Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharaghpur

Module No # 04
Lecture No # 19
Central Force Motion – III

In this lecture we are going to continue our discussions on central force field motion and we are going to look at some problems. The theory is first recapitulated in the following slides.

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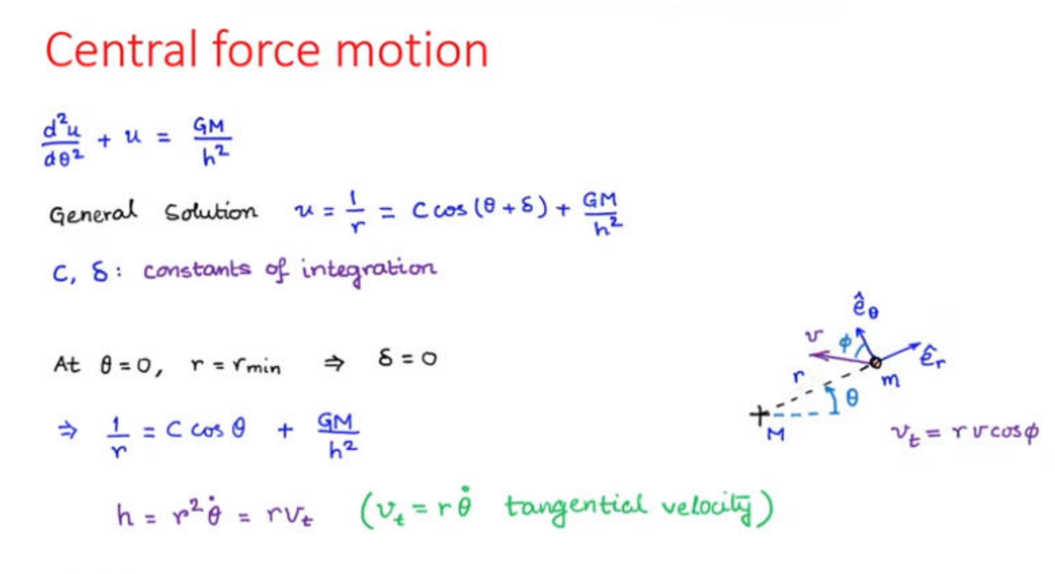

In the above slide the equation of motion of a particle in a central force field is presented along with the solution of the motion.

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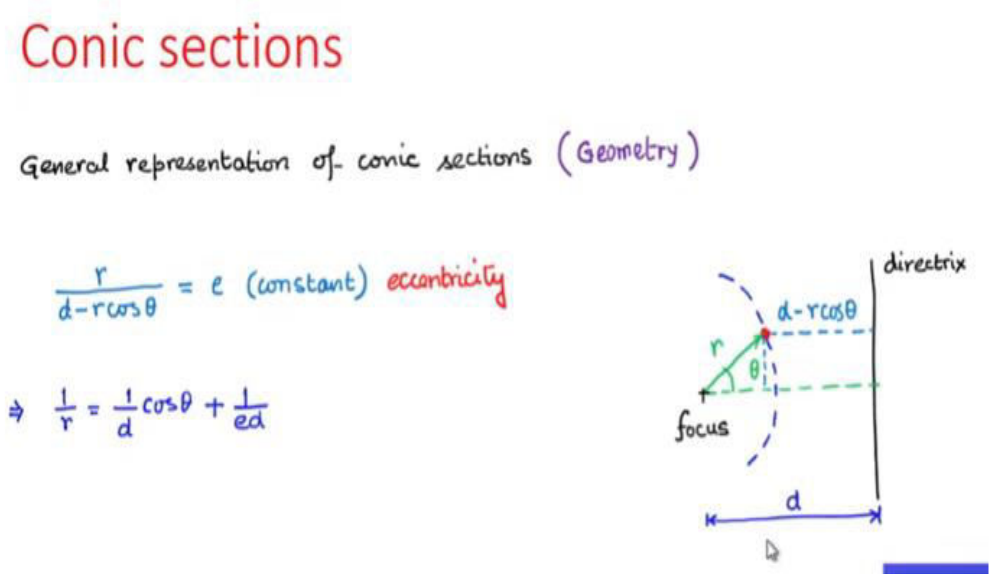

The mathematical representation of conic sections is presented in the above slide. A similarity of this representation with the structure of solution of a massive particle in a central force field is clearly observed.

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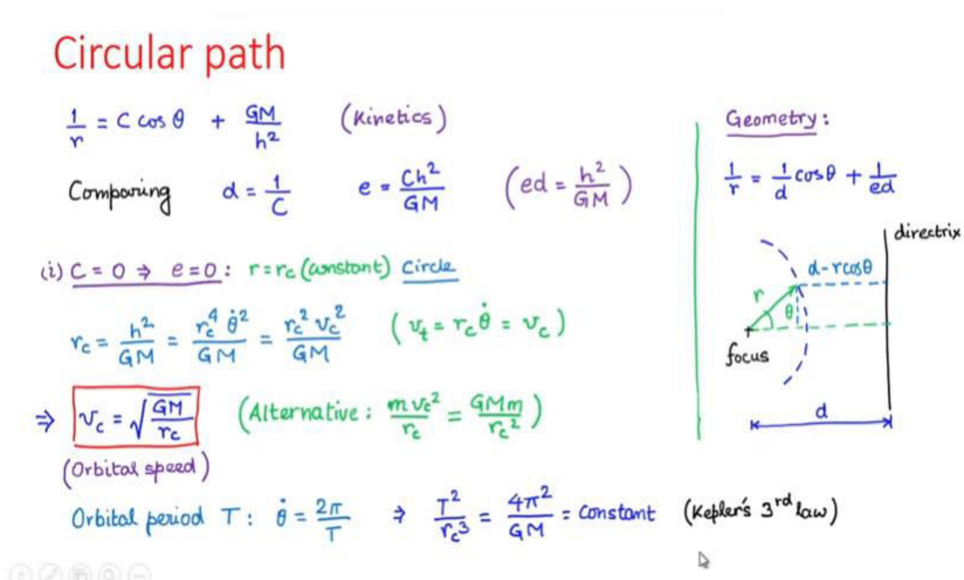

A comparison between the conic section and the solution is presented above. The characteristics of a circular trajectory is also shown.

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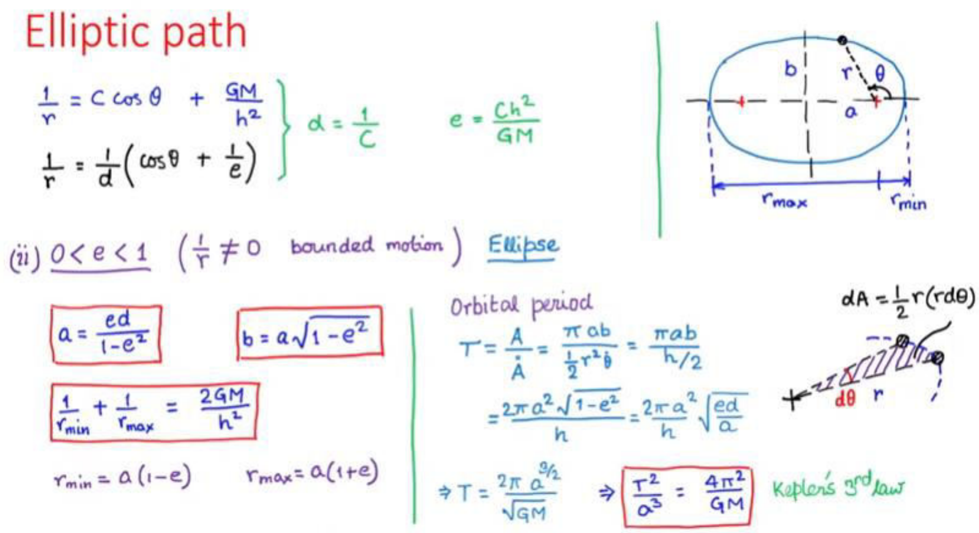

Characteristics of the elliptic path are presented above.

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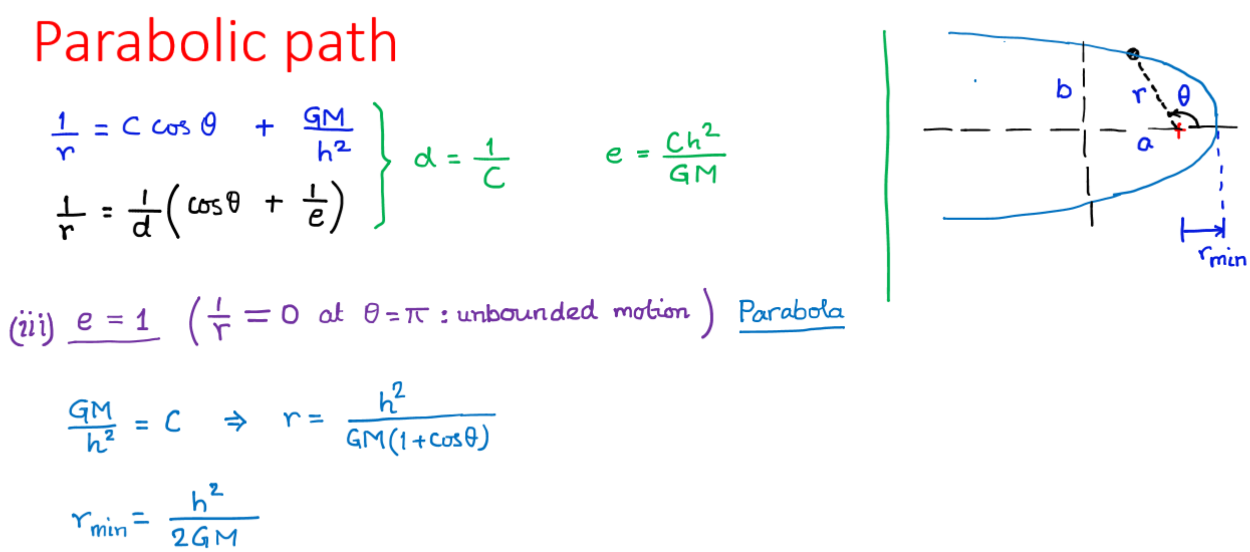

Characteristics of the parabolic path are presented above.

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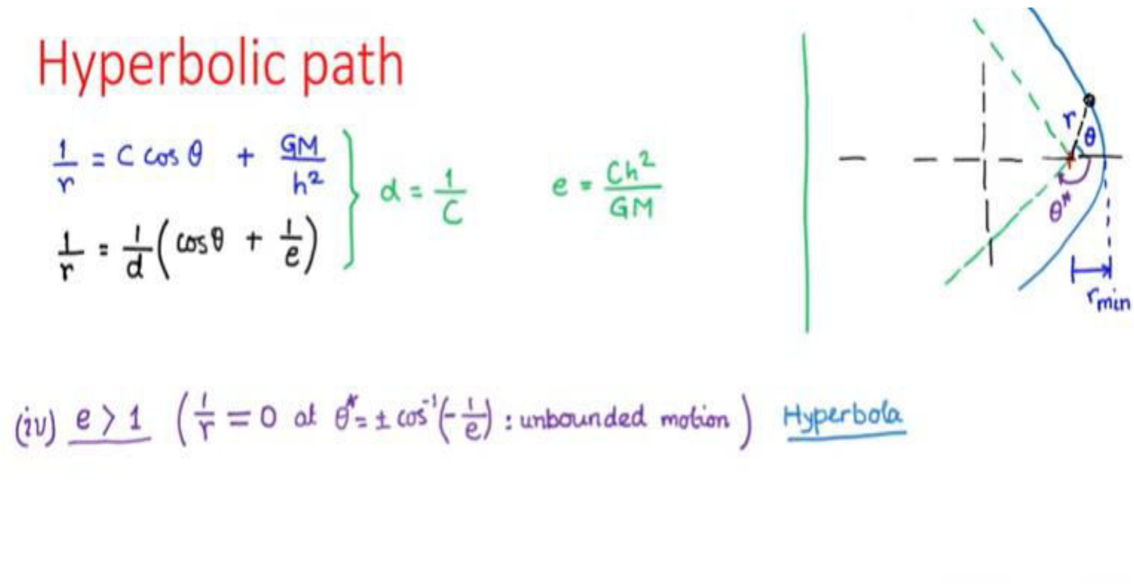

Characteristics of the hyperbolic path are presented in the slide above.

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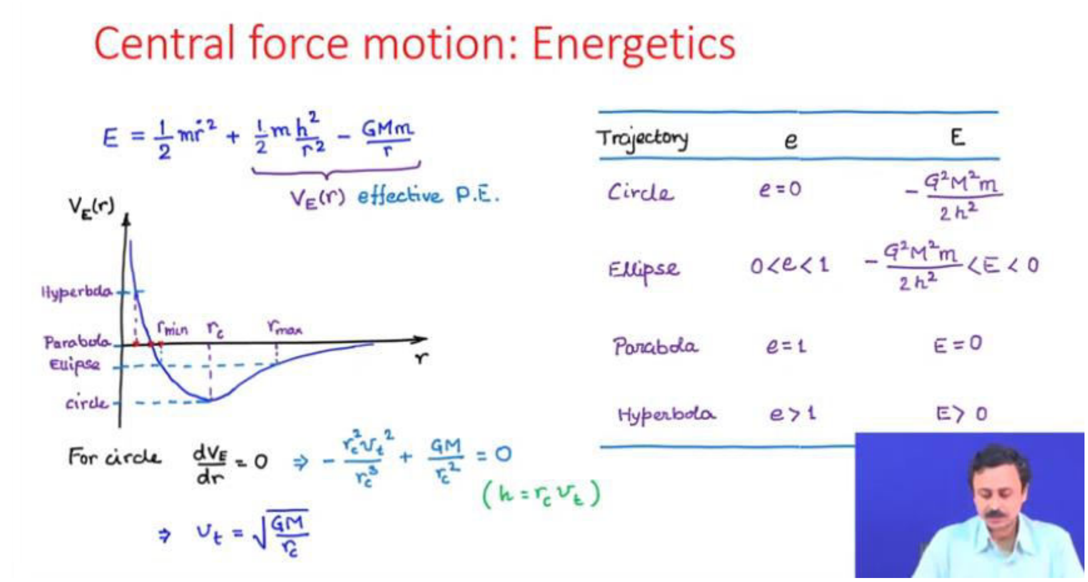

The energetics of motion of a massive particle in a central force field is presented in the slide above. We plotted and discussed the effective potential energy of the particle. We found that, for bounded trajectories, the effective energy is negative, while for unbounded trajectories it is 0 or greater. We also calculated escape velocity etc.

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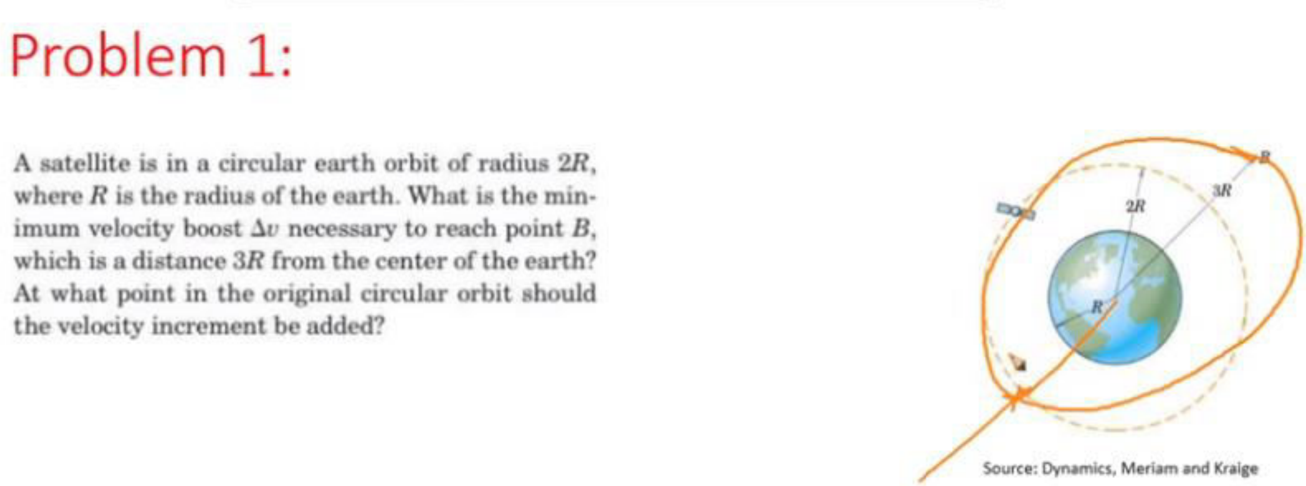

We consider the above problem. An important consideration is the determination of minimum velocity boost. This will happen when the satellite will reach B as the furthest point on the new trajectory. Thus, the new trajectory should be an ellipse with B as the apogee. And therefore the diametrically opposite point should be the perigee.

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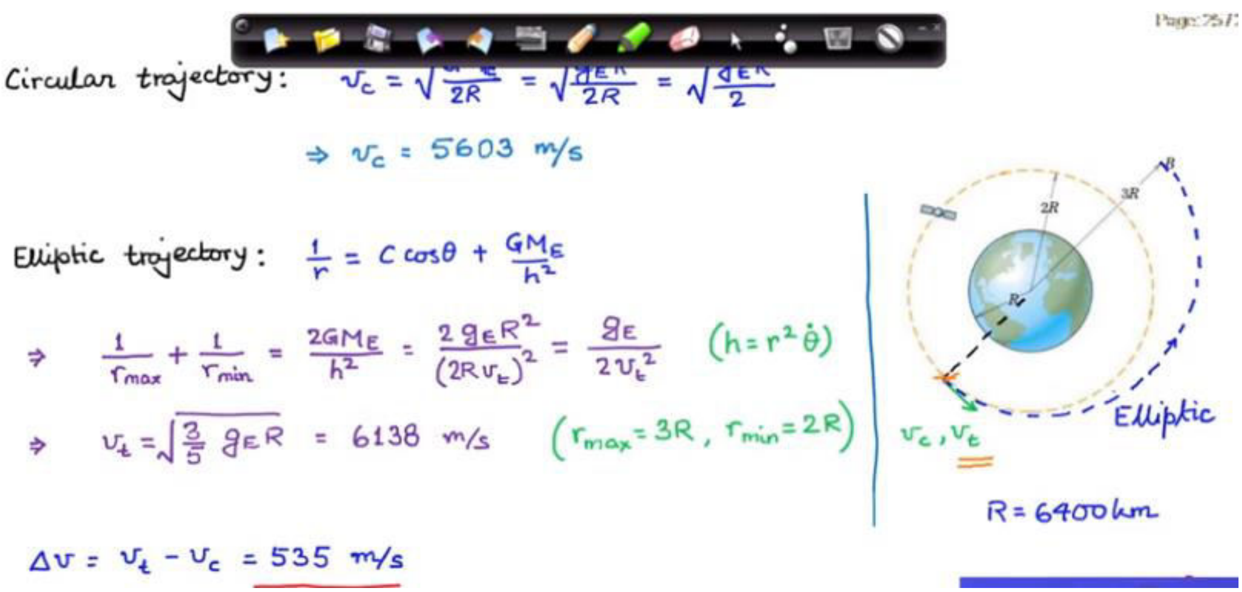

With these considerations, the solution is presented in the slide above.

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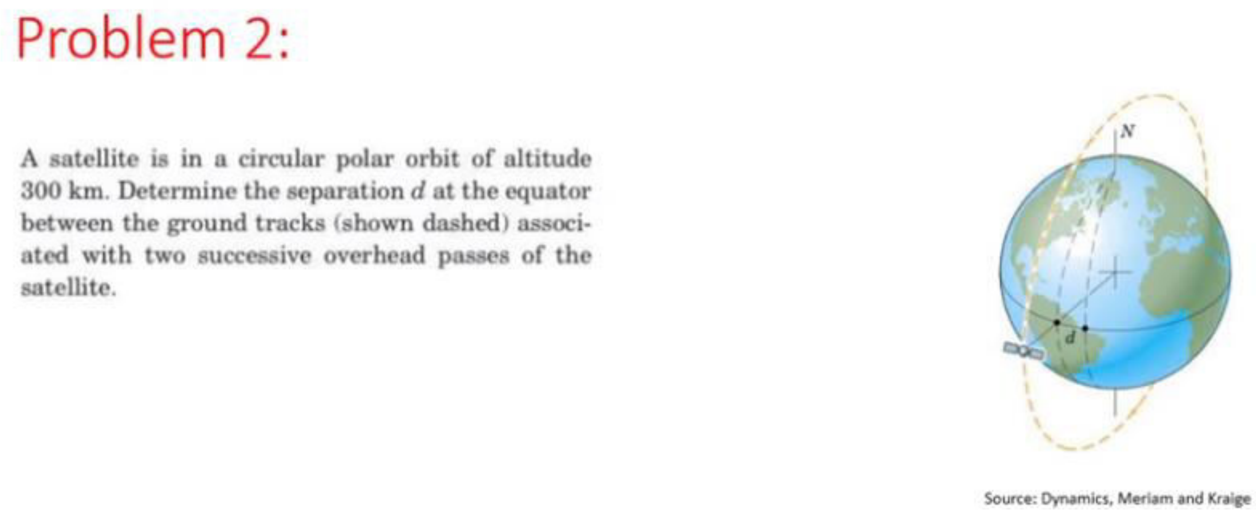

The next problem is presented in the slide above.

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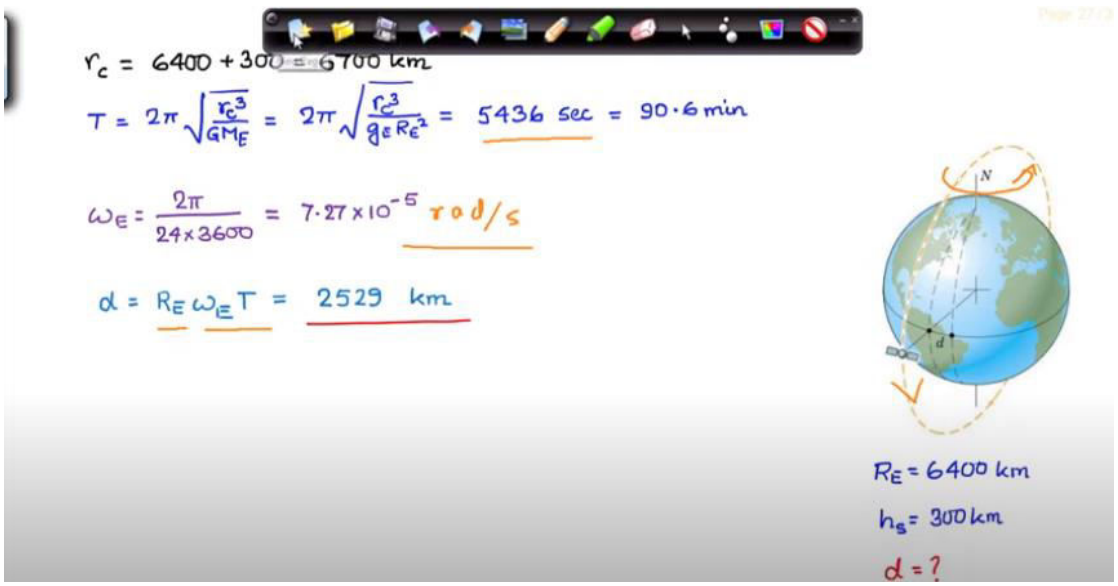

The solution is presented in the slide above.

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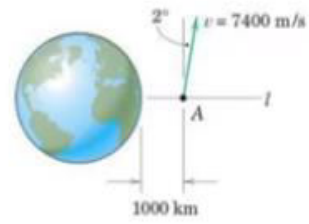
Problem 3:

A spacecraft in an elliptical orbit has the position and velocity indicated in the figure at a certain instant. Determine the semimajor axis length a of the orbit and find the acute angle α between the semimajor axis and the line l. Does the spacecraft eventually strike the earth?



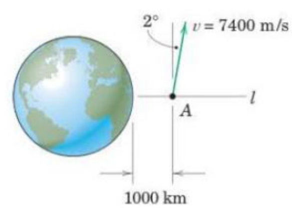
Source: Dynamics, Meriam and Kraige

The next problem is shown in the slide above. We have to calculate the trajectory of the space craft and also check whether it strikes earth or not. It will strike the earth if the minimum separation for the center of the earth falls below the radius of the earth.

$$h = r^2\dot{\theta} = r v_t$$

$$\Rightarrow h = (74\times10^5)\, 7400 \cos 2^\circ = 5.473\times10^{10} \ m^2/s$$

$$\dot{r} = v \sin 2^\circ = 258.26 \ m/s$$



$R_E = 6400$ km

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{g_E R_E^2}{r} m$$

$$= -2.69\times10^7 m \quad (\text{constant})$$

At perigee and apogee

$$E = \frac{1}{2} m \frac{h^2}{r^2} - \frac{g_E R_E^2}{r} m = -2.69\times10^7 m$$

$$\Rightarrow \quad 2.69\times10^7 r^2 - 4.018\times10^{14} r + 1.4978\times10^{21} = 0$$

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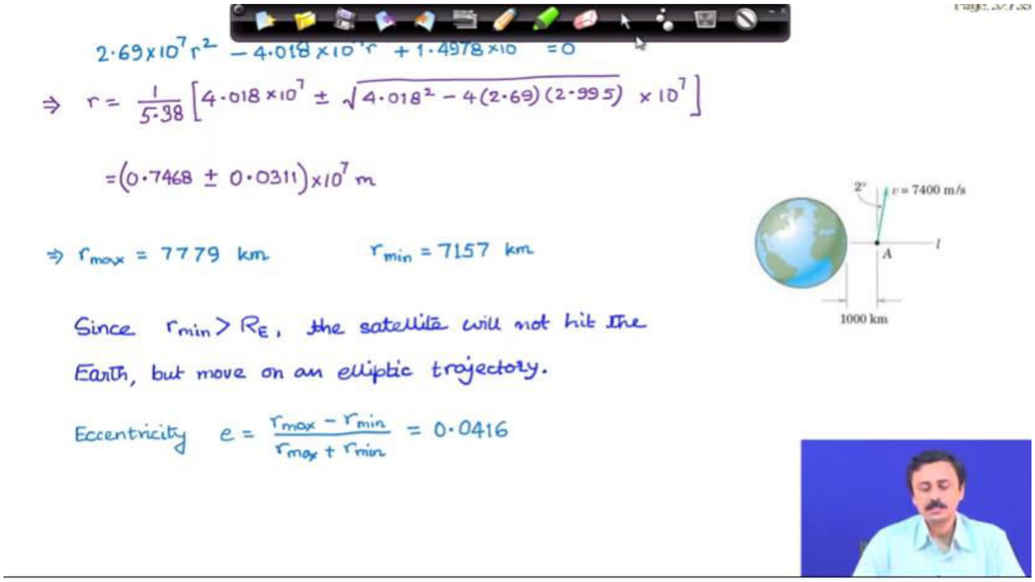

The solution is presented in the 2 slides above.

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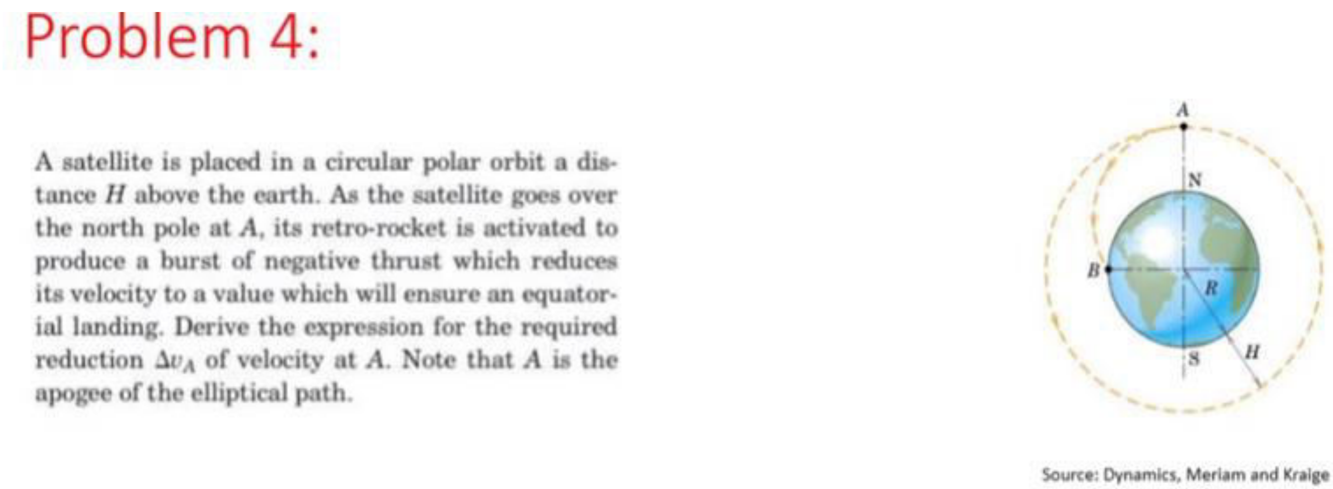

The next problem is presented above. It is to be understood that the path will be elliptic with A as the apogee.

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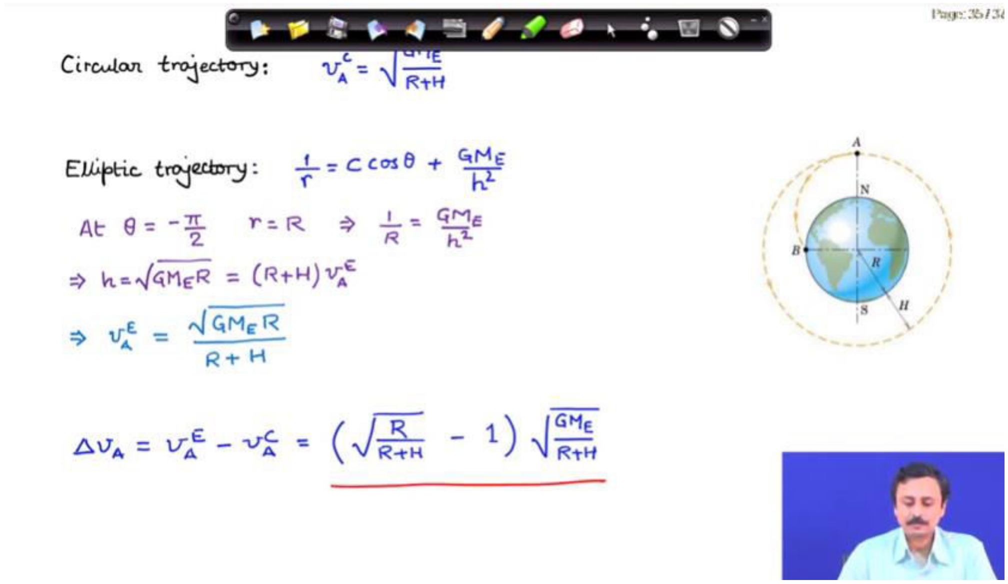

The solution is presented in the slide above.

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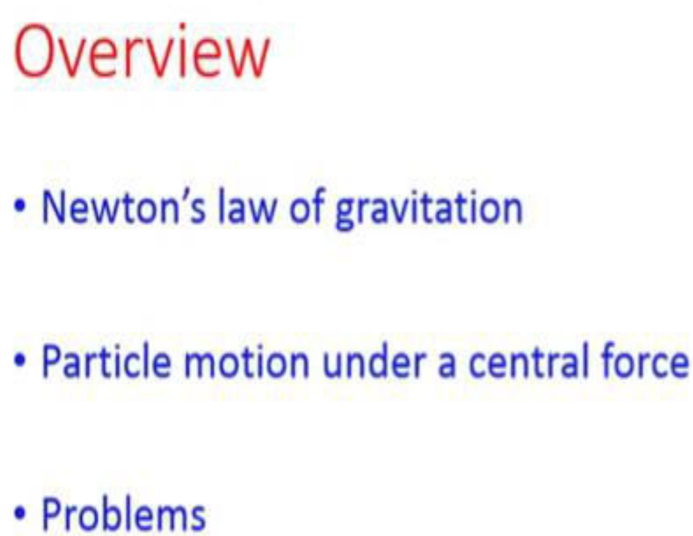

The summary is presented in the slide above.