

Advanced Dynamics
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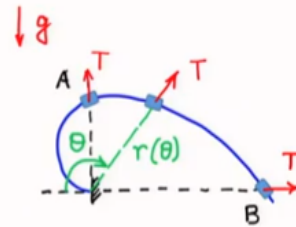
Module No # 03
Lecture No # 12
Work-Energy Relation – II

Let us continue our discussion on the work energy relation.

(Refer Slide Time: 00:18)

Problem 1:

A 0.5 kg collar slides with negligible friction along a fixed spiral rod from A to B in the vertical plane under the action of a constant radial force $T = 10$ N. The rod shape is described by $r(\theta) = 0.3\theta$, where r is in meter and θ is in radian. Calculate the velocity of the collar when it reaches B if it starts from rest at A.



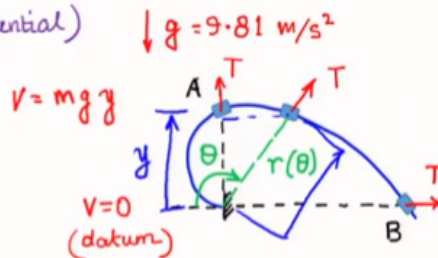
We are going to look at the application of work energy relation in certain problems. The problem statement is shown in the slide above.

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Work-energy relation: $\Delta E = W_{A-B}$

$$\Delta E = T_B + V_B - (T_A + V_A) \quad (V: \text{gravitational potential})$$

$$= \frac{1}{2} m v_B^2 + 0 - \left(0 + m g r(\theta) \sin \theta \right) \bigg|_{\theta=0}^{\theta=\frac{\pi}{2}}$$



$$T = 10 \text{ N}$$

$$r = 0.3\theta \text{ (m)}$$

$$v_B = ?$$

We solve the problem using the work energy relation as explained above. We define a datum where $V = 0$ in this problem, which is shown above.

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Work-energy relation: $\Delta E = W_{A-B}$

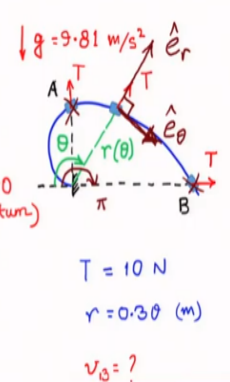
$$\Delta E = T_B + V_B - (T_A + V_A) \quad (V: \text{gravitational potential})$$

$$= \frac{1}{2} m v_B^2 + 0 - (0 + m g r(\theta) \sin \theta) \Big|_{\theta=\pi/2}$$

$$W_{A-B} = \int_A^B \vec{T} \cdot d\vec{r} \quad \left\{ \begin{array}{l} \vec{T} = T \hat{e}_r \\ d\vec{r} = dr \hat{e}_r + r d\theta \hat{e}_\theta \\ r = 0.3\theta \Rightarrow dr = 0.3 d\theta \end{array} \right. \quad \hat{e}_r \cdot \hat{e}_\theta = 0 \quad v=0 \quad (\text{datum})$$

$$\Rightarrow W_{A-B} = \int_A^B T dr = \int_{\pi/2}^{\pi} 0.3 T d\theta = \frac{3\pi}{2} \text{ J}$$

From work-energy relation

$$\frac{1}{2} (0.5) v_B^2 - 0.5 (9.81) 0.3 \left(\frac{\pi}{2} \right) = \frac{3\pi}{2} \text{ J} \Rightarrow \underline{v_B = 5.3 \text{ m/s}}$$


$g = 9.81 \text{ m/s}^2$
 $T = 10 \text{ N}$
 $r = 0.3\theta \text{ (m)}$
 $v_B = ?$

Now the work done by the external force T as the collar moves from A to B is $T \cdot dr$. I have to now represent these vectors to find out the work done because the path is a spiral path. These vectors are given by

$$\vec{T} = T \hat{e}_r$$

$$d\vec{r} = dr \hat{e}_r + r d\theta \hat{e}_\theta$$

Hence, the work done is obtained as

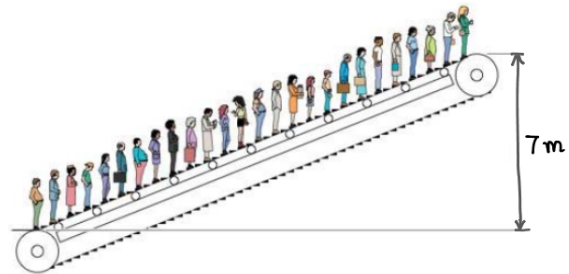
$$W_{A-B} = \int_A^B T dr = \int_{\pi/2}^{\pi} 0.3 T d\theta = \frac{3\pi}{2} \text{ J}$$

Finally, from the work-energy relation

$$\frac{1}{2} (0.5) v_B^2 - 0.5 (9.81) 0.3 \left(\frac{\pi}{2} \right) = \frac{3\pi}{2} \text{ J} \Rightarrow \underline{v_B = 5.3 \text{ m/s}}$$

Problem 2:

A department-store escalator handles a steady load of 30 people per minute in elevating them from the first floor to the second floor through a vertical rise of 7 m. The average person has a mass of 65 kg. If the motor that drives the unit delivers 3 kW, calculate the mechanical efficiency e .



Source: Dynamics, Meriam and Kraige

Next we consider the above problem.

(Refer Slide Time: 13:01)

Work-energy relation (differential form)

$$\frac{dE}{dt} = \vec{F} \cdot \vec{v} = P \quad (E = T + V)$$

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dV}{dt}$$

+0 (assuming people get on/off the elevator at its steady speed)

$$\Rightarrow \frac{dE}{dt} = \frac{dV}{dt} = \dot{m}gh$$

$$\dot{m} = \left(\frac{\text{ppm}}{60} \times 65\right) \text{ kg/s} = \frac{30}{60} \times 65 = 32.5 \text{ kg/s}$$

$$\text{Rate of work output} = \frac{dE}{dt} = 32.5(9.81)(7) \text{ W} = 2.232 \text{ kW}$$

$$\text{Efficiency } e = \frac{2.232 \text{ kW}}{3 \text{ kW}} = 0.744$$

Persons per minute (ppm) = 30
average mass per person = 65 kg
input power = 3 kW

Mechanical efficiency

$$e = \frac{\text{Rate of}}{\text{input}}$$

The solution is given in the slide above. The important step here is to realize that the rate of doing work by the escalator is the rate of change of potential energy, since the kinetic energy remains constant. Further, this rate of change of potential energy is given by

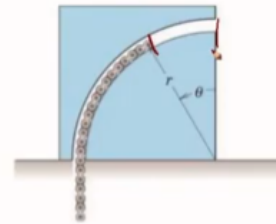
$$\frac{dE}{dt} = \frac{dV}{dt} = \dot{m}gh$$

where dm/dt is the rate at which mass is transported up, and not rate of change of mass. This is a mass flow problem.

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Problem 3:

A flexible bicycle chain of length $\pi r/2$ and mass per unit length ρ is released from rest in a smooth circular guide with $\theta = 0$. Calculate the velocity of the chain when $\theta = \pi/2$.



Source: Dynamics, Meriam and Kraige

Next we look at the problem of a flexible chain like a bicycle chain. The problem statement is shown above.

(Refer Slide Time: 21:14)

Work-energy relation: $\Delta E = W_{A-B}$

$\Delta E = T_B + V_B - (T_A + V_A)$ (V : gravitational potential)

$\theta = \frac{\pi}{2}$ $\theta = 0$

$W_{A-B} = 0$

$T_B = \int_0^l \frac{1}{2} \rho v^2 ds = \frac{1}{2} \rho \frac{\pi r}{2} v^2 = \frac{1}{4} \pi \rho r v^2$

$V_B = \int_0^l \rho g ds (-s) = -\frac{1}{2} \rho g l^2 = -\frac{\pi^2}{8} \rho r^2 g$

$V_A = \int_0^{\pi/2} \rho r d\theta g (r \cos \theta) = \rho r^2 g$

Work-energy relation $\Rightarrow \frac{1}{4} \pi \rho r v^2 - \frac{\pi^2}{8} \rho r^2 g - \rho r^2 g = 0$

$\Rightarrow v = \left[\frac{4}{\pi} \left(\frac{\pi^2}{8} + 1 \right) r g \right]^{1/2}$

length $l = \frac{\pi r}{2}$

linear density: ρ

$r(\theta = \pi) = ?$

$\theta = \frac{\pi}{2}$

$V = 0$ datum

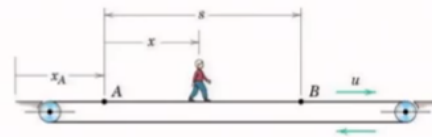
$-s$ ds f

The solution is given in the slide above. It is important to note that the calculation of the potential energy has to be done carefully when the chain is inside the curved slot by assuming that the mass is distributed uniformly.

(Refer Slide Time: 29:10)

Problem 4:

A boy of mass m is standing initially at rest relative to the moving walkway, which has a constant horizontal speed u . He decides to accelerate his progress and starts to walk from point A with a steadily increasing speed and reaches point B with a speed $\dot{x} = v$ relative to the walkway. During his acceleration he generates an average horizontal force F between his shoes and the walkway. Write the work-energy equations for his absolute and relative motions and explain the meaning of the term $mu v$.



Source: Dynamics, Meriam and Kraige

The next problem statement is shown above.

(Refer Slide Time: 30:55)

(a) Fixed observer ($v_B = u + \dot{x}$)

$$T_B - T_A = W_{A-B}$$

$$\Rightarrow \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2 = F \cdot s$$

$$\Rightarrow \boxed{\frac{1}{2}mv^2 + muv = Fs}$$

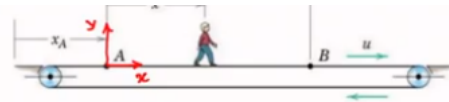
(b) Observer A (walkway fixed) ($v_B = \dot{x}$)

$$T_B - T_A = W_{A-B}$$

$$\Rightarrow \frac{1}{2}mv^2 - 0 = F \cdot s' \quad (s': \text{distance walked on the walkway})$$

$$\Rightarrow \boxed{\frac{1}{2}mv^2 = Fs'}$$

$$\text{Difference: } F(s - s') = muv$$



$$F = ma$$

time taken to accelerate

$$v = 0 + at_a \Rightarrow t_a = \frac{v}{a}$$

Work done by walkway (on boy)

$$W_w = F \cdot (ut_a) = F \left(\frac{uv}{a} \right) = muv$$

$$\boxed{Fs' + muv = Fs}$$

Work done by: Boy Walkway

We look at this problem from the eyes of an observer who is fixed to the ground and watching this boy, and another observer who is fixed to the walkway. The solution is shown above.

It is noted that the net work done by the boy is measured by the walkway fixed observer. While, the ground-fixed inertial observer can see the work done by the boy and the work done by the walkway when the boy is accelerating on the walkway, and the walkway motor is doing additional work in maintaining the speed of the walkway.

(Refer Slide Time: 39:39)

Summary

- Work-energy form of Newton's second law of motion (scalar relation)
- Integral of Newton's second law of motion
- Conservation of energy
- Problems

To summarize we have looked at the applications of work energy form of the Newton's second law.