Advanced Dynamics
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Module No # 04
Lecture No # 17
Central Force Motion – I

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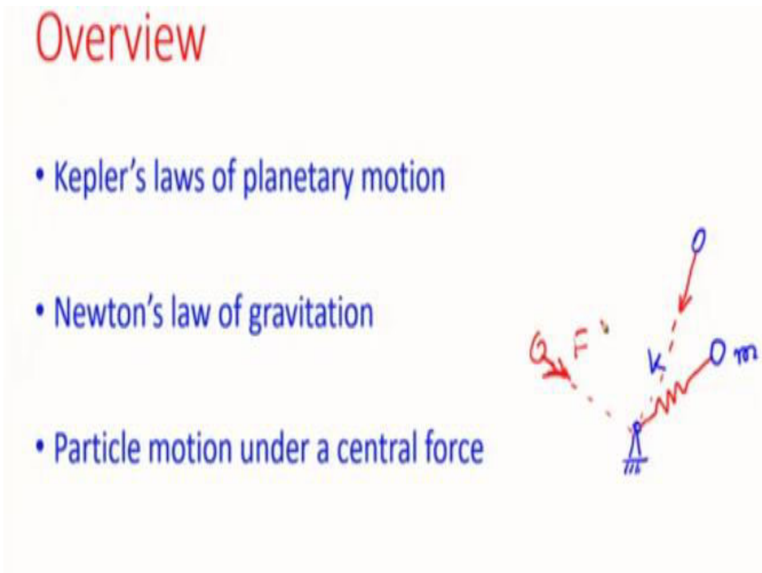

In this lecture we are going to start our discussions on central force motion now the first question that arises what is central force motion? Central force motion is the motion of a massive particle moving in a central force field. A central force field is a force field in which if you place a particle the force that acts on the particle is towards the fixed part in the inertial space.

We are going to start with Kepler's law of planetary motion then we are going to look at Newton's law of gravitation. And then we are going to study the dynamics of a particle in a central gravitational force field.

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Central-force motion

Motion of a particle under the influence of a fixed central gravitating body

Tycho Brahe (1546 – 1601): Accurate astronomical data

Johannes Kepler (1571 – 1630): Laws of planetary motion (kinematic)

Issac Newton (1642 – 1727): Universal law of gravitation (kinetic), generalized trajectories

So central force motion as I mentioned motion of particle under the influence of a fixed central gravitating body so central gravitational body is considered and that is considered to be fixed. Now all this started with Tycho Brahe. One of his interests was collecting astronomical data. Johannes Kepler was, assistant of Tycho Brahe, and had inherited the data were collected by Tycho Brahe. From these very accurate observations of Brahe, Kepler observed some regularity in the motion of planets which he could formulate as his 3 laws. These laws were purely kinematic.

Then came the master and gave the universal law of gravitation. In one stroke, using his laws and the universal law of gravitation, the problem entered the realm of kinetics. This approach also predicted paths like hyperbolic or parabolic paths which are very difficult to observe.

The Kepler’s laws are shown below.

Kepler’s laws of planetary motion

- Every planet moves in an elliptic path with Sun as one of the foci
- The Sun-planet line segment sweeps out equal areas in equal times
- The ratio of square of the orbital time period and the cube of the semi-major axis of the elliptic path is same for all planets

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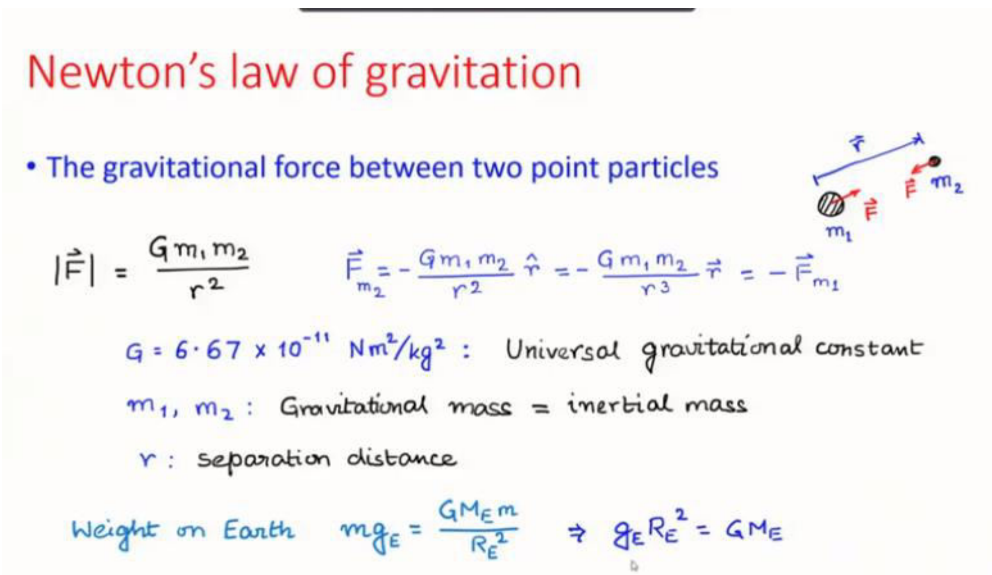

Newton’s law of gravitation is shown above.

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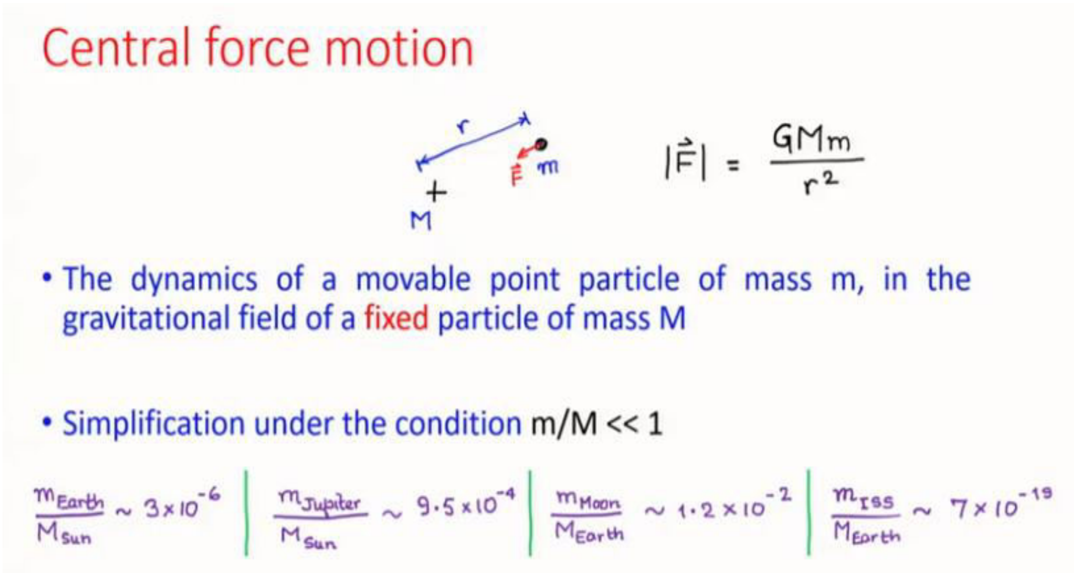

The condition under which the central gravitating mass can be considered fixed is discussed above. This depends on the mass ratio of the moving mass and the central mass which generates the gravitational field.

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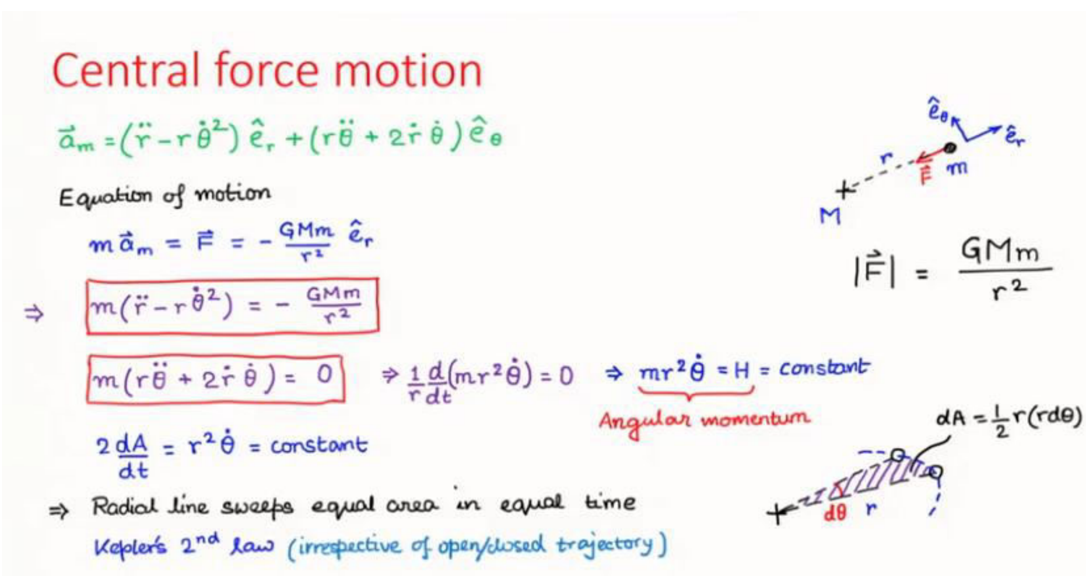

The above slide shows the formulation of dynamics of a particle in a central force field. This leads to the major conclusion of conservation of angular momentum. Further, this leads to the Kepler's 2^{nd} law of motion.

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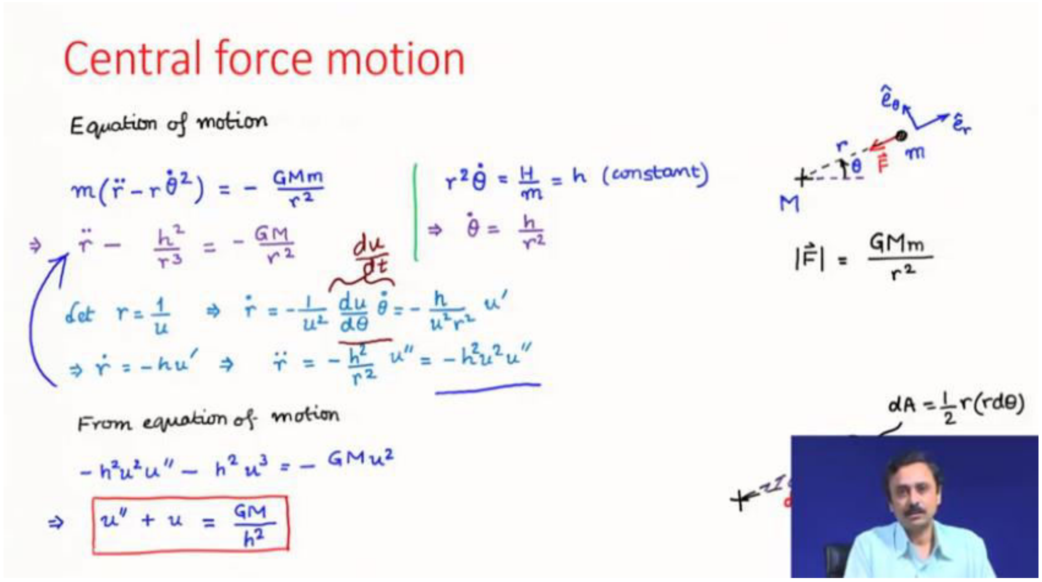

The above slide derives the equation of motion of the particle in terms of a new variable u=1/r. This is a second order linear ordinary differential equation with a constant right hand side.

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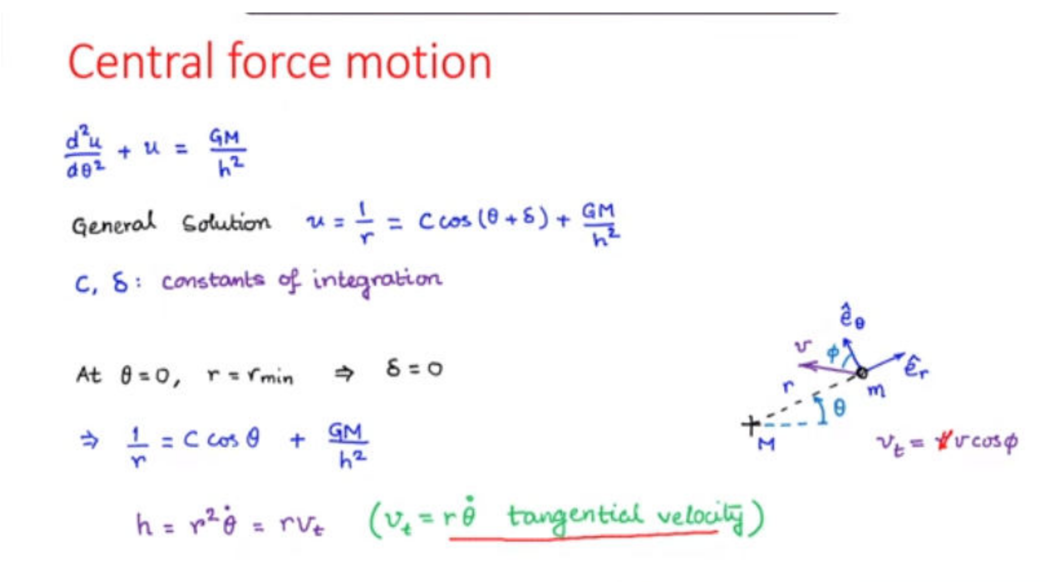

The general solution form is presented in the above slide. The constant phase is set as zero with the consideration that the datum of the angular coordinate is taken to be the line joining the force center and the point of nearest distance r_{min} of the particle.

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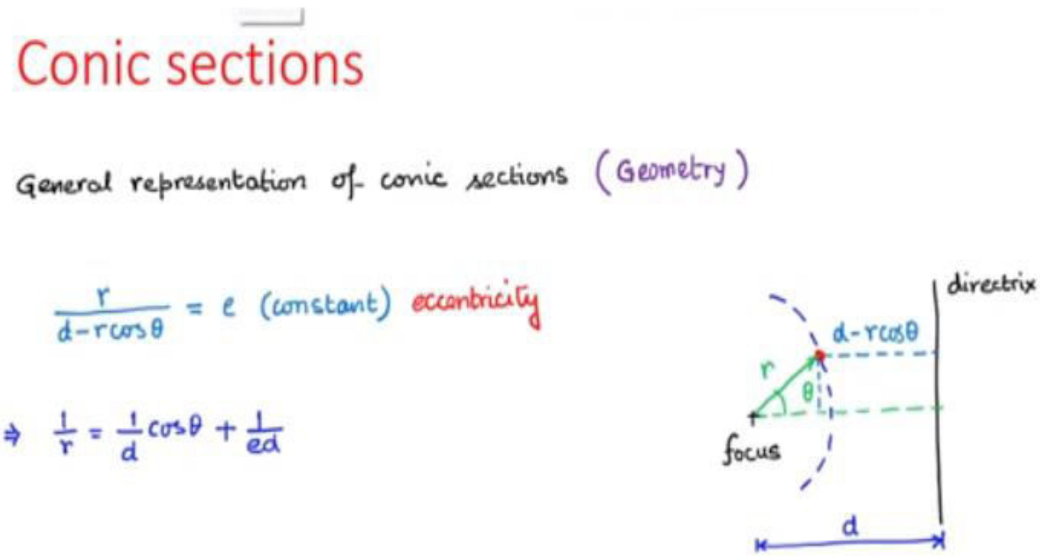

The general representation of conic sections is presented in the slide above.

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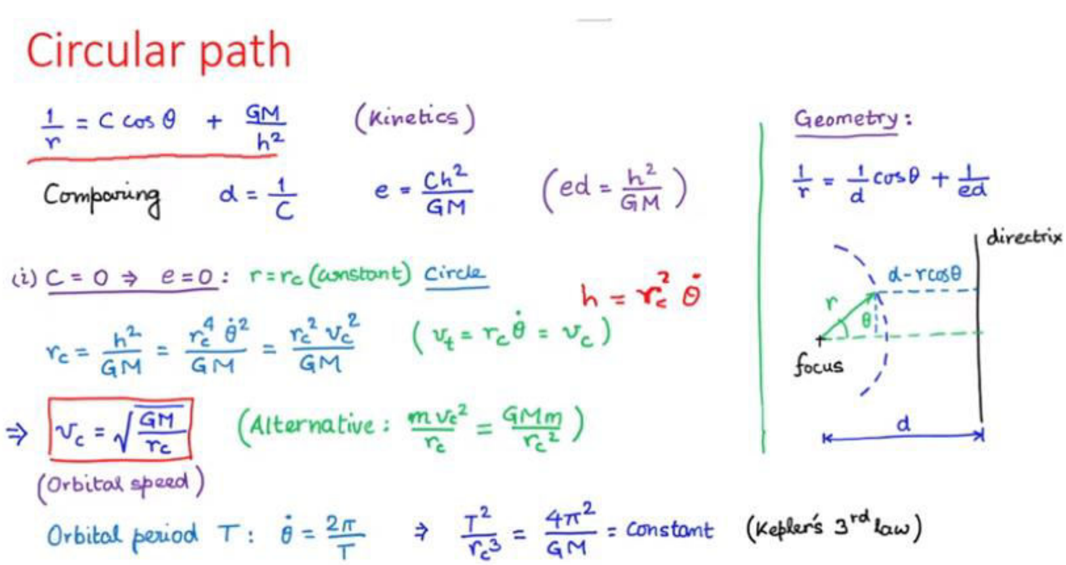

A comparison of the mathematical representation of conic sections with the solution obtained for a particle moving in a central force field clearly shows a striking similarity. The relations of constants of the two forms are shown in the slide above. Also, the characteristics of a circular trajectory are presented and discussed. A special case of Kepler's 3rd law is also derived.

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