Advanced Dynamics
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Module No # 04
Lecture No # 20
Central Force Motion – IV

We will continue out discussion on central force motion in this lecture.

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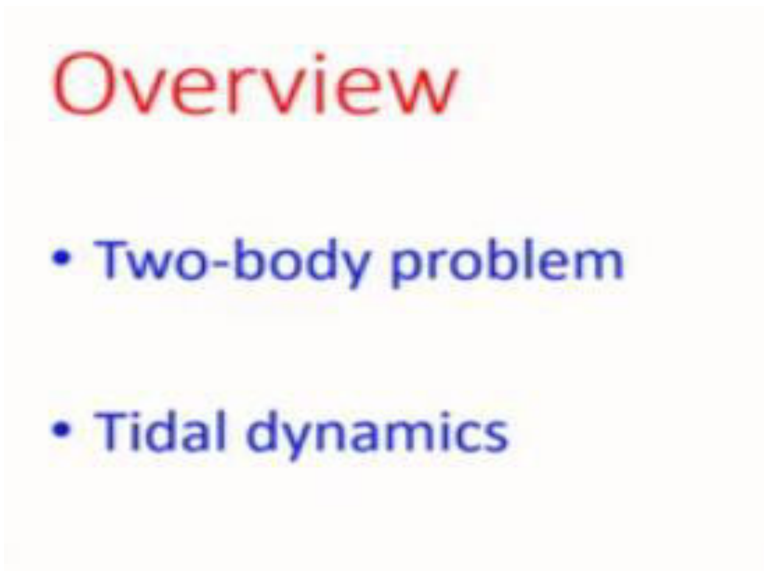

And we are going to now look at the 2 body problem. Till now what we have considered that the central gravitating mass is fixed. Here, we will discuss the 2 body problem considering that both the bodies are free to move. After that, we will discuss tidal dynamics: how tides form, why do they form, and what are the characteristics of tides.

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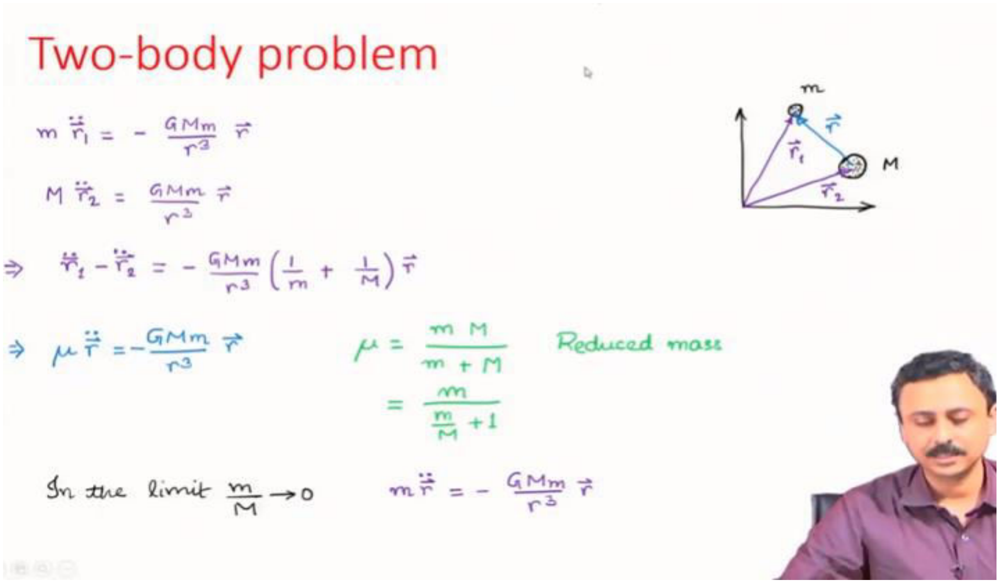

Consider two gravitating bodies of mass m and M, as shown in the slide above. The equations of motion, and finally a single equation of relative motion are derived. It is observed that a 2-body problem can reduce to a body in a central force field if m/M goes to zero.

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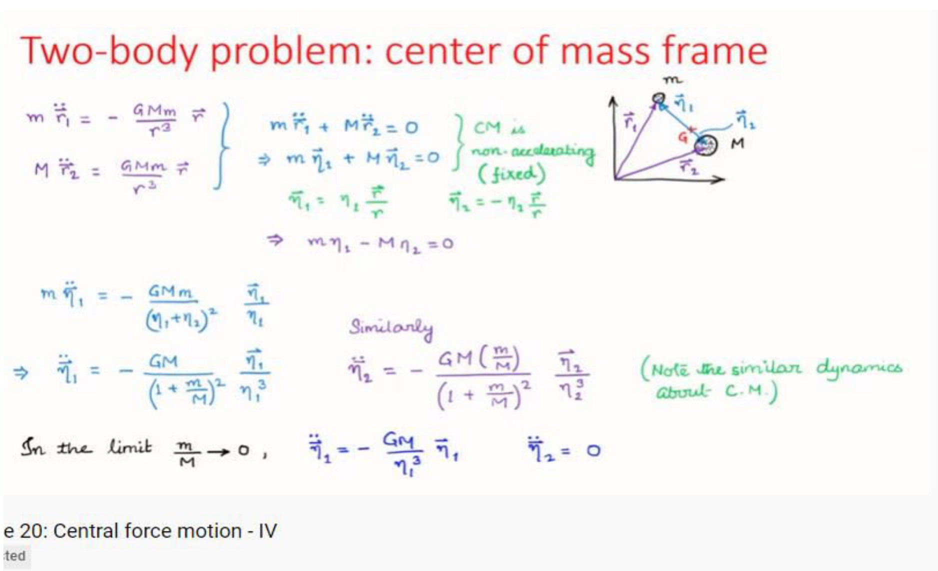

The equations of motion of 2 gravitating bodies in the center of mass frame is presented in the above slide.

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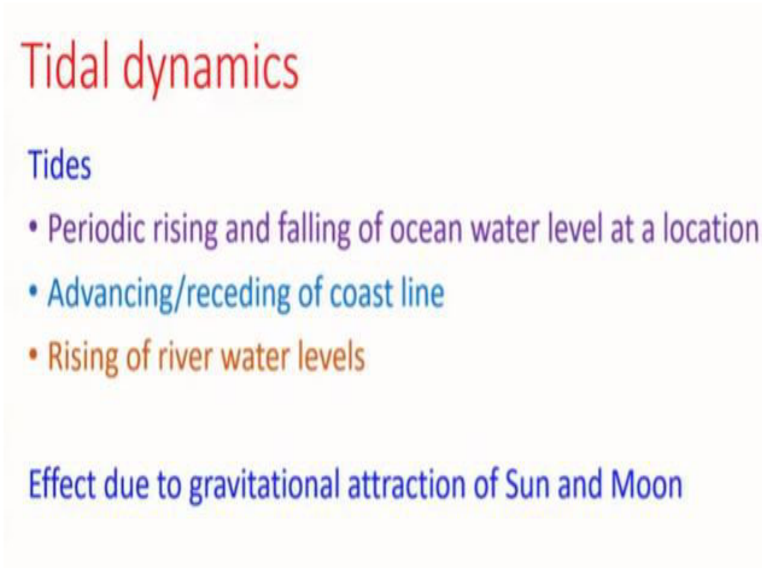

We now move to tidal dynamics. Those who live or have visited the sea shore will note that there is periodic rise and fall of the water level. This periodic rising and falling of ocean level at a particular location in the sea is known as tide. Another effect that might be seen in many places is the advancing or receding of the shore line.

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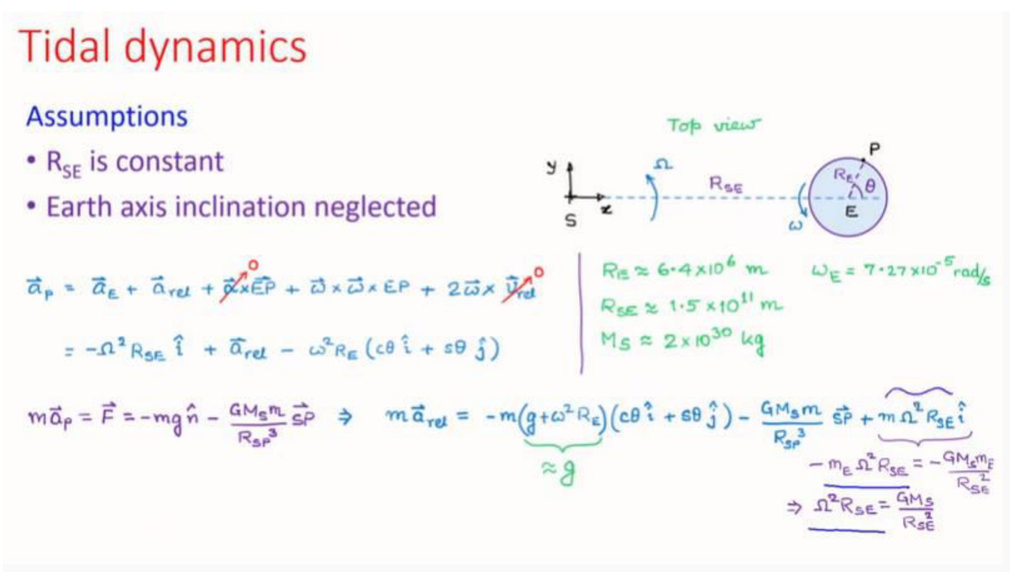

Now to analyze this complex problem, we are going to make some simplifying assumptions. The first assumption that we make is that the sun, earth distance is fixed it is constant. In other words the earth is rotating on a circular path around the sun. The second simplification is that earth's axis of inclination, which is roughly 23.4 degree, will be neglected. The figure in the slide above shows the top view of the earth revolving around the sun. The circle representing the earth is the equatorial circle. Some of the important numbers involved in this problem are given in the slide above. The smallness of the angular speed, v_{rel}, a_{rel} etc. leads to some simplification in the expression of acceleration of a small volume element of water, as presented above.

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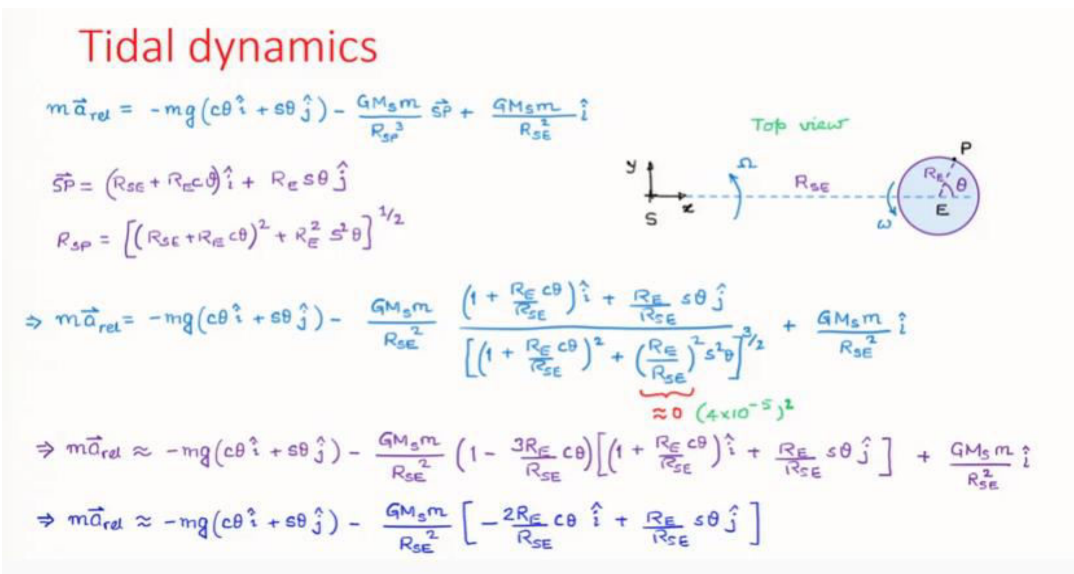

The application of Newton's 2nd law to the small volume element of water is shown in the slide above. Making the approximations as shown above, we finally arrive at the equation of motion as

$$m\vec{a}_{rel} = -mg\left(c\theta\,\hat{i} + s\theta\,\hat{j}\right) - \frac{GM_s m}{R_{SE}^2}\left[-\frac{2R_E}{R_{SE}}c\theta\,\hat{i} + \frac{R_E}{R_{SE}}s\theta\,\hat{j}\right]$$

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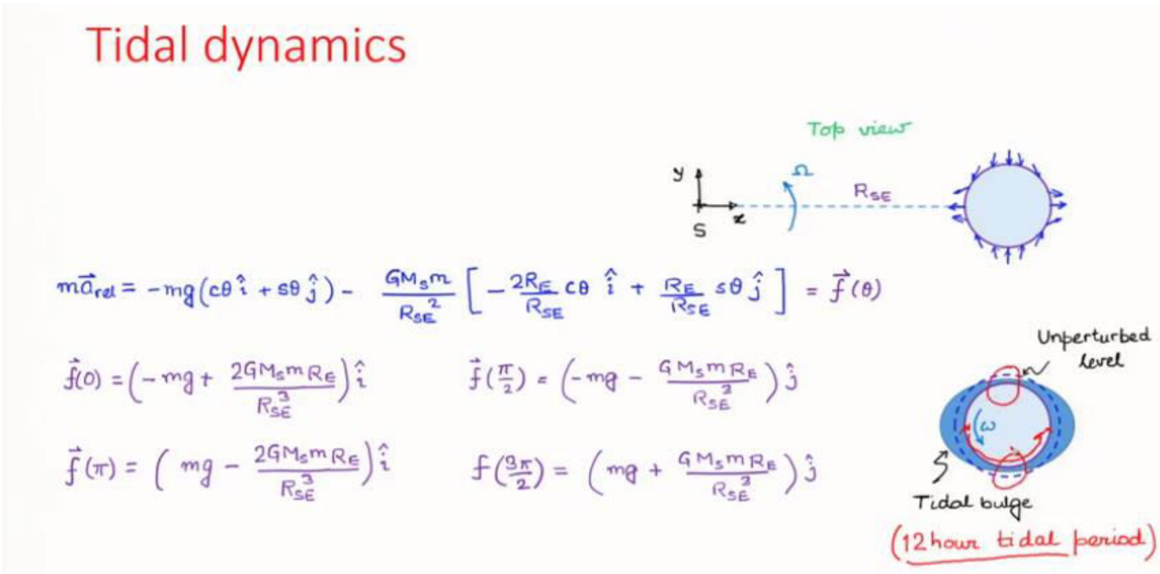

Apart from the weight of the small volume element of water, it is found that there are other force terms as shown in the slide above. This leads to bulging of the ocean level on the sun-earth line and depression in the perpendicular directions, as shown in the figure above.

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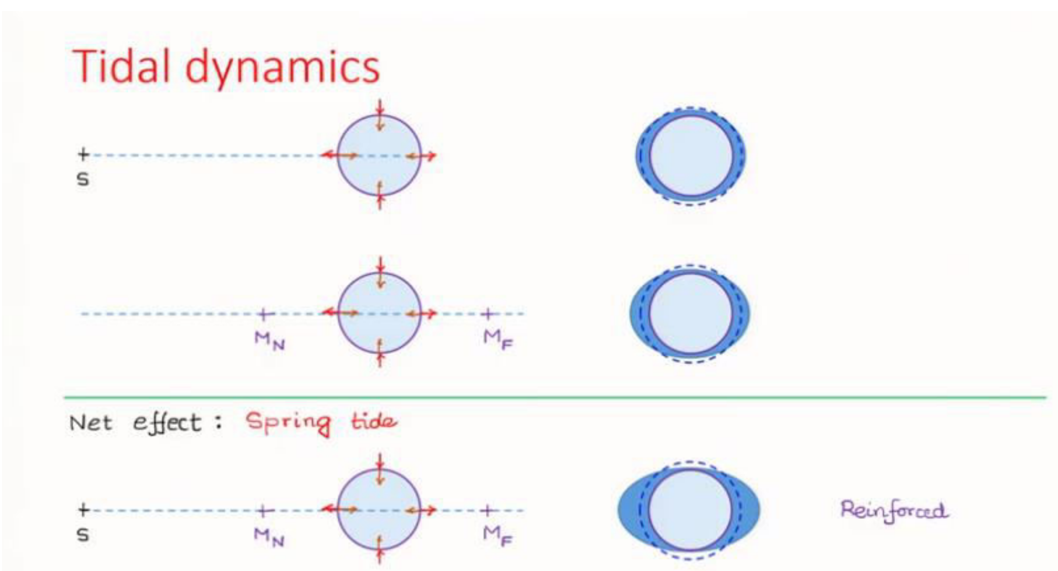

Now we look at the effects of the sun and the moon taken together. Because of the moon, the tidal bulge very similar to that of the sun. Because of the nearness of the moon to the earth, the effect the tidal bulge is more due to the moon. On a new moon and full moon configuration, when the moon, earth and the sun are approximately on the same line, both the effects reinforce each other to form the highest tidal bulge (and the lowest tidal depression in the perpendicular directions). This high-tide is called the spring tide.

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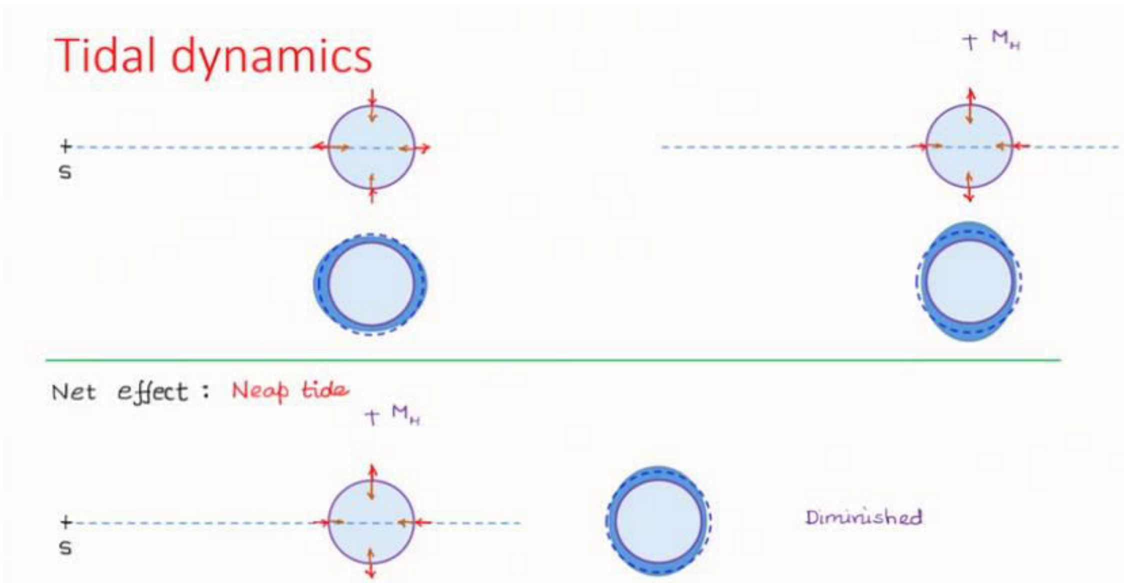

On the other hand, when the moon-earth line makes 90 degree angle with the sun-earth line, the tidal effects of the sun and the moon are in opposition. The moon tends to pull the tidal bulge towards itself whereas the sun depresses it somewhat. Therefore there is this net effect is diminished, and this is known as neap tide.

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The above slide summarizes the discussions.