Advanced Dynamics
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Module No # 03
Lecture No # 13
Impulse- Momentum Relation – I

In this lecture we are going to discuss the impulse momentum form of the Newton's second law. This is also known as the impulse momentum relation.

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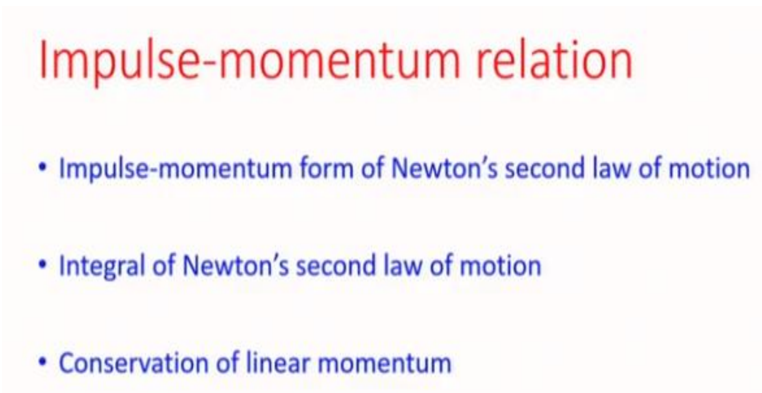

This relation is an integral of Newton's second law just like the work energy relation or work energy form of Newton's second law this is an integral of Newton's second law. Therefore it is called the impulse momentum form of Newton's second law and because it is an integral of Newton's second law it will not generate any new information which is not already present in Newton's second law. This relation will lead us to concept of conservation of linear momentum.

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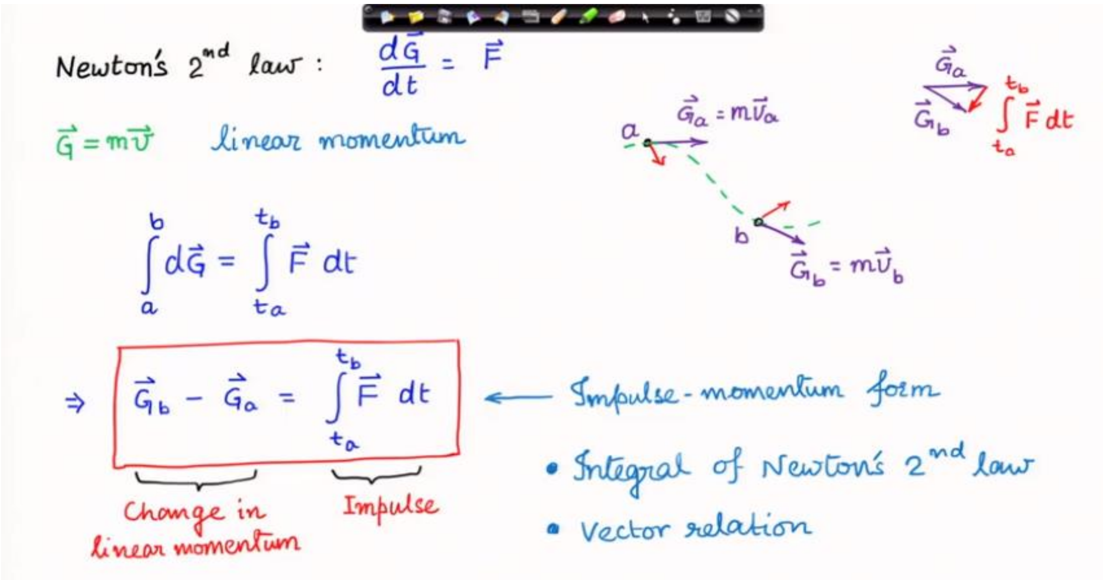

Let us start by looking at Newton's second law as shown above. The rate of change of linear momentum is equal to the force this is what is Newton's second law. Here I am using the notation G vector as the linear momentum vector. G is therefore equal to mass times velocity vector. The linear momentum vector is in the direction of the velocity vector mass being a scalar quantity. Imagine a particle under the action of a force F indicated by this red arrow in the figure. The particle is moving on a path from a to b; at position a, the linear momentum G_a. Similarly at b the linear momentum G_b. If we integrate the Newton's second law over the time interval t_a to t_b, we obtain the impulse momentum form of Newton's 2^{nd} law, as shown above. The implication of this relation is also shown above as a vector diagram. This relation is a direct time integral of Newton's second law, and yields a vector relation.

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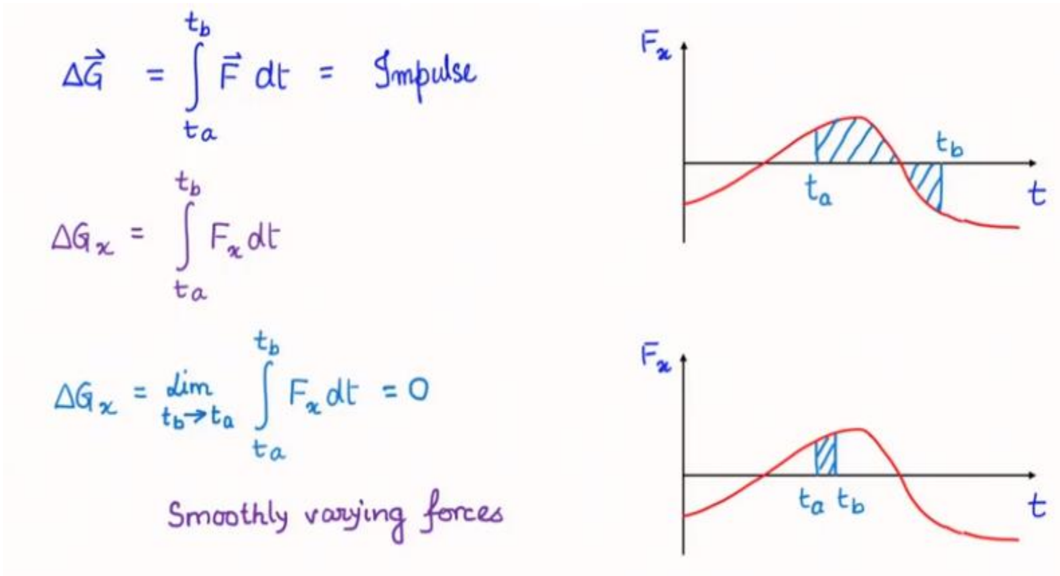

The impulse-momentum relation is written in Cartesian frame above. In the right hand side I have a figure of this force acting in the x-axis direction and I am integrating this force from t_a to t_b. Let us consider the situation when t_b tends to t_a. The integral is nothing but the area under the force time curve. When t_b tends to t_a the area (impulse) tends to 0. In other words, there is no change in momentum in the x-axis direction. But this area is 0 only when the forces are smooth there are example of forces for which the area (impulse) does not tend to zero even when t_b tends to t_a.

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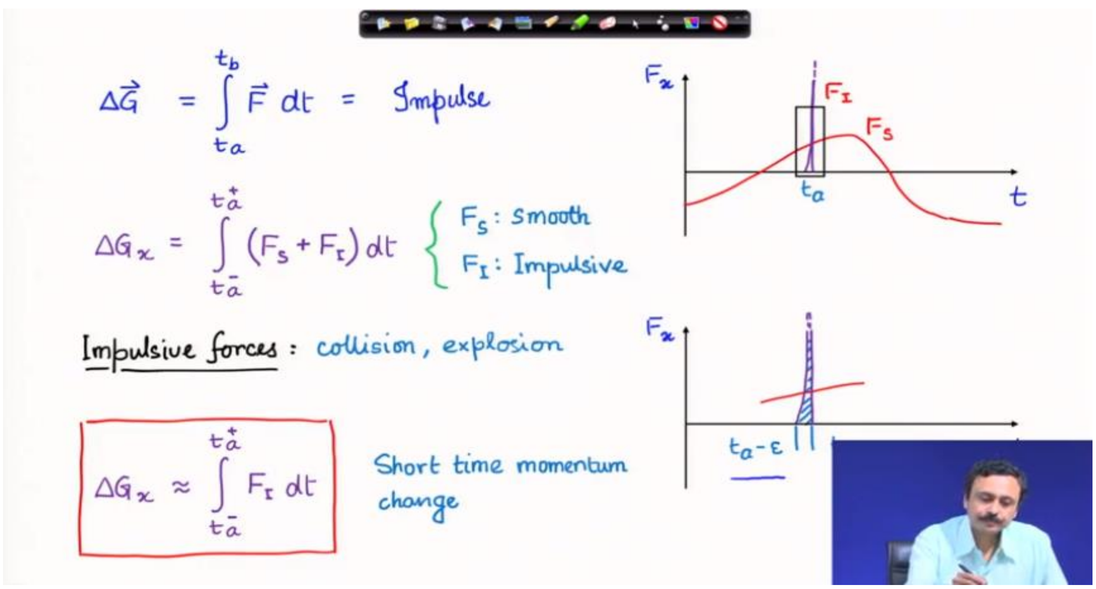

Such forces are called impulsive forces. In the above slide, we have the situation when 2 kinds of forces are acting smooth forces F_S, and impulsive forces F_I. The impulsive force acts for a very short time at a location. Suppose it acts at t_a, then this impulsive force exists during the time interval $t_a - \varepsilon$ to $t_a + \varepsilon$. When we perform the time integration of the net force over this very short time interval, then only the impulsive force will have significant contribution while the smooth forces hardly make any difference, or are negligibly small compared to the impulsive force contribution. Therefore the change in momentum of the particle will come only from the impulsive force, and the smooth forces do not contribute appreciably over this very short time interval.

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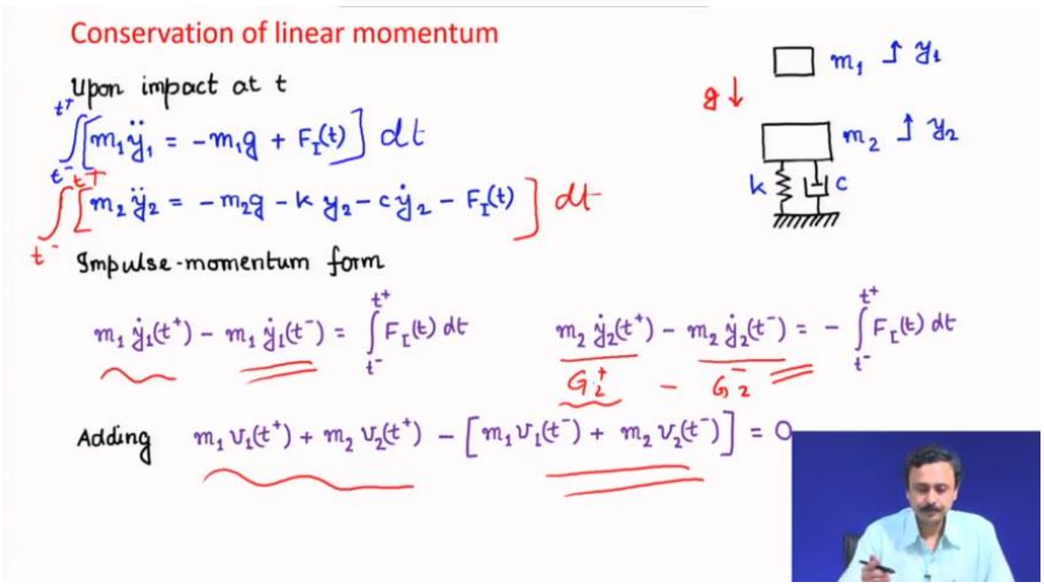

Let us try to understand this thing with this example of a mass falling on top of a mass spring dash spot system in a uniform gravitational field, as shown above. The equations are written, and integrated individually to obtain

$$m_1 \dot{y}_1(t^+) - m_1 \dot{y}_1(t^-) = \int_{t^-}^{t^+} F_I(t)\, dt$$

$$m_2 \dot{y}_2(t^+) - m_2 \dot{y}_2(t^-) = - \int_{t^-}^{t^+} F_I(t)\, dt$$

Adding these two impulse-momentum forms for the two blocks leads to

$$m_1 v_1(t^+) + m_2 v_2(t^+) - \left[m_1 v_1(t^-) + m_2 v_2(t^-) \right] = 0$$

This tells us is the final linear momentum of the whole system just after the impulsive contact minus the linear momentum of the whole system just before the impulsive contact force is equal to 0. In other words, the total linear momentum is conserved over the very short time duration of the impact.

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Conservation of linear momentum

If the net external force on a particle is zero in a particular direction over an interval of time, linear momentum is conserved in that direction over that time interval

In the presence of an impulsive force between two particles, the short time total linear momentum change of the total system is zero

The above slide enumerates the conditions of conservation of linear momentum.

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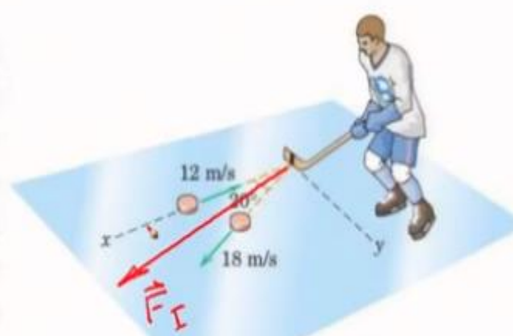
Problem 1:

The ice-hockey puck with a mass of 0.20 kg has a velocity of 12 m/s before being struck by the hockey stick. After the impact the puck moves in the new direction shown with a velocity of 18 m/s. If the stick is in contact with the puck for 0.04 s, compute the magnitude of the average force **F** exerted by the stick on the puck during contact, and find the angle β made by **F** with the x-direction.



Source: Dynamics, Meriam and Kraige

We consider the problem shown above.

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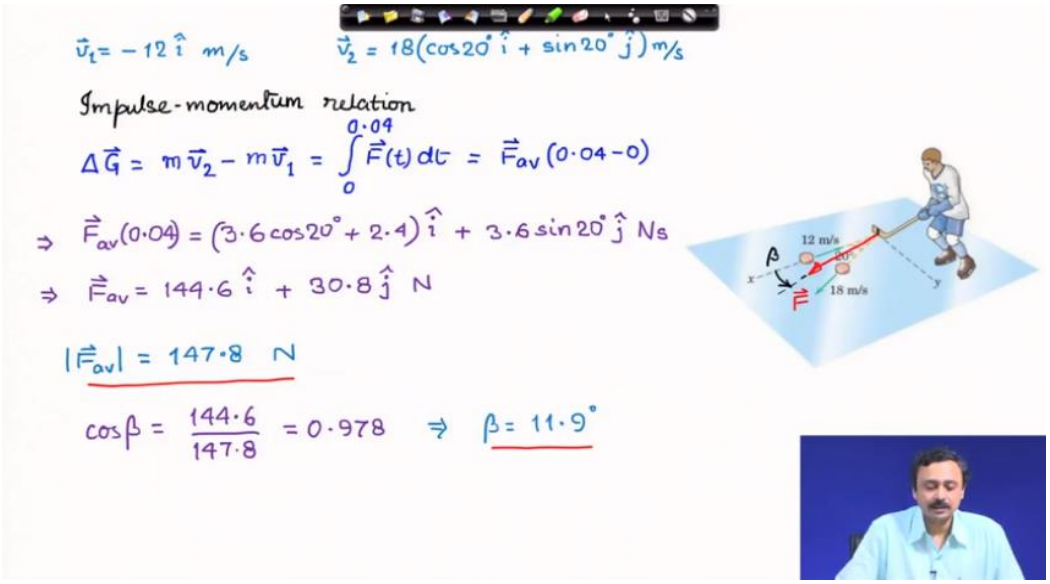

The detailed solution is presented in the slide above. This involves application of the vector form of the impulse-momentum relation as demonstrated above.

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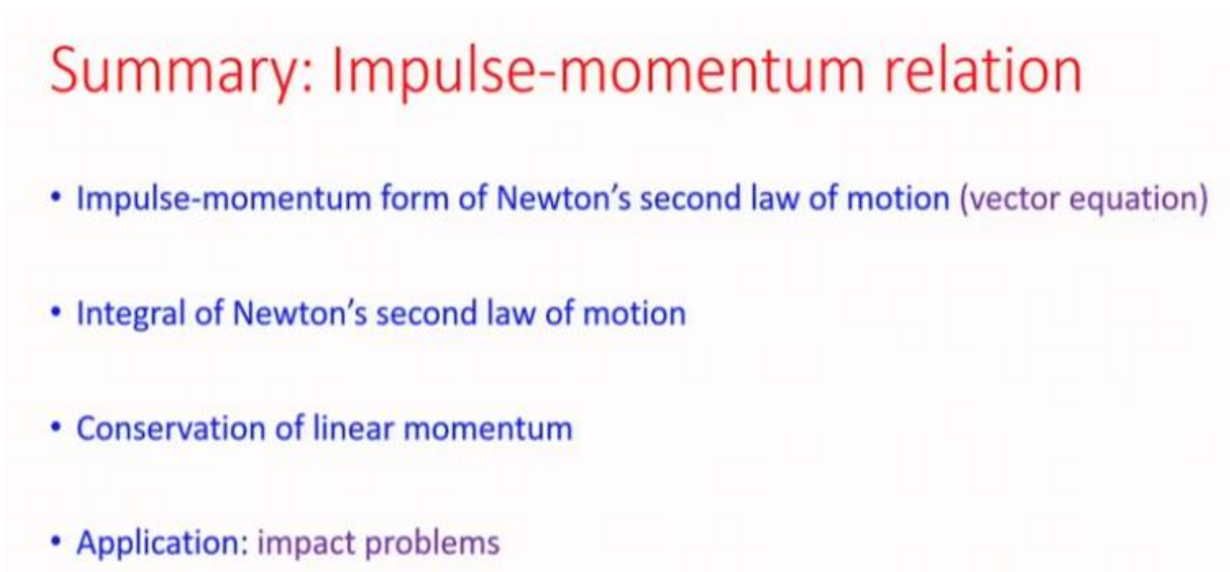

The lecture is summarized above.