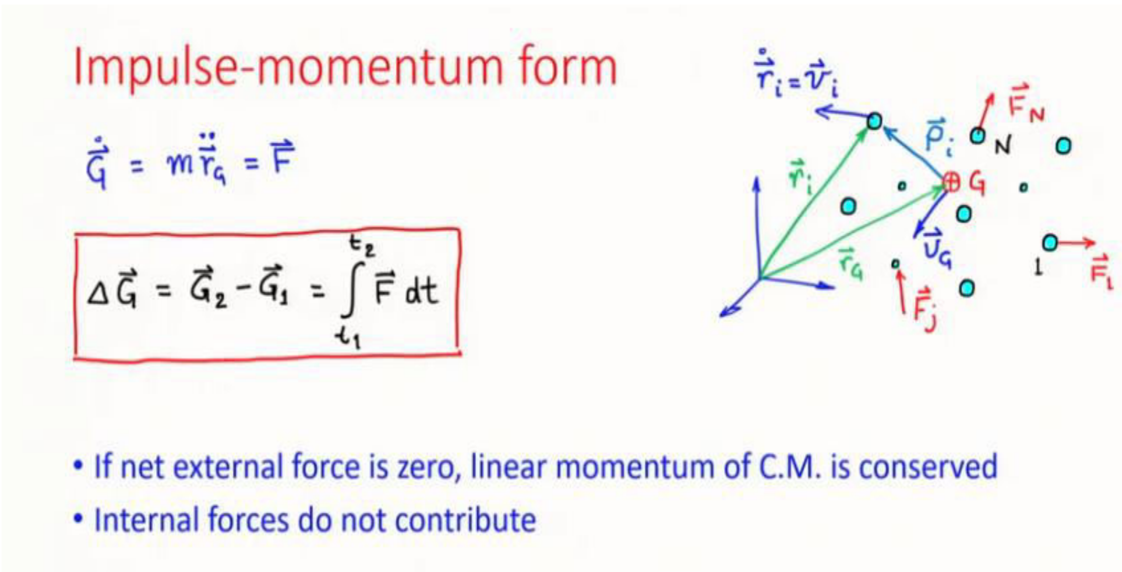

As shown above, if there is no external force on the system of particles, then the linear momentum of the center of the mass is conserved. Here we notice once again that internal forces do not contribute.

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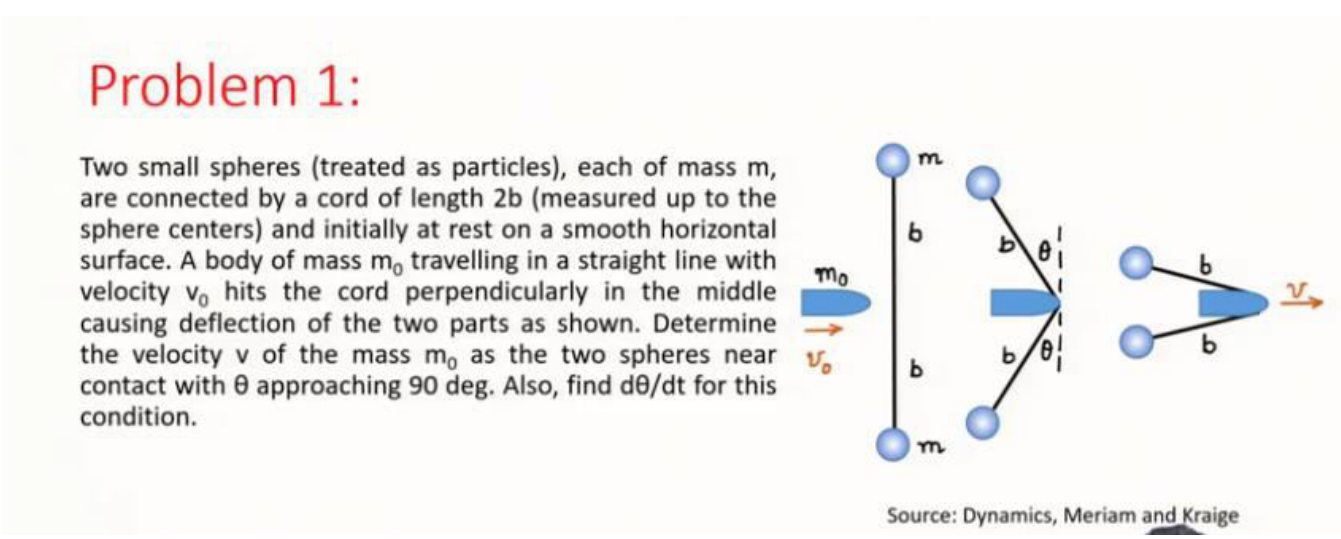

We consider the problem as shown in the slide above.

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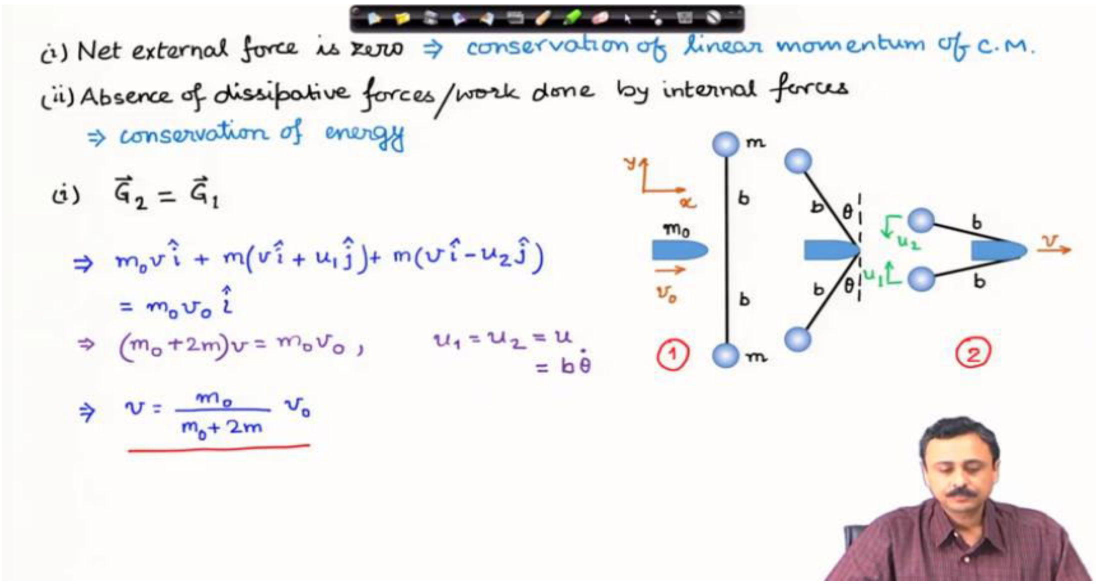

Since there is no external force acting on the system, we first use the linear momentum conservation to determine the velocities of the two balls in the folded configuration.

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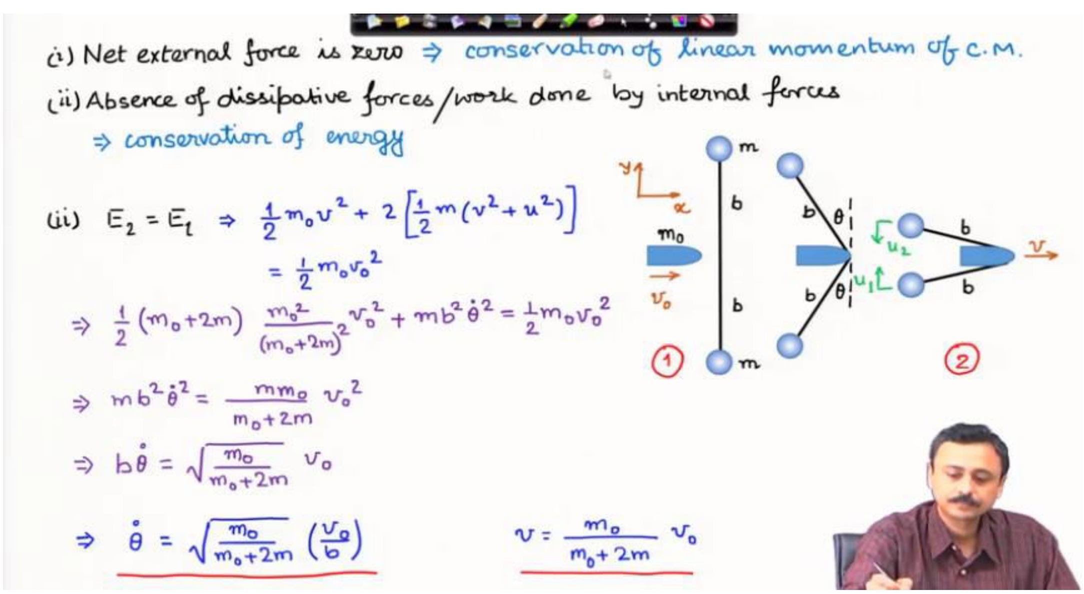

In the second step, we use the conservation of energy since there are no dissipative forces, as shown in the slide above, to completely solve the problem.

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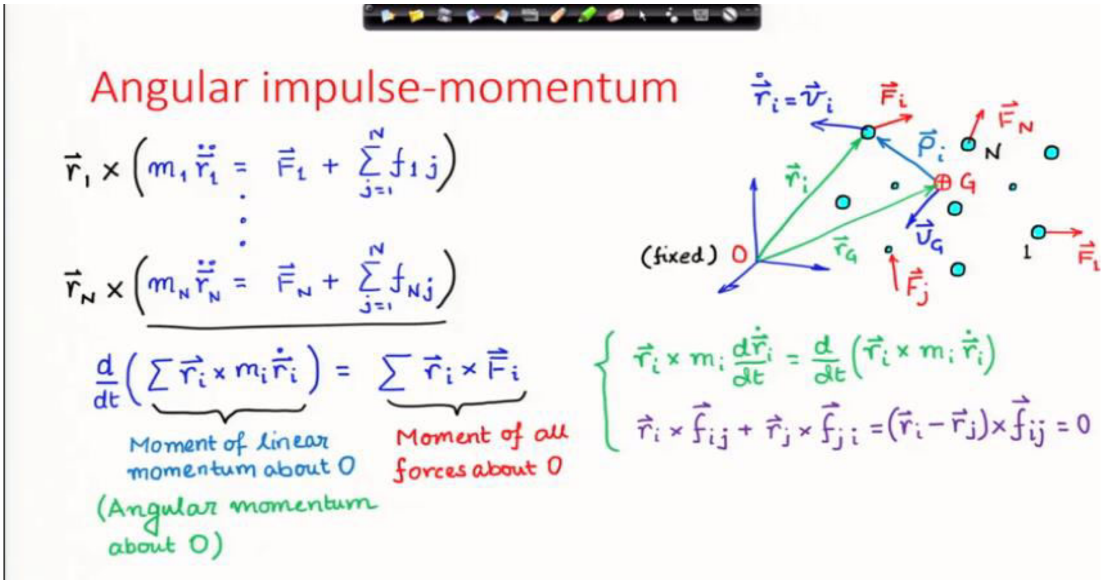

Now we will define angular momentum and look at angular impulse momentum relation. We start by writing the Newton's 2nd law for the individual particles, taking a cross product of the ith equation with the position vector of the ith particles as shown in the slide above, and summing over all the particles. This finally leads to

$$\frac{d}{dt}\left(\sum \vec{r}_i \times m_i \dot{\vec{r}}_i\right) = \sum \vec{r}_i \times \vec{F}_i$$

Moment of linear momentum about O (Angular momentum about O)

Moment of all forces about O

$$\Rightarrow \quad \boxed{\frac{d\vec{H}_o}{dt} = \vec{M}_o}$$

It is observed that the internal forces do not contribute to this equation.

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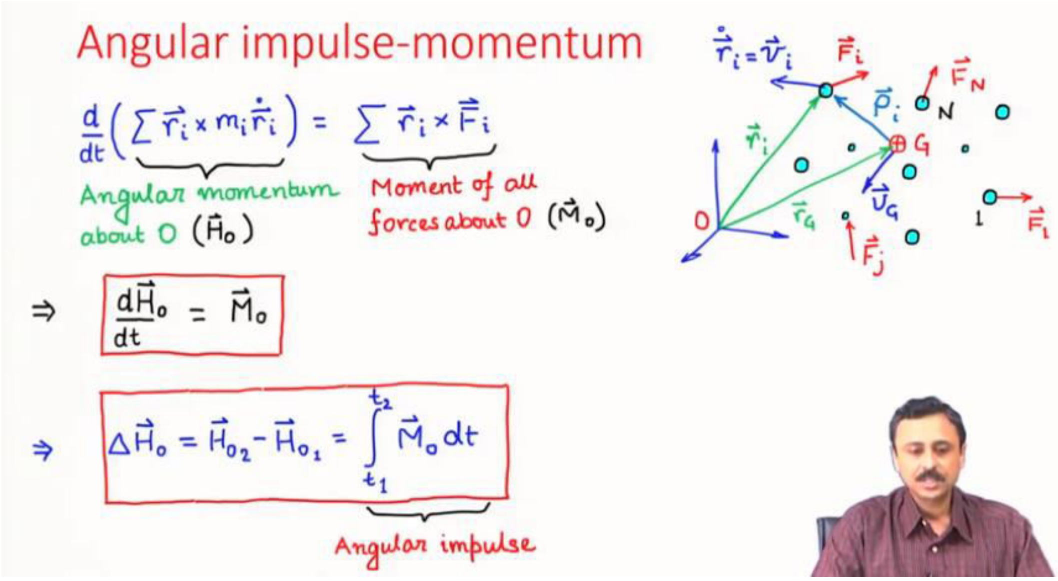

Integrating the angular momentum equation over time between two time instants t_1 and t_2, we obtain the angular impule – angular momentum relation as presented in the slide above.

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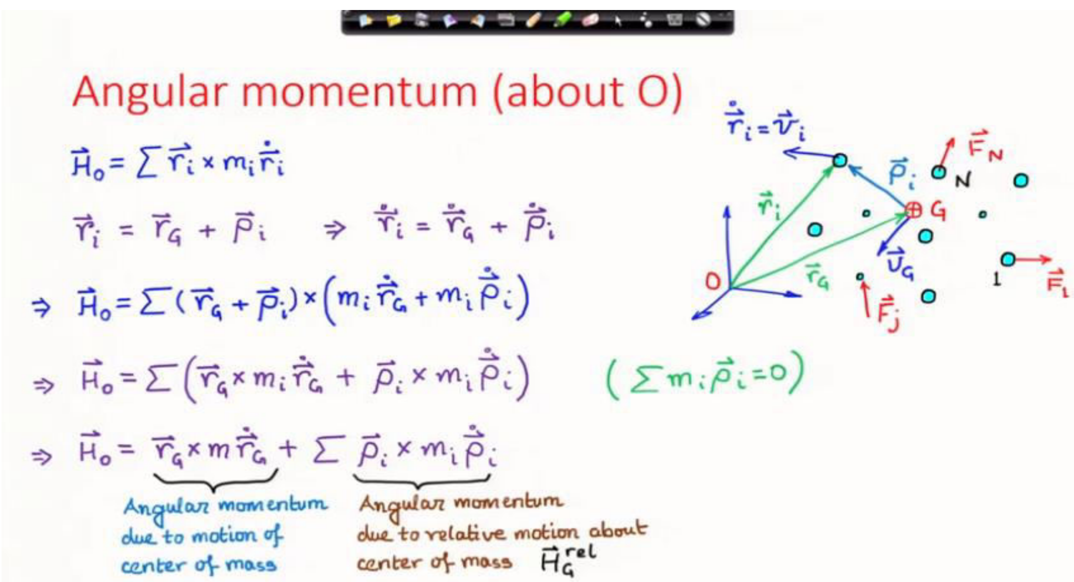

The angular momentum about a fixed point O is calculated in the slide above.

It is observed that the total angular momentum of the system about O is the sum of the angular momentum because of the motion of the center of mass with the whole mass concentrated at the center of mass at G and the angular momentum because of the relative motion of the individual particle about the center of mass.

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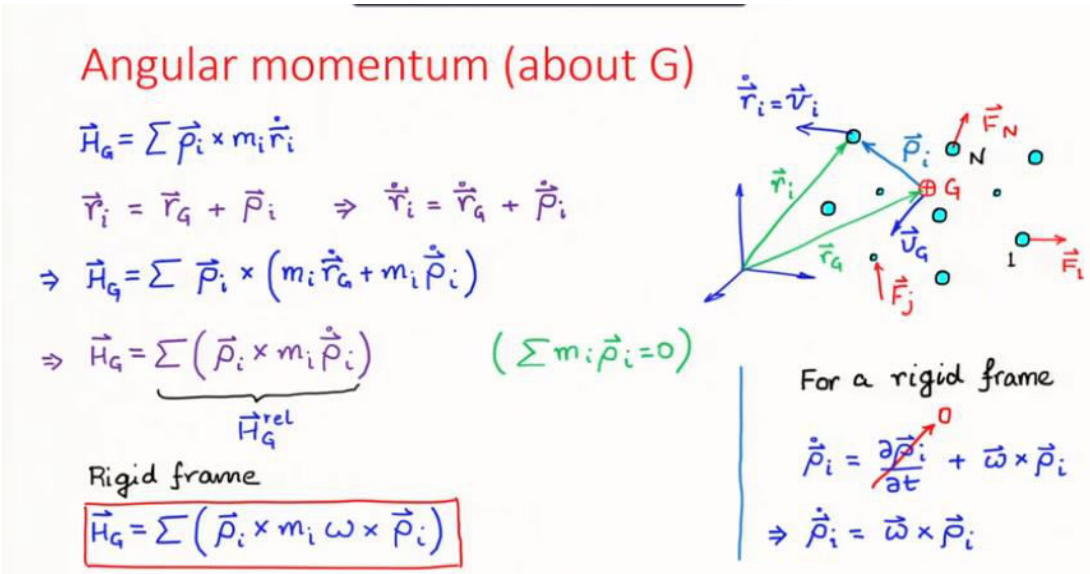

In the above slide, it is shown that the angular momentum about the center of mass G is equal to the relative angular momentum about G. It is to be noted that, in the former, the absolute velocities of the particles are used, while in the latter, the relative velocities of the particles with respect to G are used. The angular momentum expression for a system of particles held by a rigid frame is also presented in the slide above.

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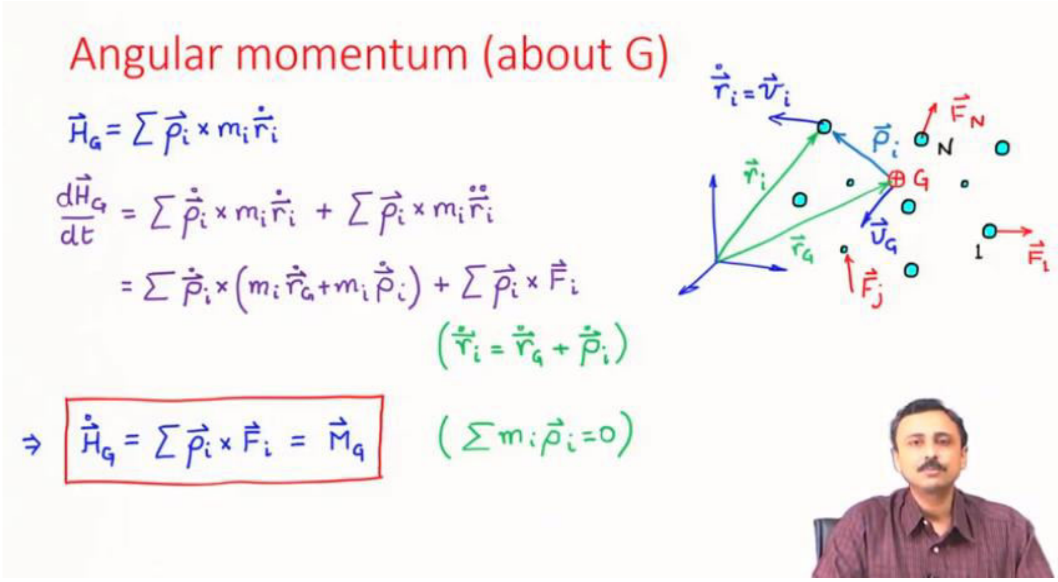

We continue with this angular momentum about the center of mass. If we take the time derivative of the angular momentum about the center of mass, we obtain

$$\dot{\vec{H}}_G = \sum \vec{\rho}_i \times \vec{F}_i = \vec{M}_G$$

This equation tells us how the angular momentum is going to change about the center of mass for a system of particles.

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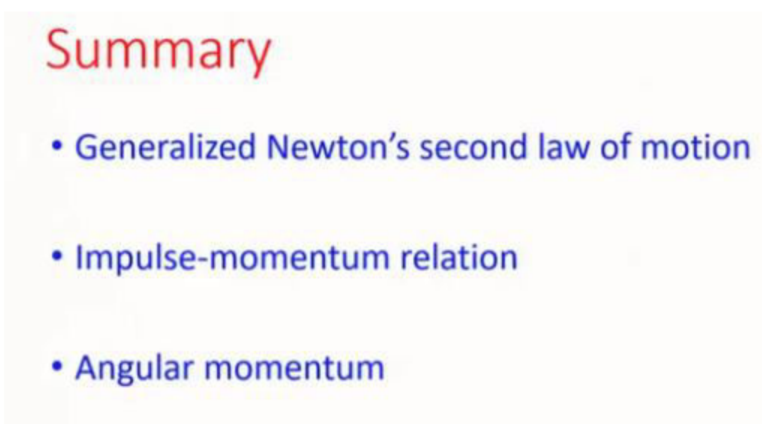

The discussions are summarized above.