

Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Module No # 05
Lecture No # 21
Systems with Mass Flow – I

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Overview

- Governing equations of systems with mass flow
- Examples

In this lecture we are going to discuss systems with mass flow.

We will start by deriving the governing equation of motion of systems with mass flow.

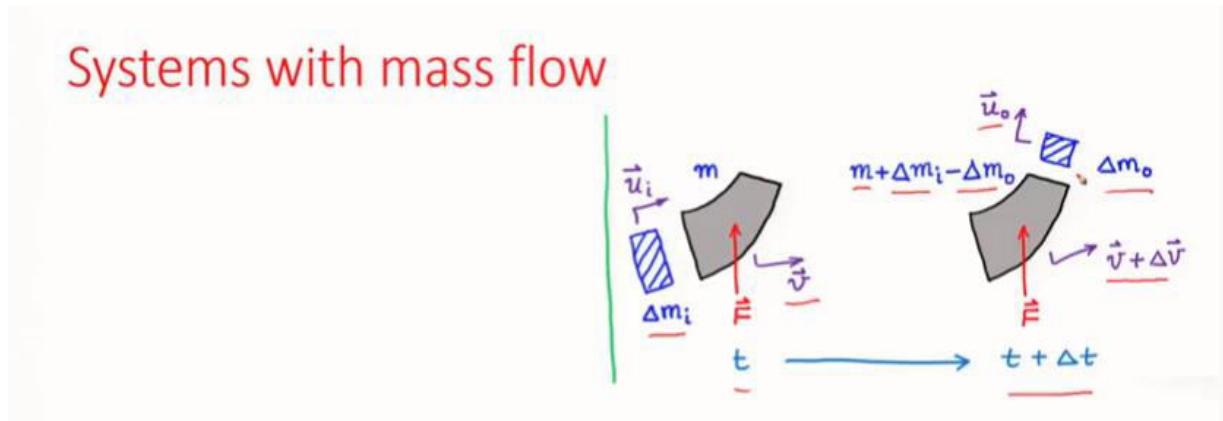
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Systems with mass flow

- Systems with steady flow: jet engine, water jet ski, turbine
- Systems with variable mass: rocket, motion of chains

Some examples of systems with mass flow are given in the slide above.

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Now to derive the equation of motion of system with variable mass, we consider a body which ingests and ejects small quantities of mass as shown above in a short duration of time. The absolute velocities of the body and the two mass blobs are shown. The body is acted upon by a non-impulsive force external force F .

(Refer Slide Time: 06:44)

Systems with mass flow

$\vec{u}_i, \vec{u}_o, \vec{v}$: absolute velocities
 \vec{F} : non-impulsive forces (other than forces due to flow)

Newton's 2nd law: $\lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{G}}{\Delta t} = \vec{F}$

$\Rightarrow \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[(m + \Delta m_i - \Delta m_o)(\vec{v} + \Delta \vec{v}) + \Delta m_o \vec{u}_o - (m\vec{v} + \Delta m_i \vec{u}_i) \right] = \vec{F}$

$\Rightarrow \boxed{m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}}$ (Ignoring second order small terms)

$\vec{a} = \frac{d\vec{v}}{dt} \quad \dot{m}_i = \frac{dm_i}{dt} \quad \dot{m}_o = \frac{dm_o}{dt}$

relative velocity at inlet relative velocity at exit

We write down Newton's second law for this system, as shown in the slide above and take the limit of the time interval to zero. Dropping second order small quantities, we obtain

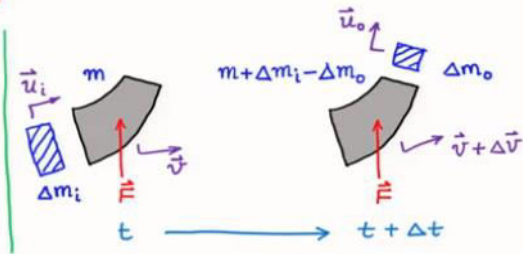
$$m\vec{a} - \underbrace{\dot{m}_i(\vec{u}_i - \vec{v})}_{\text{relative velocity at inlet}} + \underbrace{\dot{m}_o(\vec{u}_o - \vec{v})}_{\text{relative velocity at exit}} = \vec{F}$$

This equation will be referred to as the master equation for mass flow systems.

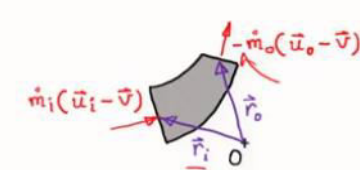
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Systems with mass flow

$$m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}$$

$$\Rightarrow m\vec{a} = \underbrace{\vec{F}}_{\text{non-flow forces}} + \underbrace{\dot{m}_i(\vec{u}_i - \vec{v}) - \dot{m}_o(\vec{u}_o - \vec{v})}_{\text{forces due to mass flow}}$$


Moment due to flow about O :

$$\vec{M}_O = \vec{r}_i \times \dot{m}_i(\vec{u}_i - \vec{v}) - \vec{r}_o \times \dot{m}_o(\vec{u}_o - \vec{v})$$


We rearrange the master equation to bring the additional terms on the left hand side to the right hand side, as shown in the slide above. These additional terms represent the flow forces due to inflow and outflow. One can also calculate the moment of the flow forces on the body about a fixed point as shown in the slide above.

(Refer Slide Time: 13:43)

Systems with mass flow

$$m\vec{a} - \dot{m}_i(\vec{u}_i - \vec{v}) + \dot{m}_o(\vec{u}_o - \vec{v}) = \vec{F}$$

Mass conservation : $\dot{m} = \dot{m}_i - \dot{m}_o$

(i) Steady mass flow: $\dot{m}_i = \dot{m}_o$

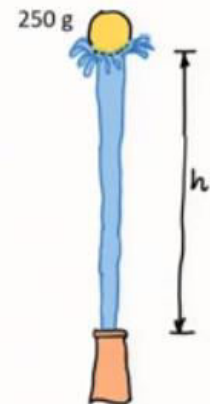
(ii) Variable mass: $\dot{m}_i \neq \dot{m}_o$

Now let us look at the mass balance of the system. This is presented in the slide above.

(Refer Slide Time: 14:55)

Problem 1:

A 250 g ball is supported by the vertical stream of fresh water which issues from a 12 mm diameter nozzle with a velocity 10 m/s. Calculate the height h of the ball above the nozzle. Assume that the stream remains intact and there is no energy lost in the jet stream.



Source: Dynamics, Meriam and Kraige

The above slide presents a problem.

(Refer Slide Time: 16:01)

Coordinate system and FBD

Equation of motion:

$$m \ddot{x} - \underbrace{\dot{m}_i (\vec{u}_i - \vec{v}) + \dot{m}_o (\vec{v}_o - \vec{v})}_{-T} = \vec{F} = -mg \hat{j}$$

Velocity of water particles at height h is u_i

$$u_i^2 - u^2 = -2gh \Rightarrow u_i(y) = \sqrt{u^2 - 2gh} = \sqrt{100 - 2(9.81)h}$$

$$\dot{m}_i = \rho A u = 1000 \frac{\pi (0.012)^2}{4} (10) \text{ kg/s (at the nozzle)}$$

From equation of motion

$$-\rho A u (u_i \hat{j}) = -mg \hat{j} \Rightarrow 1000 \frac{\pi (0.012)^2}{4} (10) \sqrt{100 - 2(9.81)h} = 0.25(9.81) \text{ N} \quad | \quad h?$$

$$\Rightarrow \underline{h = 4.86 \text{ m}}$$

We consider our system as the ball and its surroundings. Since the ball is stationary at the balanced position, its velocity is zero. We consider that the water particles enter the system boundary at a certain velocity and come to momentary rest against the ball. This leads to some simplification of the master equation for this case, as presented in the slide above. The detailed solution is also presented in the above slide.

(Refer Slide Time: 23:12)



A simple experiment demonstrating the balance of a small plastic ball by blowing through a straw is shown above.

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Summary

- Governing equations of systems with mass flow
- Examples

The above slide summarizes the discussions.