Advanced Dynamics
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Module No # 05
Lecture No # 22
Systems with Mass Flow – II

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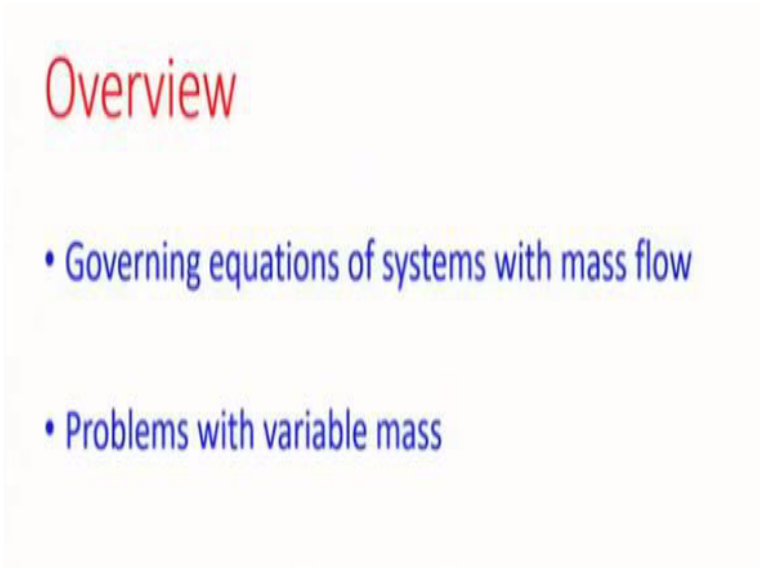

In this lecture, we are going to look at more problems, and this time with variable mass flow.

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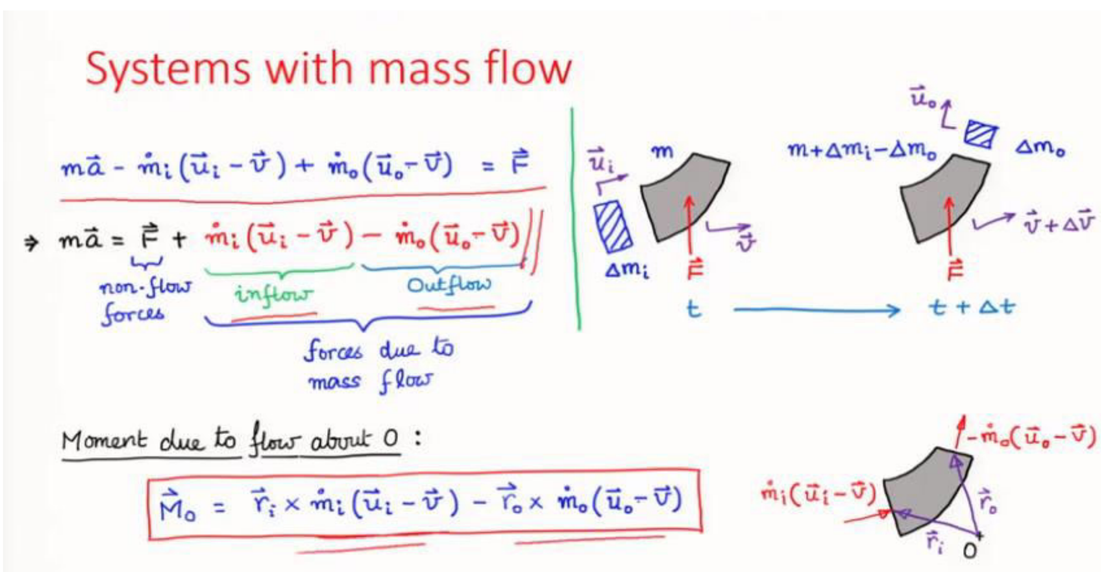

The derivation of the master equation is presented in the slide above.

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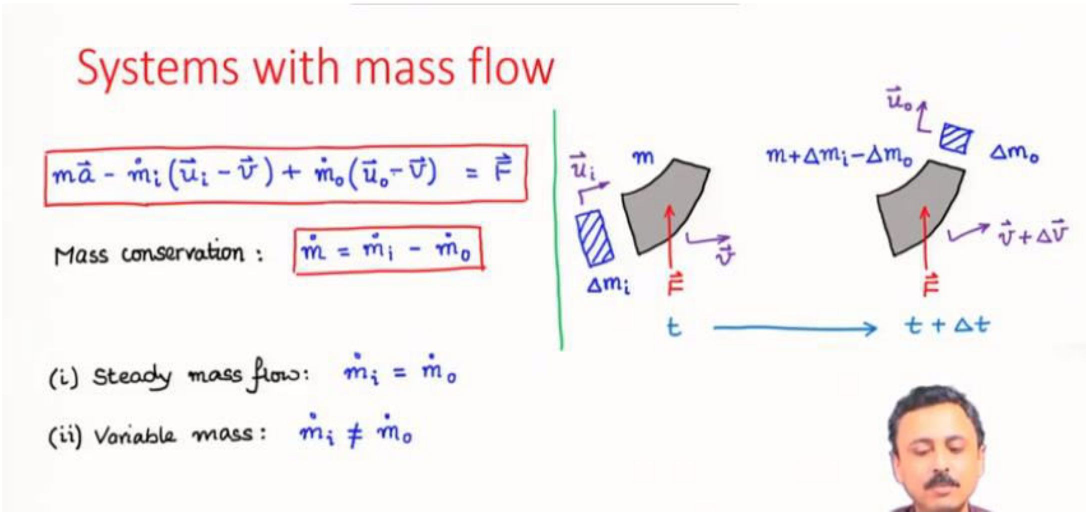

The mass conservation relation and its specialization to steady and variable mass problems is presented in the slide above.

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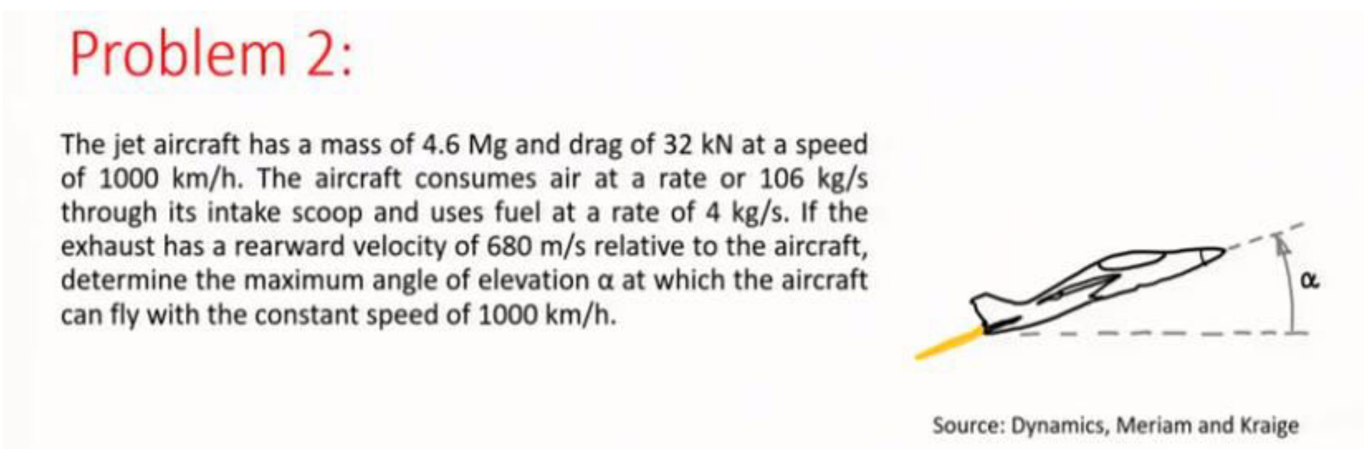

The problem statement is presented in the slide above.

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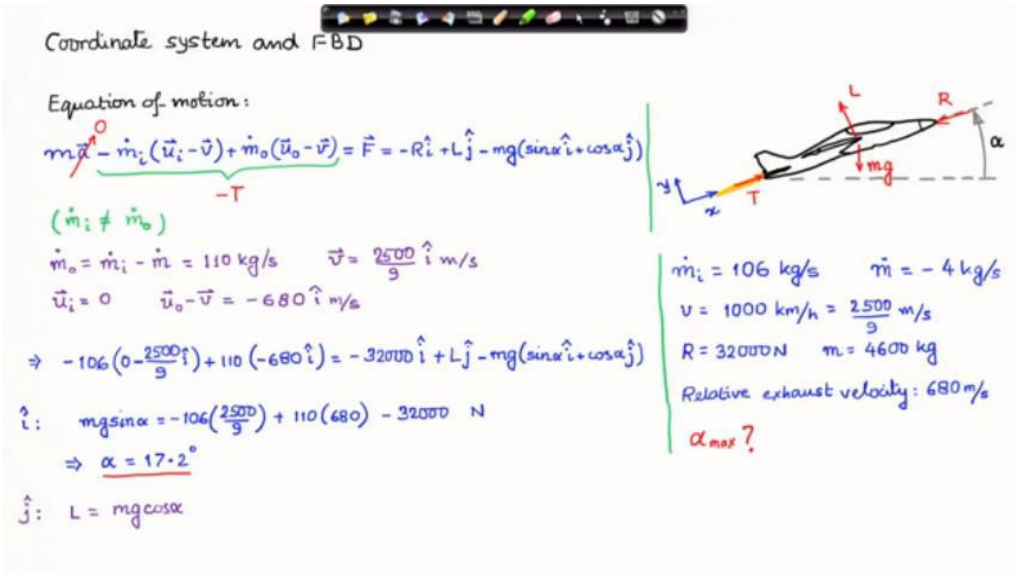

The coordinate system and free body diagram are shown in the slide above. The acceleration term in the master equation for this problem drops out since the aircraft is flying at a constant velocity. We plug in the values of the various terms in the master equation and obtain the maximum elevation angle of the aircraft as shown above.

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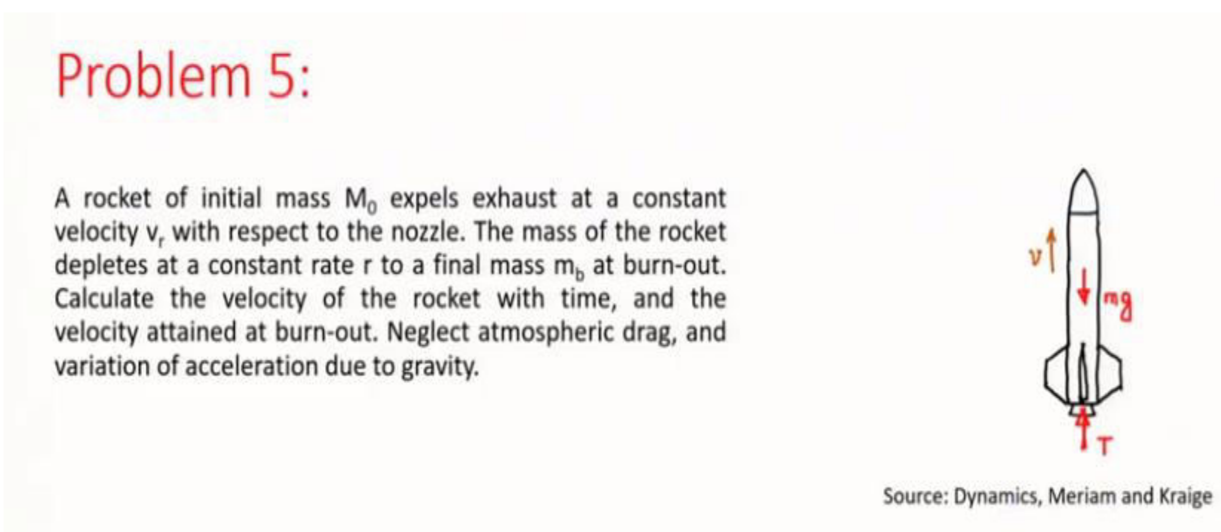

A problem of a rocket is stated above. This is a system with variable mass.

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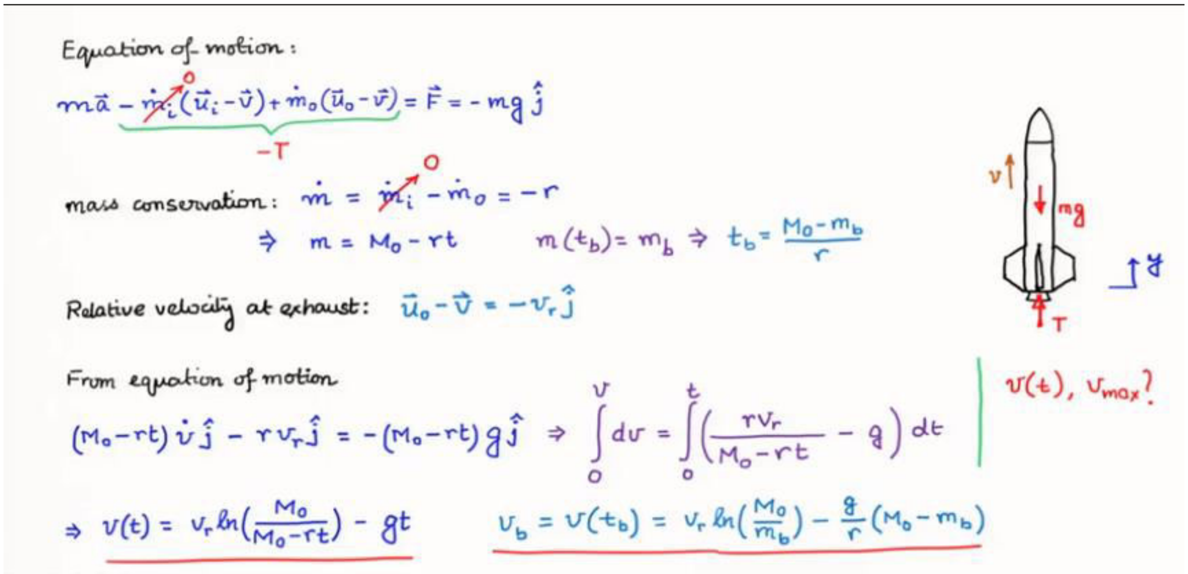

The coordinate system and free body diagram are shown in the slide above. From the mass conservation equation, we can find the fuel burn-out time as shown. By integrating the master equation, we determine the velocity at any time t, and the velocity at burn-out is also determined.

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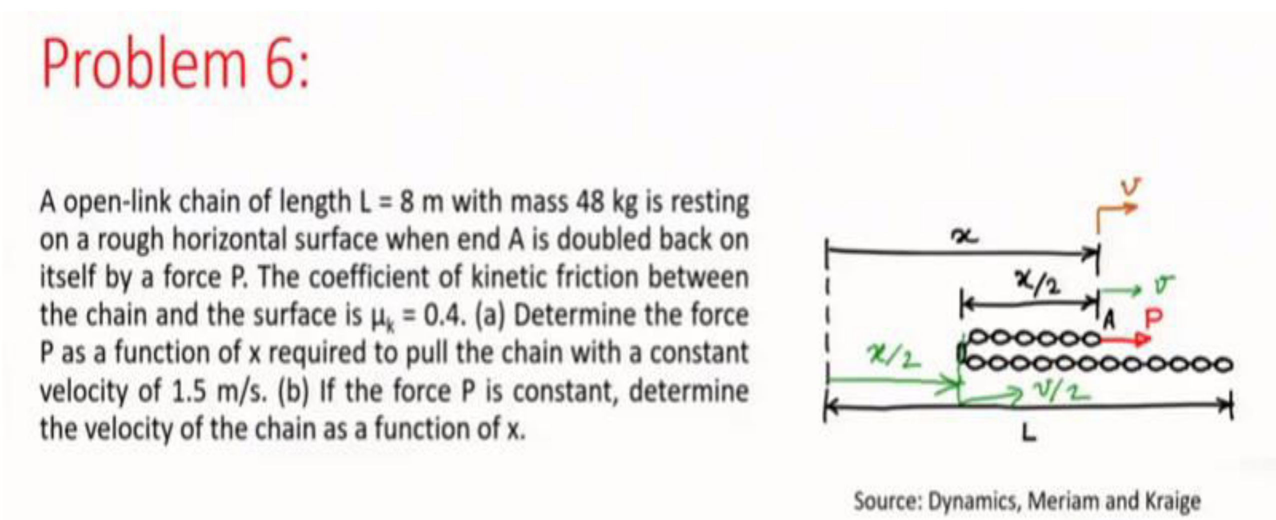

We now move to a problem of a chain as stated above.

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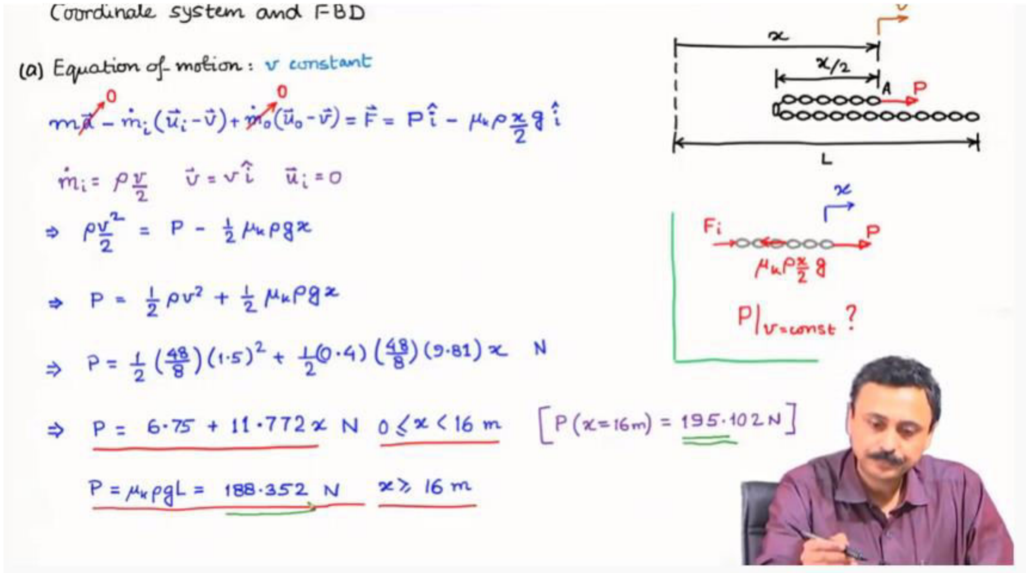

The coordinate system and free body diagram for the moving part are shown. The moving part of the chain constitute the system to which . This is a variable mass system.

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System velocity is v i cap and v is constant as I mentioned that this link on the static side had0 velocity suddenly it gets added to the moving part which has velocity v. The intake velocity the absolute velocity intake velocity of that link last link on the static side is 0. Therefore the rate at which the velocity at which the mass is being added is 0 the absolute velocity of the link just before it is getting it gets added to the system is 0.

So u i is 0 I plug in all these expressions in my master equation I have rho v by 2 is m i dot this is 0 and –v is this. So this minus this becomes plus so rho v square by 2 is on the right hand side the applied force is P and the friction force which turns out to be this. Remember the length of the moving part of the chain is x by 2 this is an important thing to observe when A point has moved at distance x from it is starting location.

So at t = 0 A was here now it has moved a distance x the length of the moving part of the chain is x by 2. Now this I can rearrange and I can find out the force required and I can plug in the values that are provided. And I will get the force required to keep the velocity constant since the system is growing as the as the links are getting added therefore with increasing value of x this force is also growing to grow and here it is growing linearly with x.

And this is valid till x goes to 16 meter from 0 to 16 meter why 16? Because remember A will move this whole chain was of length 8 meter and A will move from 0 to the point where this last link this link gets added. Therefore the whole displacement of A from its starting position will be 16 meter. Therefore x goes from 0 to 16 just at 16 the last link is added and after that there is no mass addition.

Just let us try to calculate what is the force from this formula what is the force that would be required when x is 16 meter? So it turns out to be 195.102 Newton just with the last link is added but after the last link is added there is no more mass addition it becomes a simple straight forward problem of a body with fixed mass and what is that mass 48 KG or 42 KG so that much mass of chain is moving on a rough flow and what is the force required to keep moving it at that velocity it is just the friction force you have to just balance of the friction.

And that friction force turns out to be 188.352 Newton when x is greater than or equal to 16 meter therefore once that 16 meter is reached suddenly the required force to maintain the velocity drops from around 195 Newton to 188 Newton why because? Now that the impulsive force because of mass addition is no longer present. It is only a constant mass system moving on the rough floor and what we have to provide is just the friction force and everything should be balance because it is moving at a constant speed constant velocity.

Therefore the force required will be just the friction force so there will be a jump in the force requirement when the last link of the chain will added to the system.

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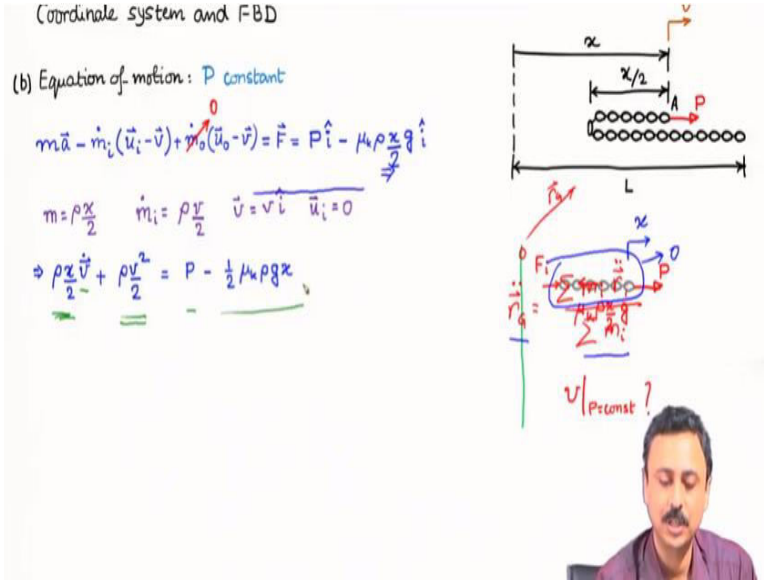

The second part of the question is how will the chain move if P is constant? Again we write down the master equation there is no ejection of mass m o dot is 0 and here the acceleration term is there some m a the first term of the master equation is present because now the force is constant. Therefore there will be acceleration or deceleration of the system mass of the system is any point of time is rho x by 2 because at any point of after the point A has moved the distance x the length of our system is that moving part is x by 2.

Therefore mass at any point of time is rho x by 2 and you can differentiate this also to get the rate; of mass addition which will be rho v by 2. Because x dot is v velocity is v i cap the absolute velocity of a link which will be added at the end of the bend part is 0 u i is 0. So it was not moving it suddenly starts it gets attached to the moving part the system and attains a velocity v. Now plug in all these expressions into our master equation v dot is the acceleration I have written acceleration as v dot.

This is the mass at any instant at when the point A as reached or travelled a distance x this is the mass this is because of the mass addition this is the force. Now this is constant force that is applied to the end of the chain and this is the friction force acting on the moving part.

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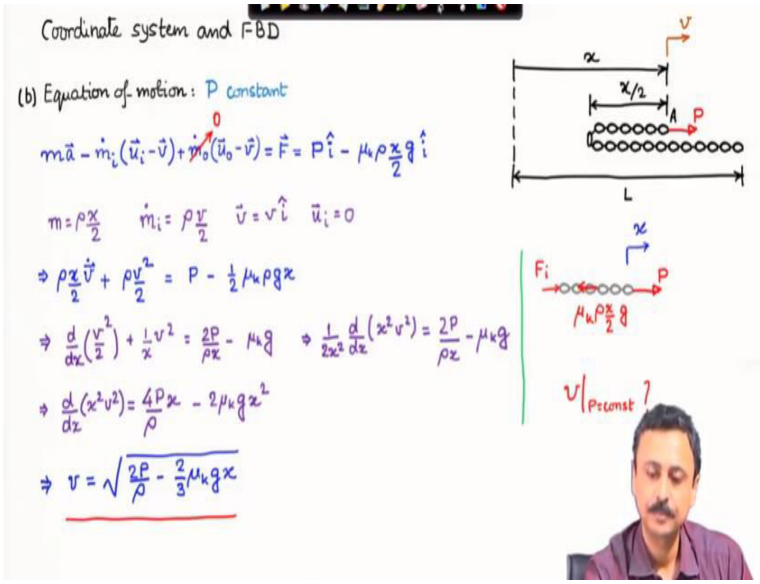

Therefore you will observe that acceleration which is v dot is a function of x if I simplify this I will get acceleration as a function of x. Therefore following our standard principle practice what we have discussed in kinematics. We express this acceleration which is v dot as v dv dx we can write this acceleration as v dv dx. Therefore I can write this as d dx of v square by 2 and this is what I have done this is 1 dimensional motion in 1 dimension therefore I mean this expression we have discussed previously for 1 dimensional motional.

And here I have therefore converted the acceleration expression in terms of d dx and I can now integrate this. If I do this integration if I do this so what I have done is the left hand side by using indicating factor I have represented in this form. And if you do this simplification then you come to this step here I have taken these terms to the right hand side these are all simple algebraic steps.

And finally I can integrate and find out the velocity as a positive square root of whatever comes inside 2 p by rho -2 by 3 mu k g x. This is straight forward integration and using some algebra to simplify things. This is how the velocity varies with x at x = 0 or 0+ when the force is applied you see the velocity will be roughly under root 2 P by rho and as it moves the velocity drops and if P is not sufficiently large.

Then at some point of x some value of x the velocity might even go to 0 and if it goes to 0 after that the change is not going to move. Therefore if the chain as to move continuously till it is last link is set to motion you can calculate what is the minimum force required to make the? I mean to bring the whole chain in motion. Whole chain means till the last link so that you can calculate by putting this velocity if you want this velocity at x = 2L.

Because the point A moves as a displacement of 2L so therefore v at x = 2L must be greater than 0 and with that you can calculate what should be minimum value of p required to set the whole chain in motion.

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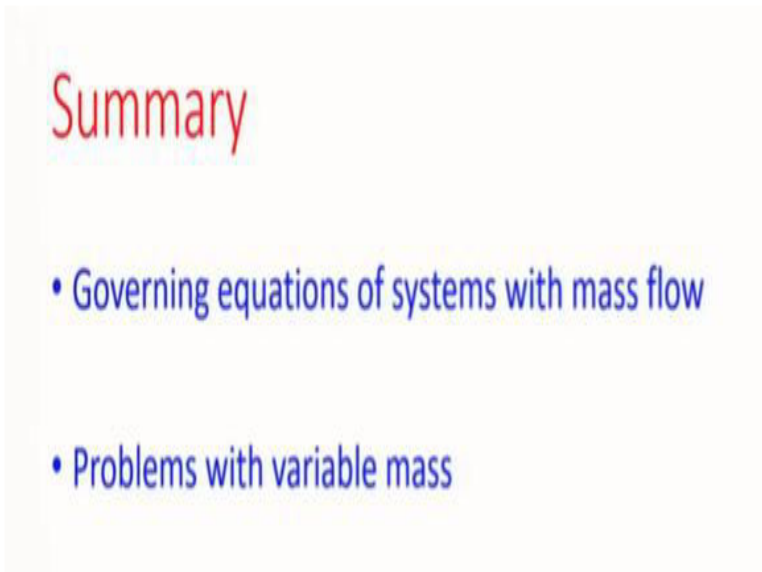

So with that I will just summarize what we have discussed we have again looked at the governing equation of motion for a system with mass flow and we have discussed problems with steady mass flow and variable mass flow. With that I will close this lecture.