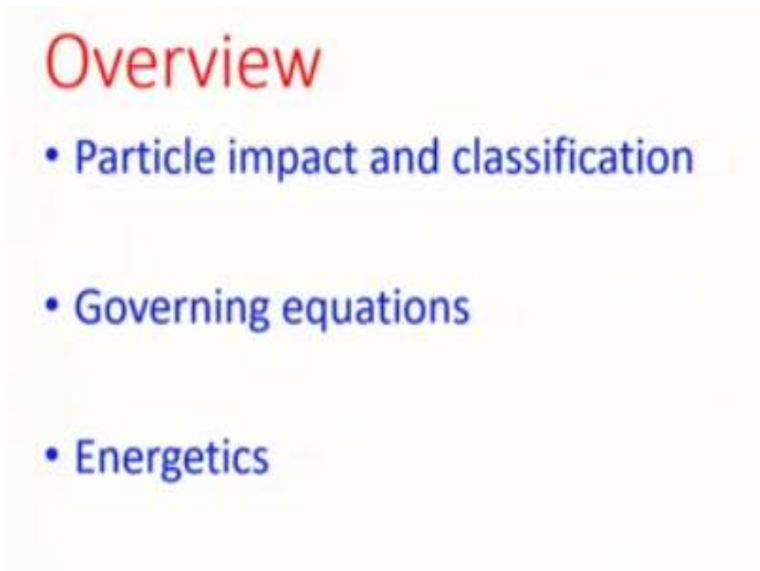
Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Module No # 03
Lecture No # 15
Particle Impact – I

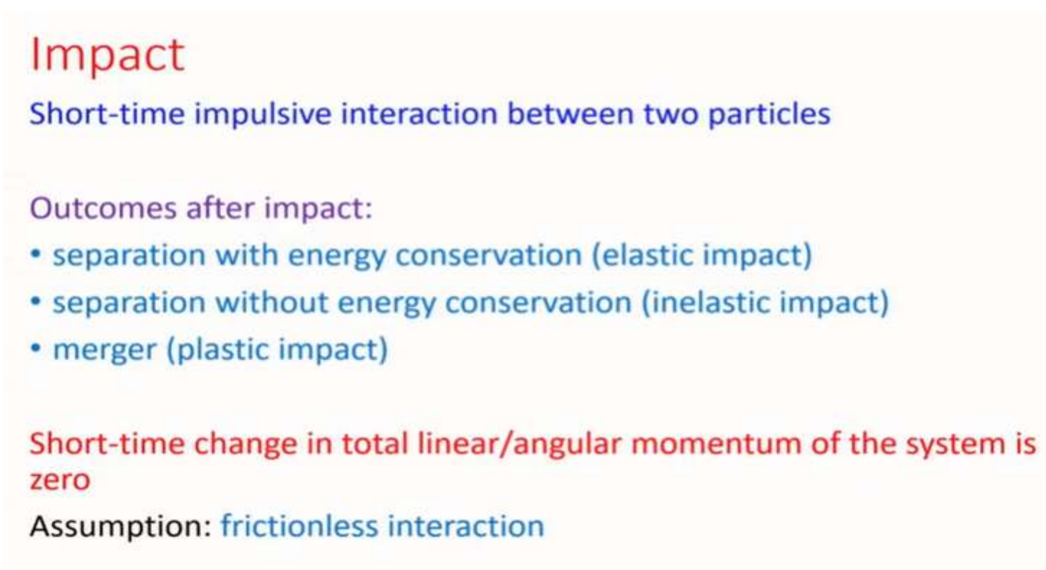
In this lecture we are going to discuss the dynamics of impacting particles.

(Refer Slide Time: 00:25)



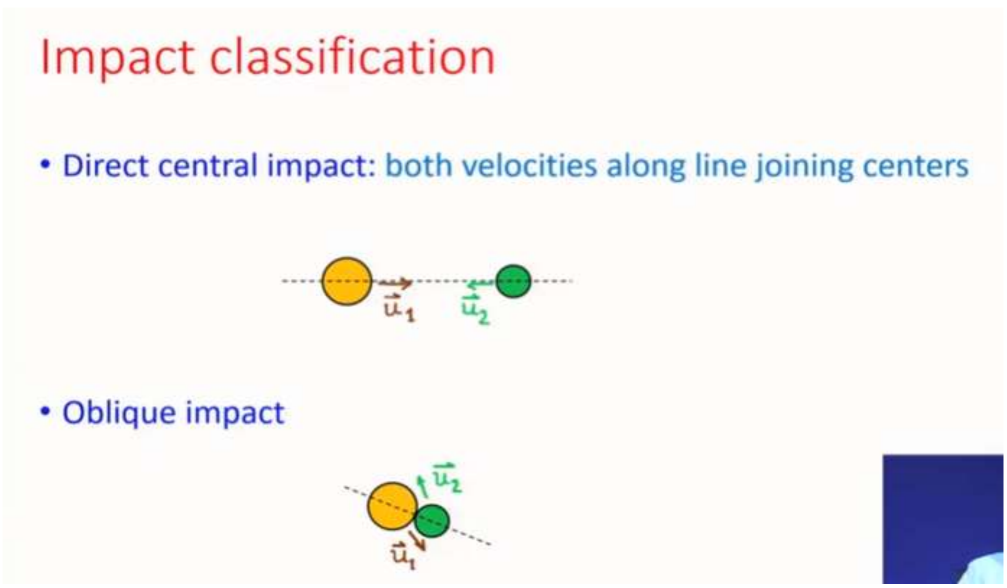
The overview of this lecture is shown above.

(Refer Slide Time: 00:46)



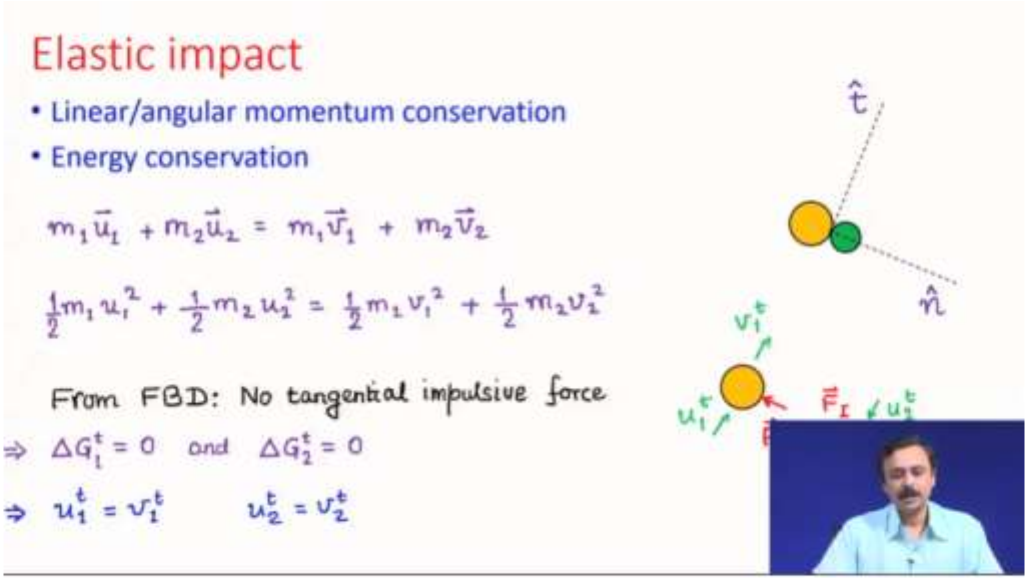
Impact is a short time impulsive interaction between particles, or a particle and a rigid surface. Here, we are going to restrict ourselves to impact of 2 particles. What could be the possible outcomes of such impacts? One can have 2 particles which were coming close, impacting and then separating out with energy conservation. Such impacts are called elastic impacts or elastic collisions. In the second case, we can have separation but some part of the energy might be lost during the impact. This is called inelastic impact. There can be a third situation where we can have merger of the particles, which is called plastic impact. The reverse of the third case is where a single particle explodes to form multiple particles. In the short time scale of the impact process, the total linear momentum of the system does not change. This we have seen in the previous lecture. Here, we will make the assumption that all contacts are frictionless.

(Refer Slide Time: 03:15)



Now we can have 2 kinds of impact as shown above: (1) direct central impact, where the incoming velocities of both the particles are on the line joining the particles, and (2) oblique impact, where the incoming velocity directions of the 2 particles at the time of impact are both not be along the line joining the 2 particles.

(Refer Slide Time: 04:27)

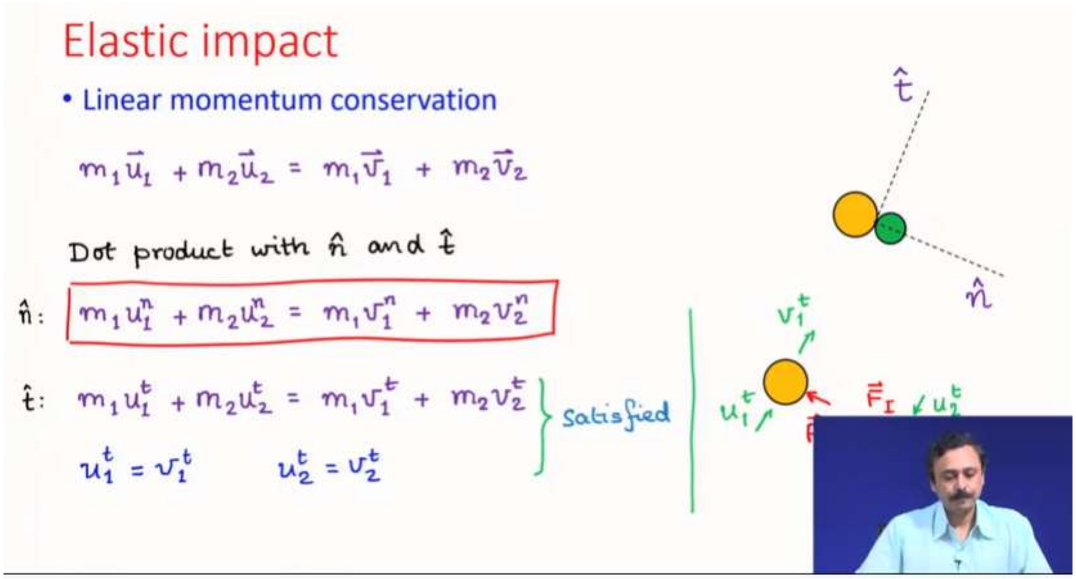


We will start by analyzing elastic impact, as shown above. We have linear momentum conservation, and energy conservation during impact. In other words, the total linear momentum and total energy of the system just before the impact, respectively, remains same as those just after the impact.

For planar impact problem, which is under consideration, we have 2 equations from linear momentum conservation, and 1 equation from energy conservation. Thus, there are 3 equations but 4 unknowns: the 2 components of the outgoing velocities after impact. The number of conditions seem to be falling short of the number of unknowns. We will now consider the equations closely.

Let us decompose the vectors using the local common normal and tangent at the point of contact. As shown in the slide above, the impulsive contact force F_I acts along the common normal. The incoming and outgoing tangential velocity components of the particles are also indicated in the figure. Since we have considered frictionless contact there is no tangential impact force, and hence the linear momentum in the tangential direction of the individual particles is conserved. This implies that, for the individual particles, the incoming and outgoing tangential velocity components will remain the same as concluded at the end of the slide above. This yields additional conditions on the system.

(Refer Slide Time: 15:22)



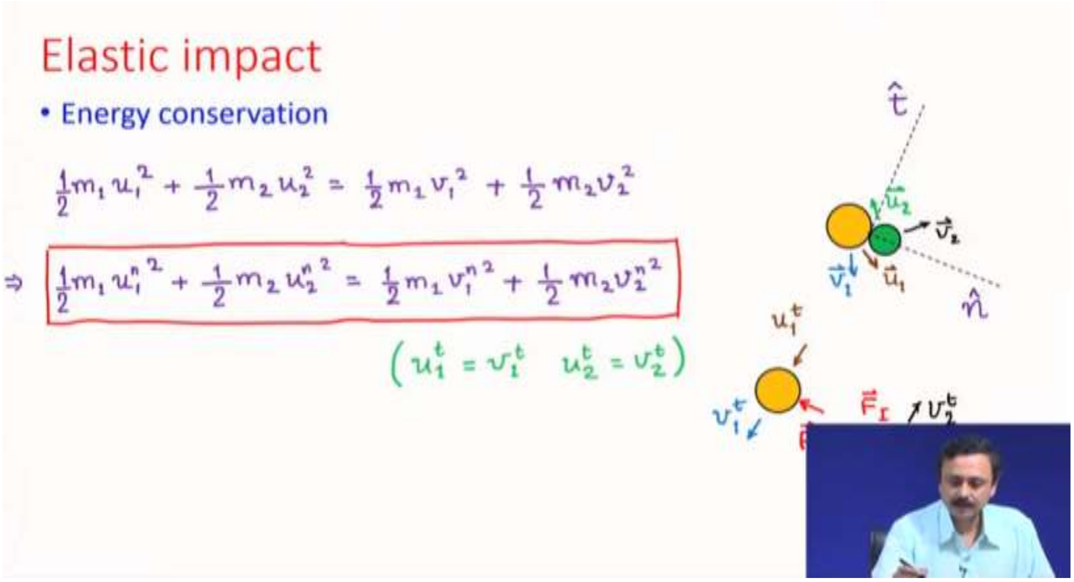
Let us now look at the components of this linear momentum conservation equation in the 2 directions normal and tangential, as shown above. If we take dot product of the linear momentum equation with the unit directions, we obtain

$$\hat{n}: \quad \boxed{m_1u_1^n + m_2u_2^n = m_1v_1^n + m_2v_2^n}$$

$$\left.\begin{aligned} \hat{t}: \quad & m_1u_1^t + m_2u_2^t = m_1v_1^t + m_2v_2^t \\ & u_1^t = v_1^t \qquad u_2^t = v_2^t \end{aligned}\right\} \text{Satisfied}$$

The tangent component is trivially satisfied considering our previous discussion. Next, we consider the energy equation as shown in the following slide.

(Refer Slide Time: 18:22)

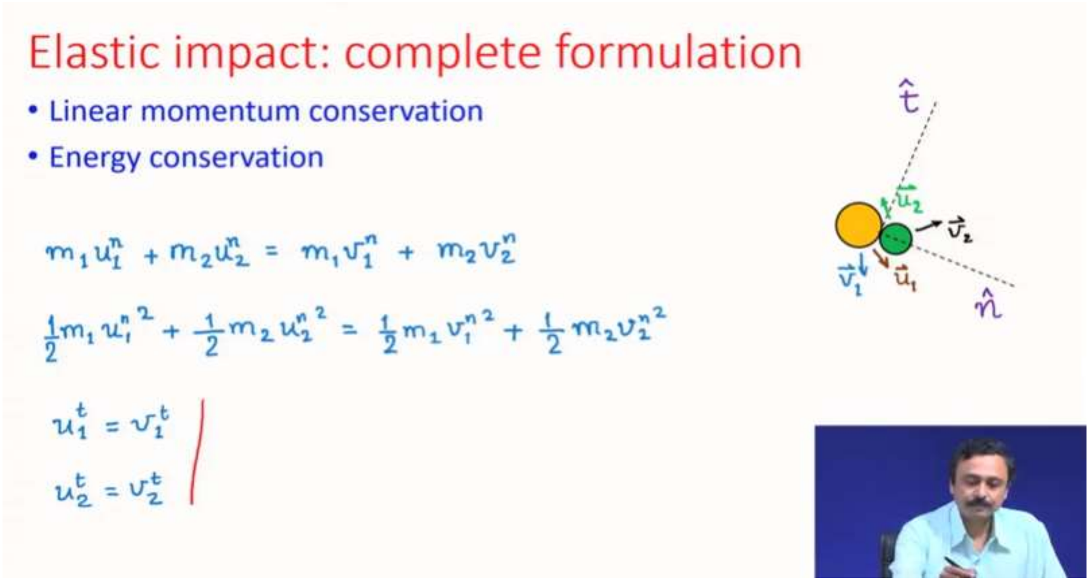


As shown in the slide above, using the equality of the tangential velocity components of the individual particles, we obtain the simplified energy condition

$$\frac{1}{2}m_1 {u_1^n}^2 + \frac{1}{2}m_2 {u_2^n}^2 = \frac{1}{2}m_1 {v_1^n}^2 + \frac{1}{2}m_2 {v_2^n}^2$$

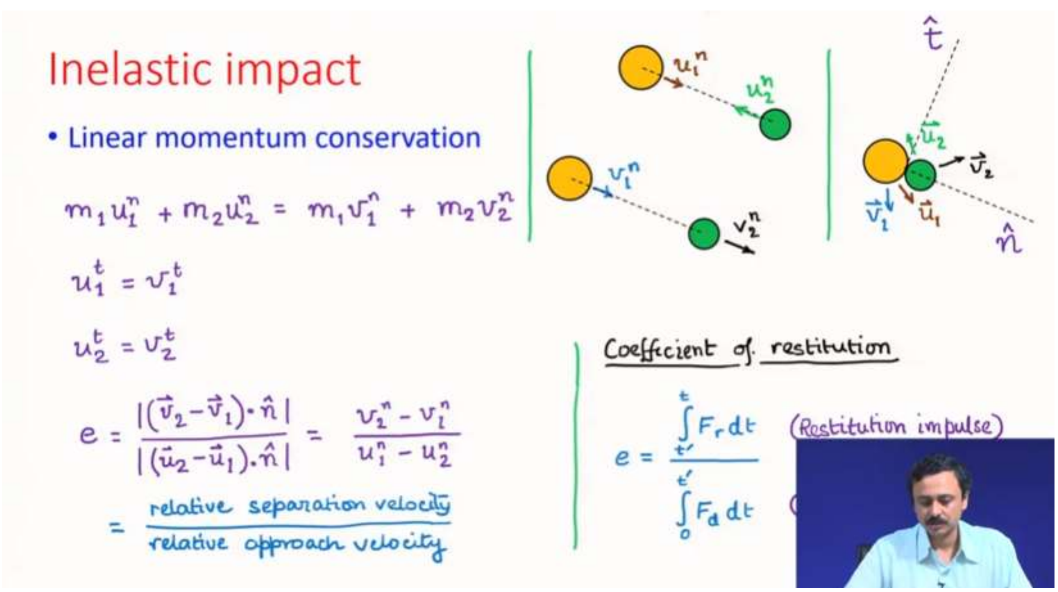
Therefore we have now 2 scalar conditions: (1) linear momentum conservation equation in the normal direction of contact, and (2) energy conservation of the system. These two scalar conditions involve only the 2 normal components of the velocity vector of the 2 particles. Thus, the system is solvable.

(Refer Slide Time: 23:08)



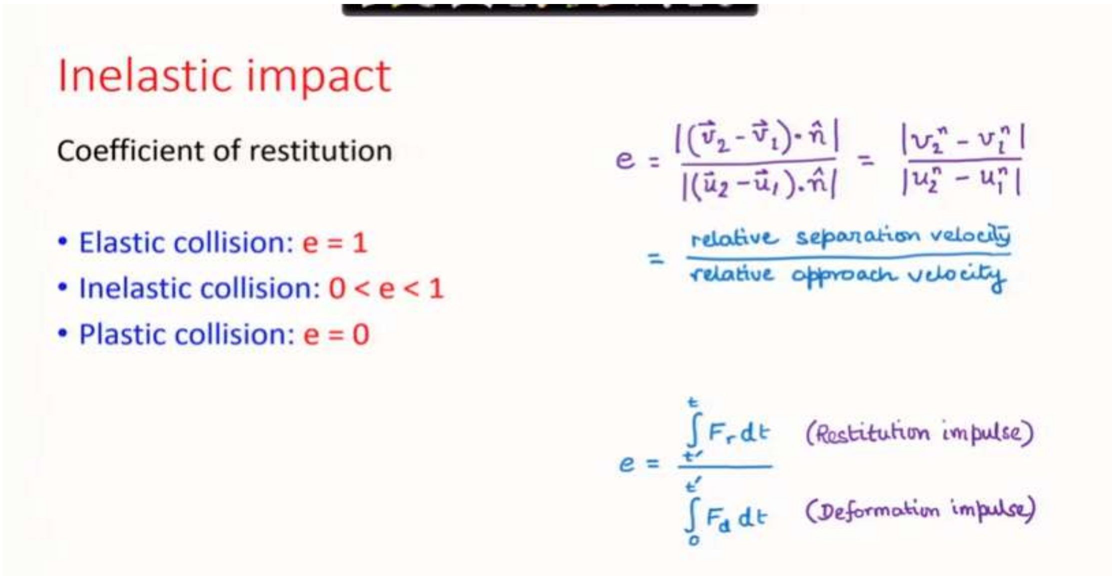
The complete formulation in terms of the equations is now presented in the slide above.

(Refer Slide Time: 24:33)



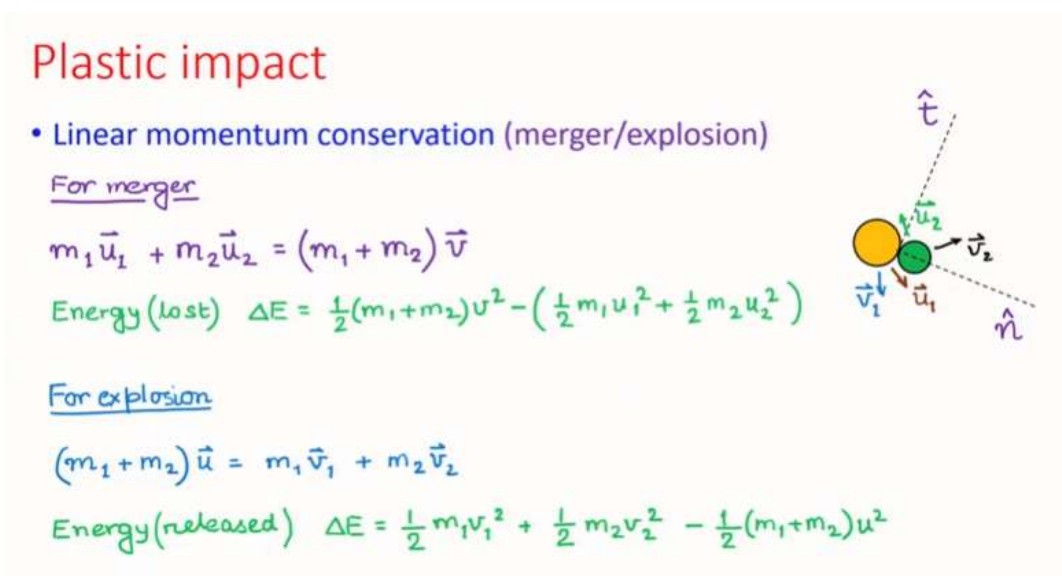
Now we discuss inelastic impact as shown above. Linear momentum conservation is always satisfied. However, the energy conservation condition cannot be used. Instead, the concept of coefficient of restitution is introduced as shown above. This is defined as the ratio of the impulse of the normal contact force during restitution and the impulse of the normal contact force during deformation. The analysis finally yields the coefficient of restitution as the ratio of relative velocity of separation and the relative velocity of approach, and represented mathematically as shown in the slide above. The coefficient itself is determined experimentally by measuring these relative velocities just after and just before the impact.

(Refer Slide Time: 30:32)



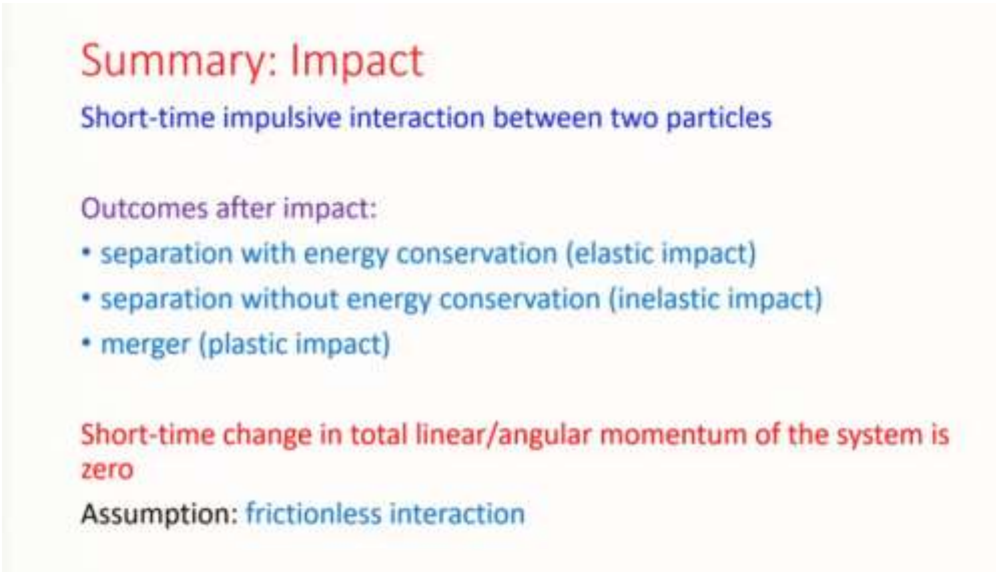
For elastic collision, the coefficient restitution e = 1, and it can be checked that this leads to conservation of energy. For inelastic collision we have e lying between 0 and 1. In plastic collision we have merger, and e = 0, which means that is no separation.

(Refer Slide Time: 31:33)



The case of plastic impact and explosion is shown in the slide above. We can also calculate the energy lost (in collision) or the energy released (in explosion).

(Refer Slide Time: 33:07)



The summary of this lecture is presented in the slide above.