

Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Module No # 03
Lecture No # 16
Particle Impact – II

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Impact

Short-time impulsive interaction between two particles

Outcomes after impact:

- separation with energy conservation (elastic impact)
- separation without energy conservation (inelastic impact)
- merger (plastic impact)

Short-time change in total linear/angular momentum of the system is zero

Assumption: frictionless interaction

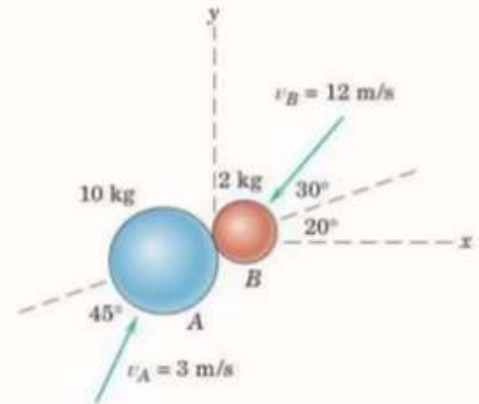


In this lecture, we are going to look at some problems and the application of the theory that we have developed.

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Problem 1:

Sphere A collides with sphere B as shown in the figure. If the coefficient of restitution is $e = 0.5$, determine the x- and y-components of the velocity of each sphere immediately after impact. Motion is confined to the x-y plane.



Source: Dynamics, Meriam and Kraige

The first problem statement is shown above. The detailed solution is shown in the following 2 slides. We solve the problem first using the tangent-normal coordinates at the contact. Later, we transform the vectors in the x-y coordinates as discussed below in detail.

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Coordinate system

$$\vec{v}_A' = v_{A't} \hat{t} + v_{A'n} \hat{n}$$

$$\vec{v}_B' = v_{B't} \hat{t} + v_{B'n} \hat{n}$$

Momentum conservation along \hat{t} and \hat{n}

\hat{t} :

$$\begin{cases} m_A v_A \sin 45^\circ = m_A v_{A't} \Rightarrow v_{A't} = \frac{3}{\sqrt{2}} \text{ m/s} \\ -m_B v_B \sin 30^\circ = m_B v_{B't} \Rightarrow v_{B't} = -6 \text{ m/s} \end{cases}$$

\hat{n} :

$$m_A \frac{3}{\sqrt{2}} + m_B (-6\sqrt{3}) = m_A v_{A'n} + m_B v_{B'n}$$

$$\Rightarrow 10 v_{A'n} + 2 v_{B'n} = 15\sqrt{2} - 12\sqrt{3} \quad \text{---(1)}$$

Normal impact along \hat{n}

$$\frac{(\vec{v}_B' - \vec{v}_A') \cdot \hat{n}}{(\vec{v}_A - \vec{v}_B) \cdot \hat{n}} = e = 0.5$$

$$\Rightarrow v_{B'n} - v_{A'n} = 0.5 \left(\frac{3}{\sqrt{2}} + 6\sqrt{3} \right) \quad \text{---(2)}$$

From (1) and (2)

$$v_{B'n} = 5.25 \text{ m/s}$$

$$v_{A'n} = -1.007 \text{ m/s}$$

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Handwritten calculations for the collision problem:

$$\hat{i} \cdot \vec{v}'_A = (2.121 \hat{e} - 1.007 \hat{n}) \cdot \hat{i} \quad \vec{v}'_B = -6 \hat{e} + 5.25 \hat{n} \text{ m/s}$$

$$= (v'_{Ax} \hat{i} + v'_{Ay} \hat{j}) \cdot \hat{i} = v'_{Ax} = v'_{Bx} \hat{i} + v'_{By} \hat{j}$$

$$v'_{Ax} = \vec{v}'_A \cdot \hat{i} = 2.121 \cos 110^\circ - 1.007 \cos 20^\circ$$

$$= -1.67 \text{ m/s}$$

$$v'_{Ay} = \vec{v}'_A \cdot \hat{j} = 2.121 \cos 20^\circ - 1.007 \cos 70^\circ$$

$$= 1.65 \text{ m/s}$$

$$v'_{Bx} = \vec{v}'_B \cdot \hat{i} = -6 \cos 110^\circ + 5.25 \cos 20^\circ$$

$$= 6.99 \text{ m/s}$$

$$v'_{By} = \vec{v}'_B \cdot \hat{j} = -6 \cos 20^\circ + 5.25 \cos 70^\circ$$

$$= -3.84 \text{ m/s}$$

Diagram labels:

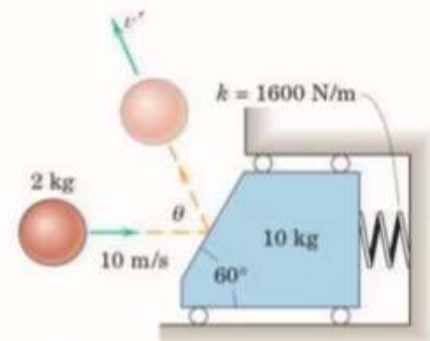
- 10 kg sphere A, 2 kg sphere B
- Initial velocity of A: $v_A = 5 \text{ m/s}$ at 45°
- Initial velocity of B: $v_B = 12 \text{ m/s}$ at 30°
- Angle of impact: 20°
- Unit vectors: \hat{e} (horizontal), \hat{n} (normal to contact surface)
- Relationships: $\hat{e} \cdot \hat{i} = \cos \phi$, $\hat{n} \cdot \hat{i} = \cos 20^\circ$

The final solution in terms of the components of the outgoing velocity vectors in the x-y coordinate system is presented in the slide above.

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Problem 2:

The 2-kg sphere is projected horizontally with a velocity of 10 m/s against the 10-kg carriage which is backed up by the spring with stiffness of 1600 N/m. The carriage is initially at rest with the spring uncompressed. If the coefficient of restitution is 0.6, calculate the rebound velocity v' , the rebound angle θ , and the maximum travel δ of the carriage after impact.



Source: Dynamics, Meriam and Kraige

The second problem statement is presented above. The detailed solution steps are presented in the next 2 following slides.

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Coordinate system

$$\vec{v} = 10 \hat{i} \text{ m/s} = 10 (-\cos 30^\circ \hat{n} + \sin 30^\circ \hat{t}) \text{ m/s}$$

$$\vec{v}' = v'_n \hat{n} + v'_t \hat{t}$$

$$\vec{v} = 0, \quad \vec{v}' = v'_t \hat{t} = v' (-\cos 30^\circ \hat{n} + \sin 30^\circ \hat{t})$$

Momentum conservation along \hat{t} and \hat{i}

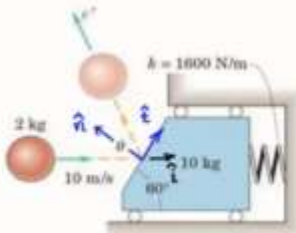
\hat{t} : $m \vec{v} \cdot \hat{t} = m \vec{v}' \cdot \hat{t} \Rightarrow v'_t = 5 \text{ m/s}$

\hat{i} : $m \vec{v} \cdot \hat{i} = m \vec{v}' \cdot \hat{i} + M \vec{V}' \cdot \hat{i}$

$$\Rightarrow 2(10) = 2 [v'_n (-\cos 30^\circ) + v'_t \cos 60^\circ] + 10 v'$$

$$\Rightarrow 15 = -\sqrt{3} v'_n + 10 v'$$

Normal impact $\frac{(\vec{v}' - \vec{v}) \cdot \hat{n}}{(0 - \vec{v}) \cdot \hat{n}} = e = 0.6 \Rightarrow v'_n + v' \cos 30^\circ = 0.6 (10 \cos 30^\circ)$

$$\Rightarrow v'_n + v' \frac{\sqrt{3}}{2} = 3\sqrt{3}$$


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$$\left. \begin{aligned} -\sqrt{3} v'_n + 10 v' &= 15 \\ v'_n + v' \frac{\sqrt{3}}{2} &= 3\sqrt{3} \end{aligned} \right\} \begin{aligned} v' &= 2.087 \text{ m/s} \\ v_n &= 3.389 \text{ m/s} \end{aligned}$$

$$\vec{v}' = 3.389 \hat{n} + 5 \hat{t} \text{ m/s}$$

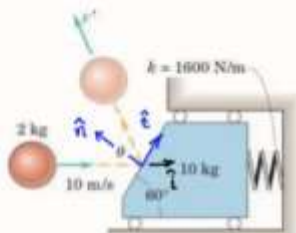
$$|\vec{v}'| = 6.04 \text{ m/s}$$

$$\cos \theta = (\hat{i}) \cdot \frac{\vec{v}'}{|\vec{v}'|} = -\frac{1}{6.04} \left[3.389 \left(-\frac{\sqrt{3}}{2}\right) + 5 \left(\frac{1}{2}\right) \right]$$

$$\Rightarrow \theta = 85.9^\circ$$

$$\vec{V}' = 2.087 \hat{i} \text{ m/s}$$

Energy conservation: $\frac{1}{2} M v'^2 = \frac{1}{2} k \delta^2 \Rightarrow \delta = \sqrt{\frac{M}{k}} v' = 0.165 \text{ m}$



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Summary: Impact

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Assumption: frictionless interaction

The summary of discussions in this lecture is presented above.