Advanced Concepts In Fluid Mechanics
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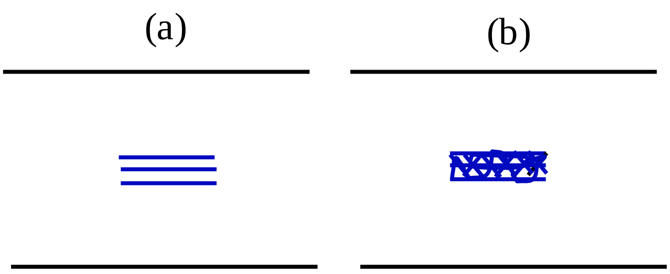
Lecture – 22
Introduction to Turbulence



Schematic of the flow through a pipe; (a) represents ordered or layered motion of a colored dye for laminar flow while (b) represents chaotic or random motion for turbulent flow.

In the present chapter we will discuss about the introduction to turbulence. Turbulent flow is one of the very fascinating topics in fluid mechanics and this is considered to be one of the most challenging problems for which the exact solution has not yet been obtained. People have scratched over the surface to get a statistical picture of what the turbulent flow is all about. There has been a reasonably good understanding in the context of the real physical picture of turbulence for some simple flows like pipe flows and channel flows. But for very complex flows, it has not yet come across.

The first question that we will address is that what a turbulent flow is. This is a very difficult question to answer because turbulent flow does not have a strict definition but it has certain characteristics. Now we will try to figure out certain important hallmarks of turbulent flow and bring our analysis in perspective to that. Let us start with something very classical. In 1883, Osborne Reynolds performed a very famous experiment which is known as Reynolds experiment.

What Reynolds did is as simple as that is shown in figure 1. There is a tube in which a colored dye is injected and the flow rate through a flow controlling valve is progressively increased. For very low flow rate, we can see that the colored dye visible as distinct lines. This shows that the fluid is moving in a very ordered and layered fashion. However, if the flow rate is increased, then this orderly nature of these colored lines is lost and the entire thing will appear to be diffused and mixed. Reynolds attributed this to a physical transition from an ordered regular motion to a random chaotic motion in the flow and he attributed that to some phenomenon known as turbulence.

If we have a fixed diameter, this transition to turbulence is occurring only for a fixed flow rate for a given fluid viscosity. But if we change the diameter, if we change the fluid property, if we change the flow velocity we will see that the transition occurs under different conditions. Reynolds wanted to analyze these results in terms of the important physical parameters. So question arises about the important physical parameters. There is a driving influence, which in the present case is the inertia force (which is the accelerating component). Inertia force is of the order of ~ ma where m is mass and a is acceleration. Also, there is a resistive force which is the viscous force. Viscous force is the product of viscous stress and area, i.e. ~ $\frac{\mu v}{l} \times l^2$ where $\frac{\mu v}{l}$ is the shear stress and l^2 is the area, v is the velocity scale, l is the length scale of the problem and μ is the fluid viscosity. We are just trying to write it in terms of various scales. Inertia force ~ ma ~ $\rho l^3 \frac{v^2}{l}$ where the term ρl^3 represents the mass in the inertia force and $\frac{v^2}{l}$ represents the acceleration part in the inertia force; $\frac{v^2}{l}$ is nothing but like accelerating component $u\frac{\partial u}{\partial x}$. So, $\frac{\text{inertia force}}{\text{viscous force}} \sim \frac{\rho l^3 \frac{v^2}{l}}{\frac{\mu v}{l} l^2} \sim \frac{\rho v l}{\mu}$. So the ratio of inertia force and viscous force becomes of the order of ~ $\frac{\rho v l}{\mu}$. Later on, to honor the contribution of Reynolds towards understanding this physical phenomenon, $\frac{\rho v l}{\mu}$ was called as Reynolds number, so, Reynolds number = $\frac{\rho v l}{\mu}$.

We need to remember that Reynolds number can be interpreted as a non-dimensional length scale or a non-dimensional velocity scale; it may be interpreted in various ways. It is not necessary to interpret it as always the ratio of inertia force and viscous force. For example, if we have a fully developed flow through a pipe, the inertia force is zero because the fluid is not accelerating. But the Reynolds number is not zero because in that case l is the diameter of the pipe; v is the average velocity, ρ is the density of the fluid and μ is the viscosity of the fluid. This combination does not give a zero Reynolds number. We need to keep in mind that Reynolds number as interpreted by the ratio of the inertia force and the viscous force is restrictive. It is restricted only under those conditions where the inertia force and the viscous forces are competing. Otherwise it may be interpreted in various ways as in terms of dimensionless velocity, dimensionless length, dimensionless time etc. But the definition Reynolds number = $\dfrac{\rho v l}{\mu}$ remains the same; that is the correct way of looking into Reynolds number.

Remarkably, in the Reynolds experiment, this transition from the ordered to the disordered flow was occurring at fixed range of Reynolds number which is roughly in the range of 2000. There is a controversy depending on experiments conducted by various people; it may range from 1800 (or 1700) to 2300. So we can take roughly the average value of 2000. That Reynolds number is called as the critical Reynolds number for pipe flow. Critical Reynolds number is a very important concept that needs to be discussed elaborately. Most of the students have a wrong idea that above the critical Reynolds number the flow is turbulent. But the correct interpretation is the other way around, i.e. below the critical Reynolds number; the flow is always laminar irrespective of the level of disturbance. So question arises about what happens when the flow becomes turbulent. When there is a disturbance in the flow, the disturbance gets amplified if the flow is turbulent but the disturbance gets dampened out if the flow is laminar. So, irrespective of the level of disturbance, the viscous forces are so strong that the disturbance is dampened out below a critical Reynolds number. But above the critical Reynolds number, the disturbances may get amplified or may not get amplified; it depends on the level of the disturbance. But below the critical Reynolds number, it will always get dampened out no matter how large the disturbances are. This is a very important concept. In the class, whenever a student is asked about the critical Reynolds number, students magically says 2000, 2300 like that. However, we need to remember that critical Reynolds number is not a magic number. Depending on whether it is a pipe flow it may be something, if it is a channel flow it may be something else or if it is a flow over a flat plate, it may be something else. So it is not the number that is important but the concept is important. It tells that below that particular Reynolds number the flow is always laminar irrespective of the level of disturbance.

As we have already mentioned, it is very difficult to have a unified definition of the turbulent flow but turbulent flow has certain characteristics like random disordered chaotic motion. But we have to understand that any chaotic motion is not turbulent flow. There has to be certain other characteristics. There is wide range of length scales and timescales. A corollary to chaotic motion is that it is hallmarked by extreme sensitivity to initial conditions. Also it is characterized by the enhanced mixing. This enhanced mixing is very important because that is the reason why turbulent flow is very important for practical engineering applications.

So we have touched upon random or disordered motions through the Reynolds experiment. But the wide range of length scales and timescales we will discuss by referring to a very interesting phenomenon in turbulent flow known as energy cascading. In a turbulent flow there are many eddies. These eddies are lumps which are physically rotating; lumps of masses which are physically rotating. We have a large eddy which can be as large as the diameter of the channel and we can also have a small eddy which is of the molecular size. So we have wide range of length scales which have its gradation form the large eddy scale to the smallest eddy scale. Now question arises about how the eddies interact. The large eddy extracts kinetic energy from the mean flow. So there is a mean flow and large eddy extracts kinetic energy from it. Then large eddy will share some of its energy to the smaller eddy; the smaller eddy will transfer the energy to even a smaller eddy. This will happen simply by momentum exchange. By simple momentum exchange the large eddy will transfer some of its energy to the small eddy, then smaller eddy, then smallest eddy. Then the energy which has been drawn from the mean flow by the large eddy will be dissipated by the smallest eddy. The smallest eddy which has a very small length scale will have much stronger viscous effect as compared to the inertial effect. Therefore, the smallest eddy will dissipate the entire energy by viscous dissipation. So large eddy will extract kinetic energy from the mean flow; this energy will be cascaded from large eddy to the small, smaller and smallest eddy and eventually it will be dissipated by viscous action by the smallest eddy. Now we have a length scale and velocity scale of the large eddy and the small eddy. Let us say that the large eddy length scale is l and velocity scale is u (just symbolically). The small eddy length scale is η and velocity scale is v. We are interested to see that how these scales are related; this is a fundamental quest in physics which we are interested to address.

To address that, we can say that the rate of extraction of turbulent kinetic energy is of the order of $\sim \frac{\frac{1}{2}mu^2}{mt}$. This is applicable for the large eddy which is evaluated per unit mass (m) and per unit time (t). So, rate of extraction of kinetic energy $\pi \sim \frac{u^2}{t}$. Now the time scale for a large eddy is the inertia time scale which is given by $t \sim \frac{l}{u}$. So, rate of extraction of kinetic energy $\pi \sim \frac{u^3}{l}$. Now the rate of dissipation of turbulent kinetic energy is given by $\varepsilon \sim \nu e_{ij} e_{ij}$ where ν is the kinematic viscosity and e_{ij} is the rate of deformation (which was discussed in the earlier chapters). The rate of deformation can be represented by the velocity gradient. So the rate of dissipation of turbulent kinetic energy is $\varepsilon \sim \nu \frac{v^2}{\eta^2}$ which is applicable for the small eddy. The rate of dissipation of kinetic energy is important for small eddy and the rate of extraction of kinetic energy is important for the large eddy. To maintain the dynamic condition, the rate of extraction of kinetic energy should be same as the rate of extraction of kinetic energy. It means that $\frac{u^3}{l}$ should be of the order of $\nu \frac{v^2}{\eta^2}$, i.e. $\frac{u^3}{l} \sim \nu \frac{v^2}{\eta^2}$. This is the first condition that we have obtained. Now in the large eddy, the length scale is large, Reynolds number is large and therefore the inertial effects are important. But when we come to the smallest eddy the viscous force has just started taking over the inertial force. It means that the viscous force for the smallest eddy has become of the same order of the inertial force and the resulting Reynolds number becomes of the order of 1. So the Reynolds number based on η (Re_η) is of the order of 1, i.e. $\frac{v\eta}{\nu} \sim 1$. This is the second condition. Since, $\frac{v\eta}{\nu} \sim 1$ we can write it as $\nu \sim v\,\eta$; so, ε becomes of the order of $\sim \nu \frac{(v\eta)^2}{\eta^4} \sim \frac{v^3}{\eta^4}$. From this, we get

$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ which is the smallest eddy length scale and called as the Kolmogorov length scale; v is called as the Kolmogorov velocity scale Although $\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ can be used as a definition of the Kolmogorov length scale, we cannot clearly assess it because we do not know ε. We know ν which is the property of the fluid but we do not know ε. To assess the Kolmogorov length scale we will utilize the condition $\nu\frac{v^2}{\eta^2} \sim \frac{u^3}{l}$. In place of v we will write $\frac{\nu}{\eta}$, so, $\frac{\nu^3}{\eta^4} \sim \frac{u^3}{l}$ or, $\frac{\eta^4}{l} \sim \frac{\nu^3}{u^3}$. So, $\frac{\eta^4}{l^4} \sim \frac{\nu^3}{u^3\, l^3}$ or, $\frac{\eta}{l} \sim Re_l^{-3/4}$ where $Re_l = \frac{u\, l}{\nu}$.

So $\frac{\eta}{l} \sim Re_l^{-3/4}$ is the relationship between the smallest eddy length scale and the largest eddy length scale which is expressed as a function of the Reynolds number $\left(Re_l\right)$. So, if the Reynolds number for example is 10000, we can calculate the ratio of these two length scales by using this relationship. Because of the presence of the power -3/4, there can be orders of difference between the two length scales. We can have at least three orders of difference between η and l (it can vary from three to six orders of difference). It means that we have a wide range of length scales because these η and l are not only length scales but correspond to the smallest and the largest eddy. In between smallest and largest eddy, there are various length scales. Now the question appears about the procedure to capture these scales and the corresponding grid resolution if one is doing CFD analysis. All these remain to be outstanding problems. The next aspect of turbulent flow which we will discuss is a very interesting aspect called as vortex stretching. To understand about the vortex stretching we will discuss about the vorticity dynamics first which is true for all types of flow. Vorticity is rotationality, because there are eddies in a turbulent flow rotational it is very important for turbulent flow. We will first write the Navier Stokes equation assuming all standard assumptions to remain valid.

The Navier-Stokes equation is given by $\rho\left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\nabla\right)\vec{v}\right] = -\nabla p + \mu\nabla^2\vec{v}$. We use the vector identity $\left(\vec{v}\cdot\nabla\right)\vec{v} = \frac{1}{2}\nabla\left(\vec{v}\cdot\vec{v}\right) - \vec{v}\times\left(\nabla\times\vec{v}\right)$ (if we recall we can find that we have used this vector identity while deriving the Bernoulli's equation). $\nabla\times\vec{v}$ is nothing but the vorticity vector $\vec{\Omega}$. Substituting the vector identity in the Navier-Stokes equation we get $\rho\left[\frac{\partial \vec{v}}{\partial t}+\frac{1}{2}\nabla(\vec{v}\cdot\vec{v})-\vec{v}\times(\nabla\times\vec{v})\right]=-\nabla p+\mu\nabla^2\vec{v}$. Now the pressure gradient is something which creates a lot of problems in terms of assessing. So, one way is to eliminate the pressure gradient term. To nullify these problems, we take curl on both sides of the Navier-Stokes equation because we know that curl of gradient of a scalar is a null vector. If we take curl on the velocity vector, it will become vorticity vector. It does not affect the time derivative because curl is a spatial derivative, it does not conflict with the time derivative. So, the curl of the term $\rho\frac{\partial \vec{v}}{\partial t}$ becomes $\rho\frac{\partial \vec{\Omega}}{\partial t}$. Again curl of gradient of a scalar is a null vector, so curl of $\frac{1}{2}\rho\nabla(\vec{v}\cdot\vec{v})$ vanishes and similarly, curl of pressure gradient vanishes. So, the simplified governing equation becomes $\rho\left[\frac{\partial \vec{\Omega}}{\partial t}-\nabla\times(\vec{v}\times\vec{\Omega})\right]=\mu\nabla^2\vec{\Omega}$. Now we will use a vector identity for the term $\nabla\times(\vec{v}\times\vec{\Omega})$ which reads as $\nabla\times(\vec{v}\times\vec{\Omega})=\vec{v}(\nabla\cdot\vec{\Omega})-\vec{\Omega}(\nabla\cdot\vec{v})+(\vec{\Omega}\cdot\nabla)\vec{v}-(\vec{v}\cdot\nabla)\vec{\Omega}$. Students should never be encouraged to remember this kind of long vector identity. Since $\vec{\Omega}$ is the curl of velocity vector, divergence of curl of a vector is zero, so, the term $\vec{v}(\nabla\cdot\vec{\Omega})$ vanishes. For incompressible flow, $\nabla\cdot\vec{v}=0$, so the term $\vec{\Omega}(\nabla\cdot\vec{v})$ vanishes. So we get, $\nabla\times(\vec{v}\times\vec{\Omega})=(\vec{\Omega}\cdot\nabla)\vec{v}-(\vec{v}\cdot\nabla)\vec{\Omega}$. Using this expression, the governing equation becomes $\rho\left[\frac{\partial \vec{\Omega}}{\partial t}+(\vec{v}\cdot\nabla)\vec{\Omega}-(\vec{\Omega}\cdot\nabla)\vec{v}\right]=\mu\nabla^2\vec{\Omega}$. We rewrite this equation in the form $\rho\left[\frac{\partial \vec{\Omega}}{\partial t}+(\vec{v}\cdot\nabla)\vec{\Omega}\right]=\mu\nabla^2\vec{\Omega}+\rho(\vec{\Omega}\cdot\nabla)\vec{v}$. This is a vorticity equation and we will see the interpretation of different terms in this if we look into physically the phenomenon of vortex stretching in turbulent flow. The large eddy has a large Reynolds number in terms of its length scale. So it is almost inviscid which means that its angular momentum remains conserved because it does not have any viscous torque acting on it (because the Reynolds number is large). So, for large eddy, $I\,\omega$ remains conserved where I is the moment of inertia and ω is the angular momentum. It means that $\frac{D}{Dt}(I\,\omega)$ for a large eddy is equal to zero. But for a small eddy, it is related to the viscous torque as

$\frac{D}{Dt}(I\,\omega) = T_v$ where T_v is the viscous torque. For a large eddy, T_v is equal to zero. The larger eddy is giving its kinetic energy to the smaller eddy. $\frac{D}{Dt}(I\,\omega) = T_v$ can also be written as $I\frac{D\omega}{Dt} + \omega\frac{DI}{Dt} = T_v$. It means that ω is decreasing for larger eddy. If ω is decreasing, in order to maintain the angular momentum conserved, I must increase. It means that the large eddy gets stretched so that its moment of inertia I increases.

This phenomenon is called as vortex stretching. In the large eddy scale because ω is getting reduced because of transfer of the kinetic energy, I becomes increased which means that the length scale gets stretched That is how it can further interact with the smaller and smaller eddies. So, we can clearly bring out this phenomenon of vortex stretching from the vorticity transport equation. The first term of this equation $\rho\left[\frac{\partial\vec{\Omega}}{\partial t} + (\vec{v}\cdot\nabla)\vec{\Omega}\right] = \mu\nabla^2\vec{\Omega} + \rho\left(\vec{\Omega}\cdot\nabla\right)\vec{v}$ is $\rho\frac{\partial\vec{\Omega}}{\partial t}$, $\mu\nabla^2\vec{\Omega}$ is the viscous term. We have also have the equation $I\frac{D\omega}{Dt} + \omega\frac{DI}{Dt} = T_v$. Comparing these two equations we can identify the $\frac{D\omega}{Dt}$ term through the $\rho\frac{\partial\vec{\Omega}}{\partial t}$ and T_v through the term $\mu\nabla^2\vec{\Omega}$. The remaining term $\omega\frac{DI}{Dt}$ is the so called vortex stretching term which is associated with the change in the total derivative of the moment of inertia of the elements that we are considering. So the term $\rho\left(\vec{\Omega}\cdot\nabla\right)\vec{v}$ therefore can be attributed to the vortex stretching, stretching of vortex elements as they give out the kinetic energy.

So energy cascading and vortex stretching are two very important phenomena that are associated with turbulent flow. In the present chapter we have discussed the basic physics of turbulent flow and some introductory aspects of turbulence. In the next chapter we will look into the statistical analysis of turbulent flow because such flows are very complicated. It is very difficult to analytically treat these flows and therefore, we have to use statistical techniques. So, in the next chapter, we will discuss about the statistical techniques associated with analyzing turbulent flow.