Advanced Dynamics
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Module No # 05
Lecture No # 23
Kinetics of a System of Particles – I

In this lecture we are going to discuss the kinetics of a system of particles.

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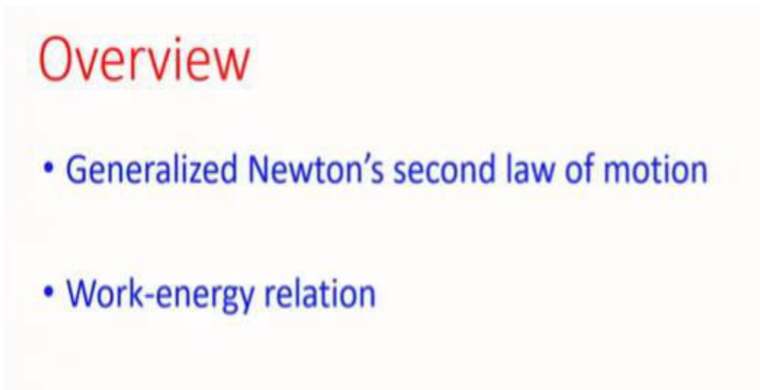

We will generalize Newton's second law of motion for a single particle to a system of particles and look at the work energy relations.

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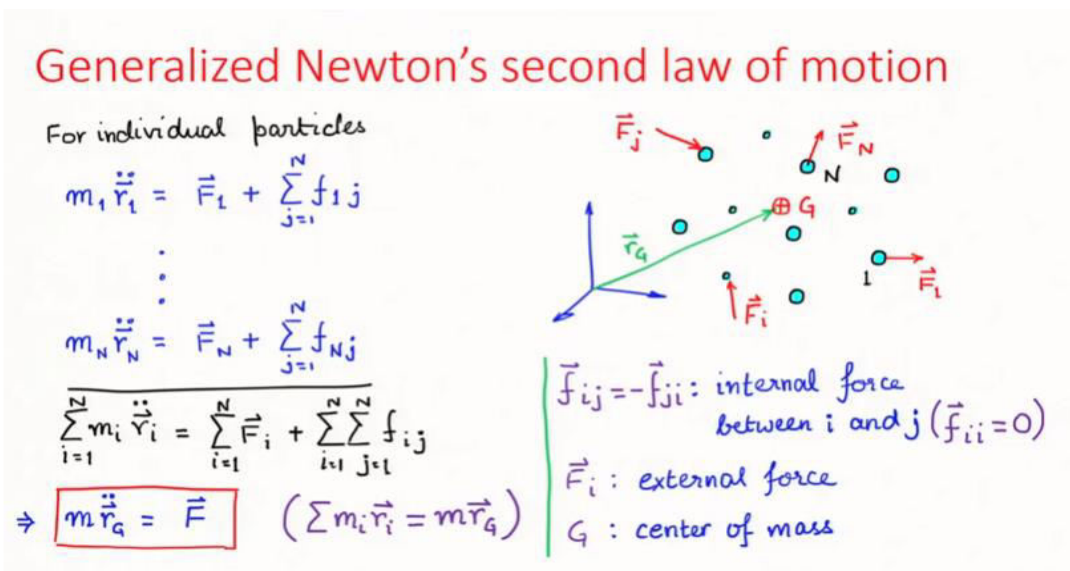

Consider a system of n particles as shown in the figure above. The position vectors of the individual particles are denoted by r_i, the interaction force on the i^{th} particle due to the j^{th} particles is denoted by f_{ij}, and the external force on the i^{th} particle is written as F_i. By Newton's 3^{rd} law, $f_{ij} = -f_{ji}$. The equations of motion of the particles are presented in the slide above and summed over all particles. This leads to the extension of Newton's 2^{nd} law for a system of particles as

$$m \ddot{\vec{r}}_G = \vec{F}$$

where r_G is the position vector of the center of mass of the system of particles, and F is the summation of all external forces acting on the system.

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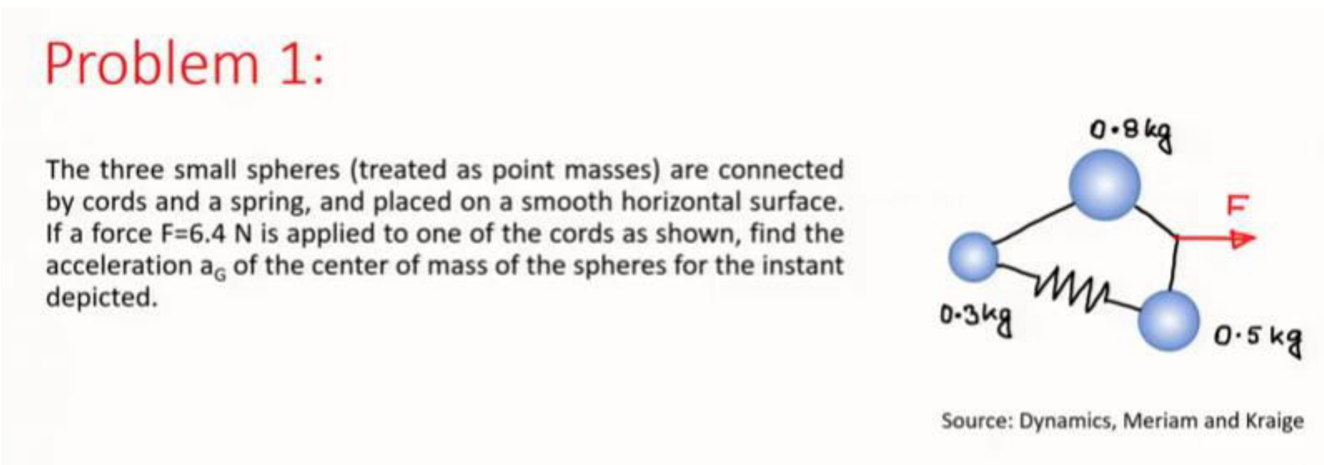

We consider the above problem.

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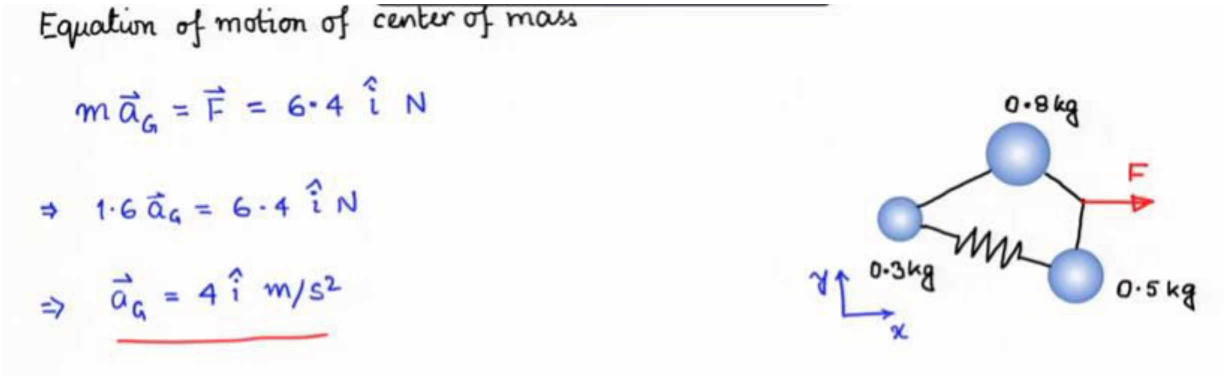

The solution is presented in the slide above.

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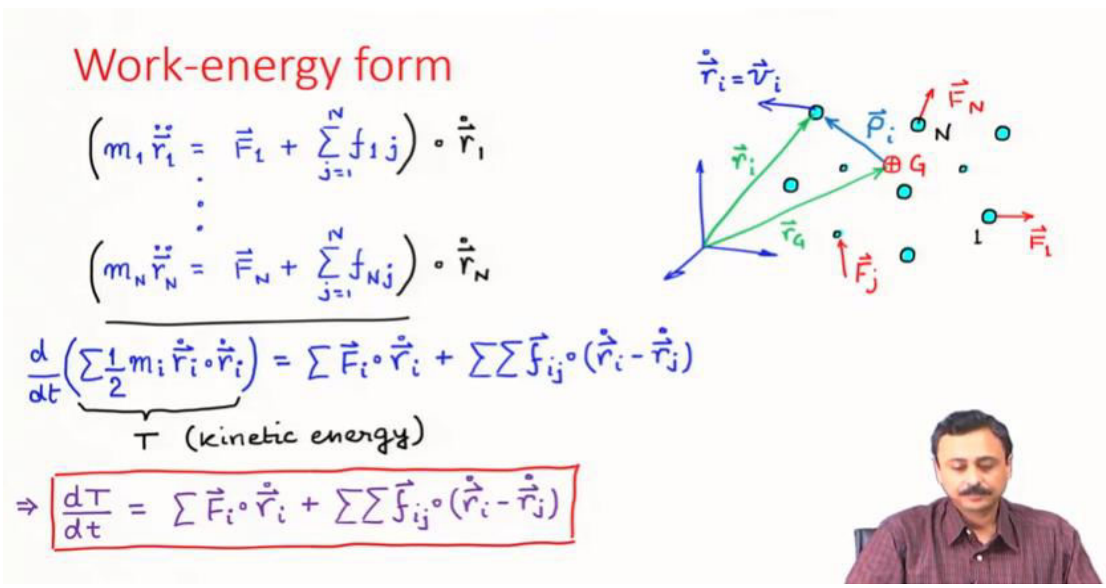

Next we will look at the work energy relation for a system of particles. First, writing out the Newton's 2nd law for the individual particles, we take dot product of the i^th particle equation with the velocity vector dr_i/dt, and sum over all particles as shown in the slide above. This leads to the rate of change of kinetic energy of the system as

$$\boxed{\frac{dT}{dt} = \sum \vec{F}_i\cdot\dot{\vec{r}}_i + \sum\sum \vec{f}_{ij}\cdot(\dot{\vec{r}}_i - \dot{\vec{r}}_j)}$$

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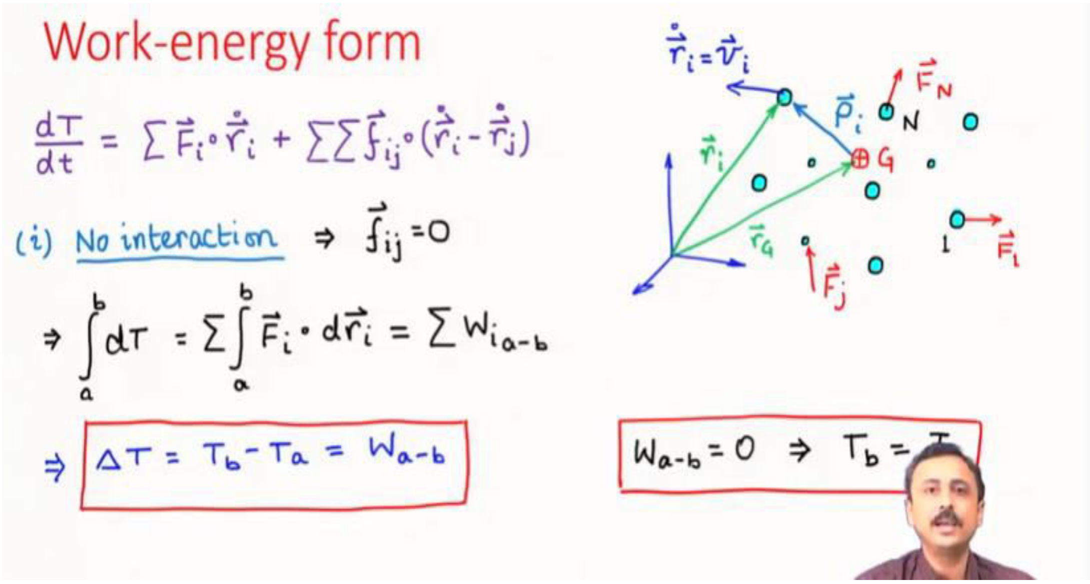

We consider two cases. The case (i) of no interaction between particles (i.e., $f_{ij}=0$) is shown in the slide above. This leads to the work-energy relation

$$\Delta T = T_b - T_a = W_{a-b}$$

If the work done by all external forces is zero, then kinetic energy of the system of particles is conserved.

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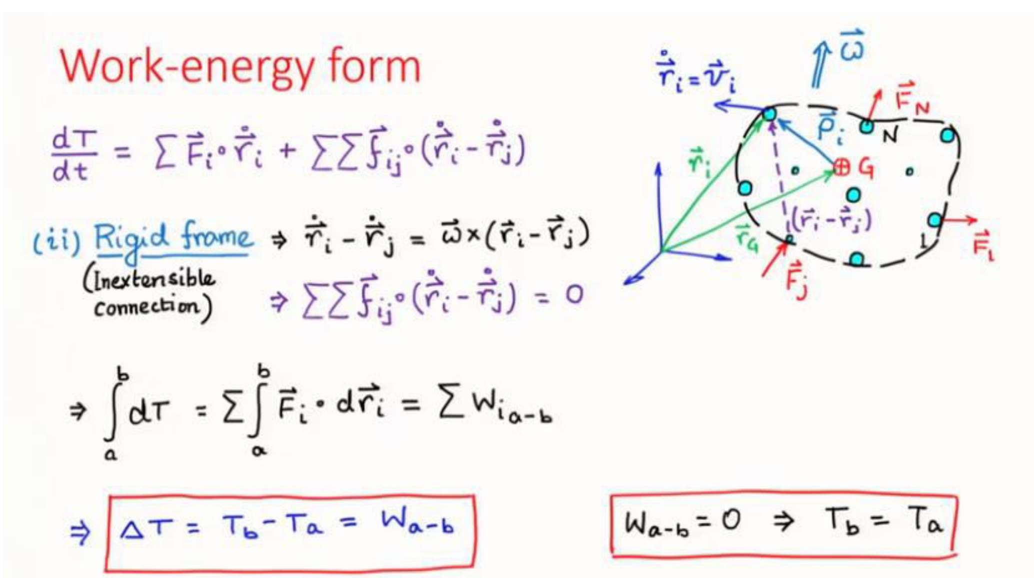

Next, we consider case (ii) of a rigid frame, as shown in the slide above. In this case, the work done term due to internal forces becomes zero. Hence, we again get back the work-energy relation

$$\Delta T = T_b - T_a = W_{a-b}$$

Now we have considered these 2 situations where the second term on the right hand side drops off. This happens when, either there is no interaction between the particles, or the particles are connected by rigid frame.

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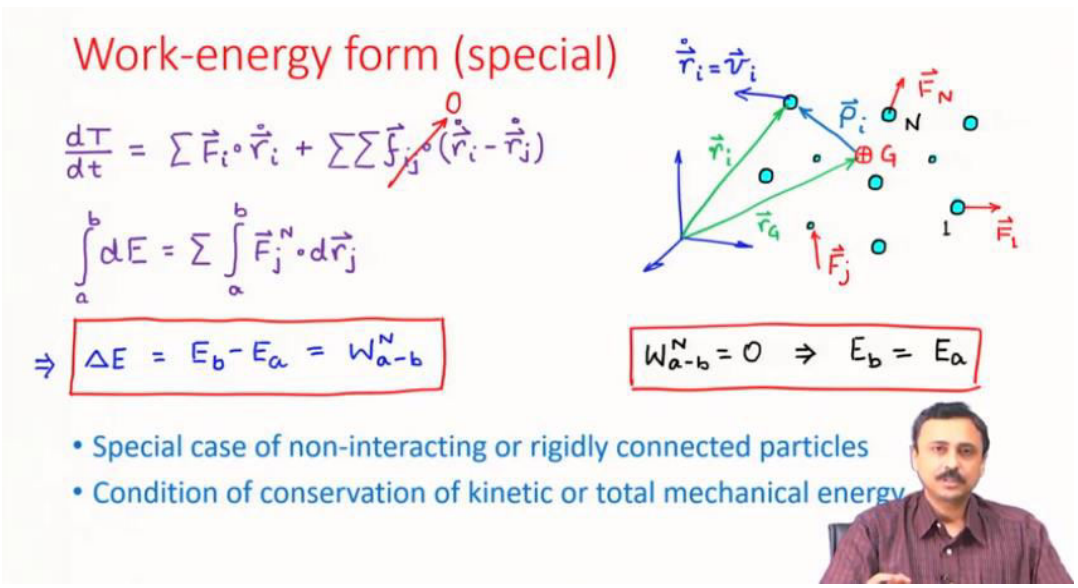

In the case when potential forces are present, we can define potential functions corresponding to each force, and the work done by such forces can be expressed as the negative of the change in the potential energy of the system as

$$\int_a^b dT = \int_a^b \Big(\underbrace{\sum \vec{F}_i^{c} \cdot d\vec{r}_i}_{-dV_i} + \sum \vec{F}_j^{N} \cdot d\vec{r}_j \Big)$$

This leads to

$$\int_a^b dT = \int_a^b \left(-dV + \sum \vec{F}_j^{\,N} \circ d\vec{r}_j\right) \quad \Rightarrow \int_a^b d(\underbrace{T+V}_{E}) = \sum \int_a^b \vec{F}_j^{\,N} \circ d\vec{r}_j$$

(Mechanical energy)

and finally, we obtain

$$\boxed{\Delta E = E_b - E_a = W_{a-b}^{N}}$$

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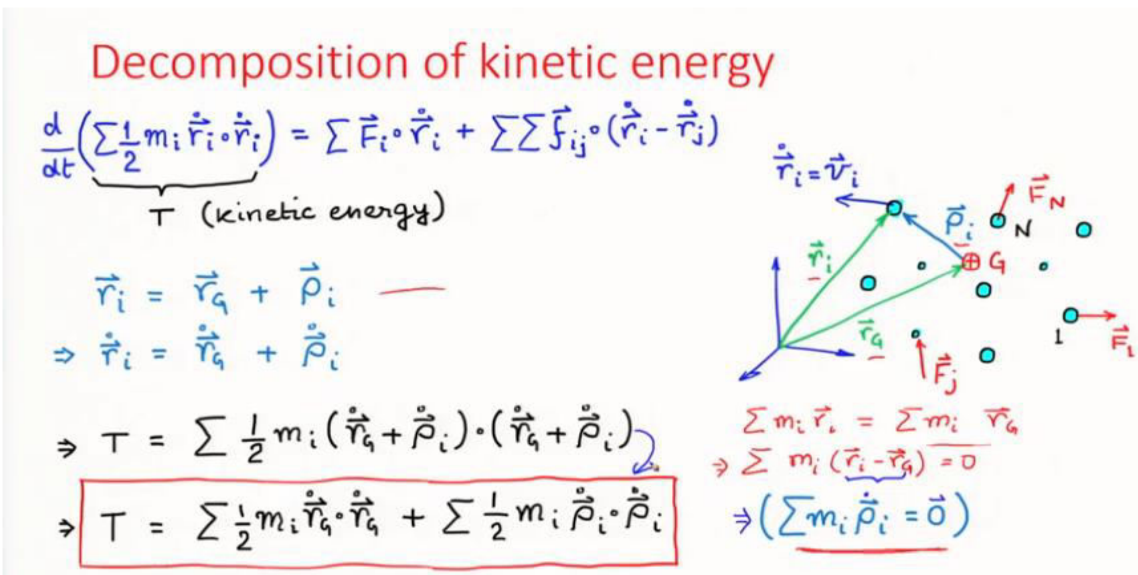

Now let us look at the decomposition of the kinetic energy as shown in the slide above.

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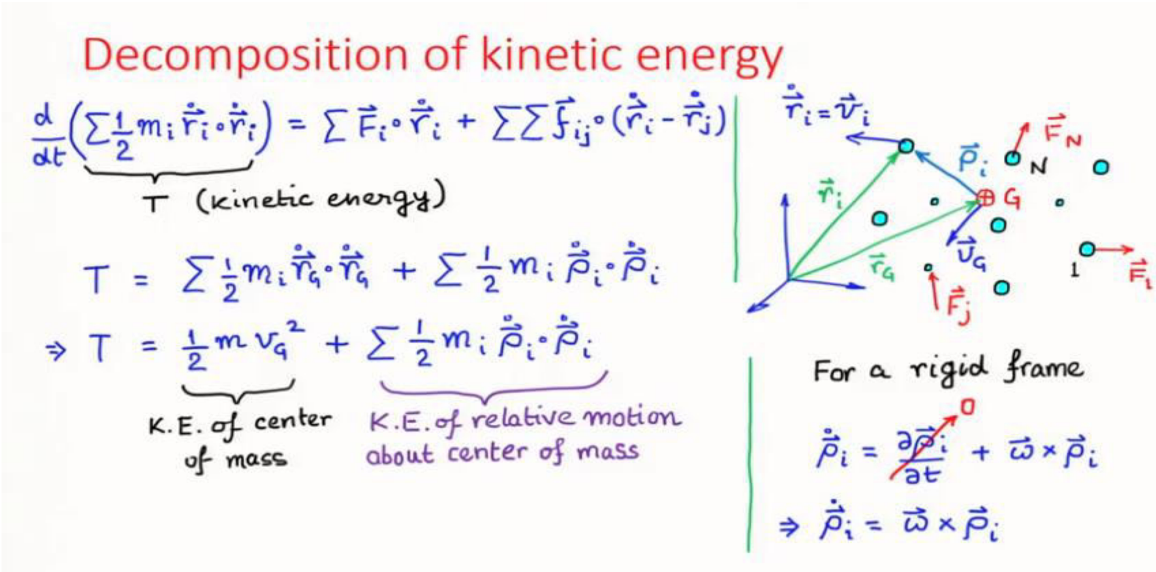

As shown in the above slide, the total kinetic energy of the system of particles is sum of of 2 parts: (i) the kinetic energy due to the motion of the center of mass with all the mass concentrated at the center of mass, and (ii) the kinetic energy of all particles moving relative to the center of mass.

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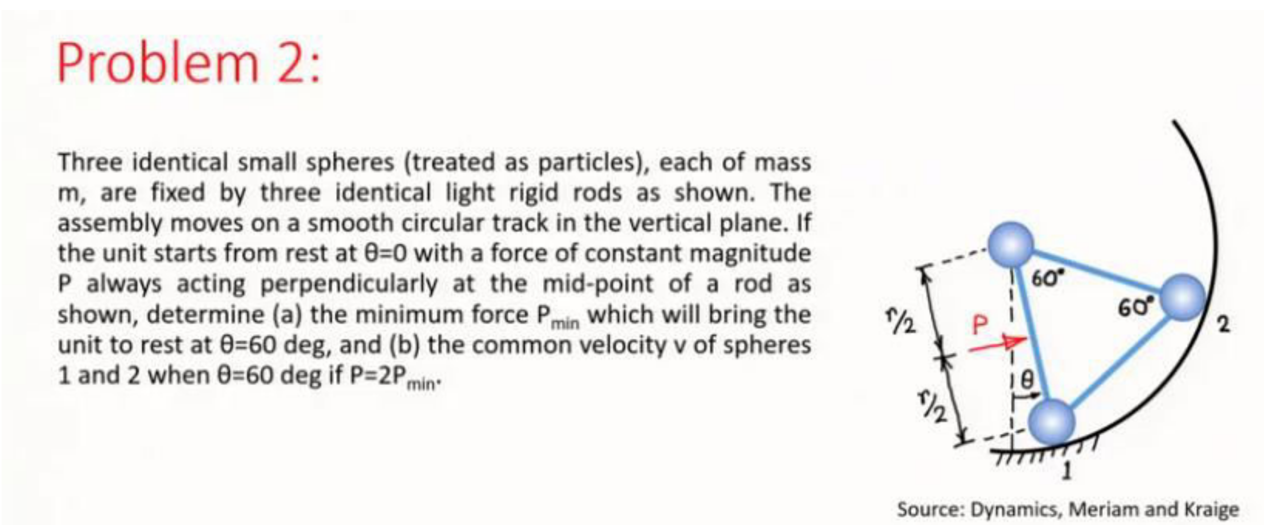

We consider the above example.

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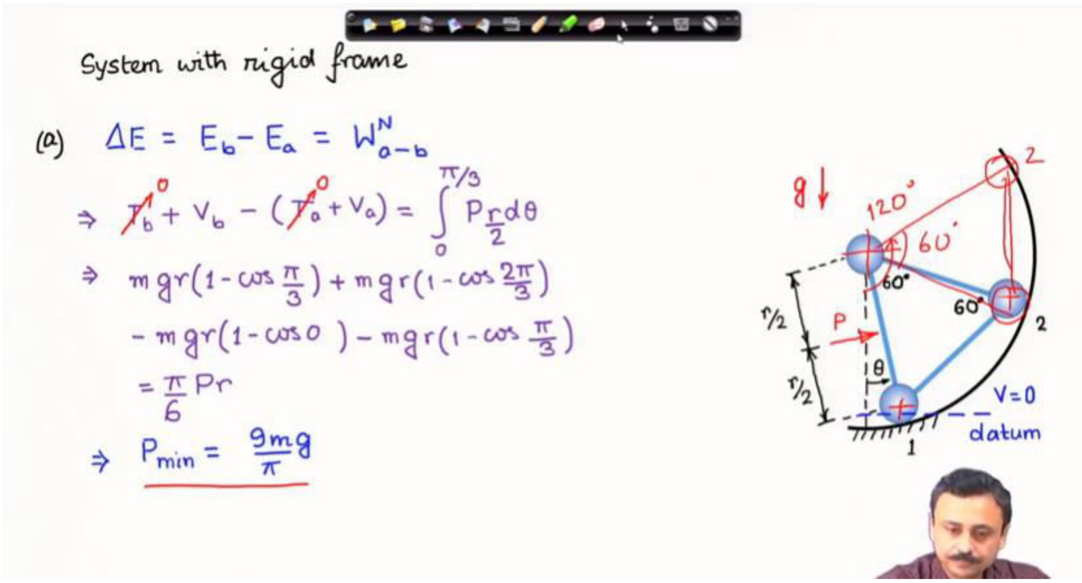

The solution is presented in the slide above using the work-energy relation to determine P_{min} required to take the system to the configuration with θ=60 degree.

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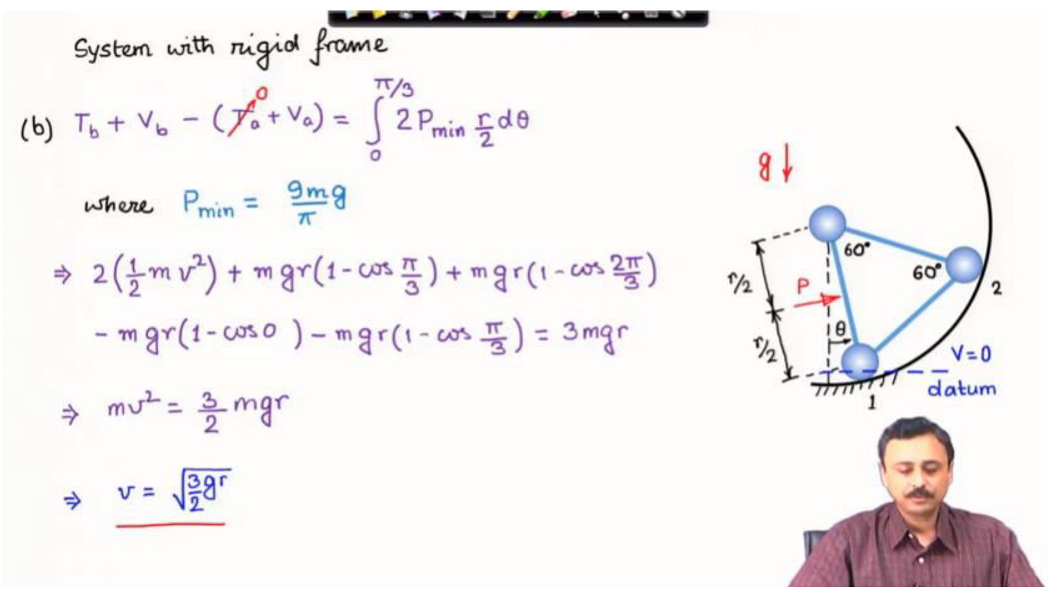

In a second part of this problem we have to find out the common velocity of the 2 particles 1 and 2 when $2P_{min}$ is applied in place of P_{min}. The solution is again approached using the work-energy relation as shown in the slide above.

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The above slide summarizes the discussion.