Advanced Concepts In Fluid Mechanics
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Lecture - 19
Exact Solution of Navier Stokes Equation in Cylindrical Polar Coordinate System

In the previous chapters we have discussed about exact solutions of Navier Stokes equation in Cartesian co-ordinate system. However, there are many problems in physics and engineering where other co-ordinate systems are important. One such important co-ordinate system is the cylindrical polar co-ordinate system. In this chapter, we will discuss about two such problems in cylindrical polar co-ordinate system which are very commonly encountered in physics and engineering.

Example 1. Fully developed laminar flow through a circular pipe (Hagen-Poiseuille flow)

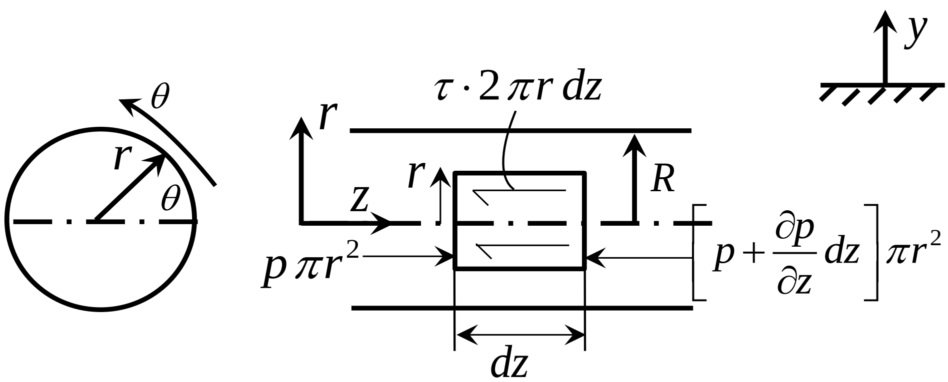

Figure 1. Schematic of the fully developed laminar flow through a circular pipe. The left hand side of the figure shows the other sectional view.

The first problem is the fully developed laminar flow through a circular pipe which is also called as the Hagen-Poiseuille flow. The schematic of this flow is depicted in figure 1.Till now we have not discussed about what the laminar flow is and what the turbulent flow is. Classically we introduce the Navier Stokes equation first and then we introduce the laminar and the turbulent flow in the subsequent chapters. Here, to give a little bit of physical understanding, by laminar flow we mean that the fluid is moving in orderly layered fashion one layer over the other and there is chaotic or random motion in the flow. Since we consider the fully developed flow, the concept of unidirectional nature of the flow is preserved. Otherwise if it is a turbulent flow, then it is a random chaotic flow.

In that case the unidirectional nature of flow may not be preserved. For simplicity let us assume the pipe is horizontal. Even if the pipe is inclined or vertical, now we know that there will be a component of gravity as a driving pressure gradient. Then we can replace the pressure by the summation of the static pressure (p) and $h\,\rho\,g$. So, $p+\rho\,g\,h$ is called as piezometric pressure. We can use the piezometric pressure instead of the static pressure if there is a component of gravity in the direction of flow. So, even if it is a horizontal pipe or an inclined pipe the method of working of the problem is still the same.

Let us assume that the pipe has a radius *R*. The co-ordinates are shown in figure 1 where *r* is the radial co-ordinate; *z* is the axial co-ordinate and *θ* is the azimuthal co-ordinate. The left hand side of figure 1 shows the other sectional view where *r* is the radial direction and the cross-radial direction is called as the *θ*-direction which is defined by the polar angle *θ*. Let us consider the flow to be fully developed. For fully developed flow, first we write a simple force balance on a rectangular element drawn in figure 1. The length of the axial element is '*dz*' and the radial element and te radial location is '*r*'. If the flow is fully developed, then all forces on this element are balanced so that there is no net acceleration. So, first we will write all the forces. In the left hand side of the rectangular element, there is a force $p\,\pi\,r^2$ where *p* is the pressure acting on the area $\pi\,r^2$. On the right hand side of this element there is a force $\left[p+\frac{\partial p}{\partial z}dz\right]\pi\,r^2$ acting in the opposite direction. Also, there will be a shear force acting over the periphery of the element which is having a surface area $2\,\pi\,r\,dz$; so, the shear force becomes $\tau\cdot\left(2\,\pi\,r\,dz\right)$ where τ is the shear stress at a radial location *r*. Since all the forces are balanced, we can write the force balance equation as

$$p\,\pi\,r^2-\left[p+\frac{\partial p}{\partial z}dz\right]\pi\,r^2-\tau\cdot\left(2\,\pi\,r\,dz\right)=0 \qquad (1)$$

From equation (1), we get an expression for the shear stress as $\tau=-\frac{r}{2}\frac{\partial p}{\partial z}$. If the fluid is a homogeneous isotropic Newtonian fluid we can write $\tau=-\mu\frac{\partial v_z}{\partial r}$. Being the fully developed flow, the velocity in the *z*-direction $\left(v_z\right)$ becomes a function of the transverse co-ordinate (r) only and therefore, $\frac{\partial v_z}{\partial r}$ can be written as $\frac{dv_z}{dr}$. This is exactly same like a flow between two parallel plates; the only difference in this equation arises because of the change in the co-ordinate system. Now question may arise because of the '-' sign in the expression of the shear stress (τ). Similar to the Cartesian co-ordinate system here also we can write $\tau = \mu \frac{du}{dy}$. Then we need to define 'y' properly which is very important; one cannot take 'y' arbitrarily. If we have a surface then 'y' is in an outward normal direction to the surface (also shown in figure 1). So, here the 'y' co-ordinate which is normal to the wall is opposite to the r co-ordinate and that is why that has been adjusted by a minus sign. Now $\mu \frac{dv_z}{dr}$ is a function of r only and $\frac{\partial p}{\partial z}$ is a function of z only. To know that whether $\frac{\partial p}{\partial z}$ becomes a function of z only or not one can refer to the r component of the momentum equation where the full Navier Stokes equation is required. In many books in the undergraduate level, for simplicity, $\frac{\partial p}{\partial z}$ is directly substituted by $\frac{dp}{dz}$ without even explaining the reason. The r-component of the momentum equation tells that it depends on two factors, the radial component of the flow v_r and the pressure gradient in the r direction $\frac{\partial p}{\partial r}$. Now for fully developed flow, the radial component of the flow v_r is equal to zero which means that the pressure gradient in the r direction $\frac{\partial p}{\partial r}$ is also equal to zero, i.e. $\frac{\partial p}{\partial r} = 0$. Then only $\frac{\partial p}{\partial z}$ can be replaced by $\frac{dp}{dz}$. So, the aforesaid aspect needs to be carefully considered, it is just like the y momentum equation for a flow between two parallel plates. Referring to that only one can tell that p becomes a function of z only and $\frac{\partial p}{\partial z}$ can be written as $\frac{dp}{dz}$. This should not be taken as an automatic and that is why just the simple force balance in the z direction may not be sufficient enough to give the clear physical understanding of the problem. We also require the force balance in the r direction to tell the complete picture. So, the expression of the shear stress becomes $\mu \frac{dv_z}{dr} = \frac{r}{2}\frac{dp}{dz}$. We can write this in the form of separated variables. If we take *r* on the left hand side, then the left hand side becomes a function of *r* only. Similarly, the right hand side will become the function of *z* only, i.e. $\frac{2}{r}\mu\frac{dv_z}{dr}=\frac{dp}{dz}$. In a mathematical way, in order to make them equal to each other, these two sides must be equal to a constant, say c. There are many ways to look into this equation, but the most important way is undoubtedly the physical way. The physical way tells that the force balance gives $\frac{\partial p}{\partial r}=0$. Now, we have $\frac{1}{r}\frac{dv_z}{dr}=\frac{1}{2\mu}\frac{dp}{dz}$; it is written in such way that we have a separate function of *r* as well as a separate function of *z*. This two expressions $\frac{1}{r}\frac{dv_z}{dr}$ and $\frac{1}{2\mu}\frac{dp}{dz}$ are equal to the constant c, i.e. $\frac{1}{r}\frac{dv_z}{dr}=\frac{1}{2\mu}\frac{dp}{dz}=c$. So, dv_z can be written as $dv_z=c\,r\,dr$. If we integrate it with respect to *r*, we get $v_z=\frac{cr^2}{2}+c_1$ where c_1 is the integration constant. Now, this expression $v_z=\frac{cr^2}{2}+c_1$ may create a dilemma that the velocity dependence in the Navier Stokes equation is a second order dependence and accordingly, there should be two integration constants. But here only one integration constant c_1 is appearing which means we can use one boundary condition to find out the velocity profile. So question may arise about the second boundary condition. The need of the other boundary condition is indeed needed in this analysis. Here, we have assumed symmetry with respect to the channel centerline. That means we have implicitly assumed that at *r* = 0, $\frac{dv_z}{dr}$ is equal to zero; otherwise we cannot write the symmetric force balance. So we have v_z which is equal to $\frac{cr^2}{2}+c_1$ where the symmetry at the channel centerline has been used already. The remaining boundary condition is the no-slip condition which means that at *r* = *R*, v_z is equal to zero. Applying this we get the expression of c_1 as $c_1=-\frac{cR^2}{2}$. So, the complete expression of the velocity v_z is given by $v_z=\frac{c}{2}\left(r^2-R^2\right)$. This is again a parabolic distribution. Now one can find out the average velocity and express the constant c in terms of the average velocity.

Now let us calculate the average velocity; the definition of the average velocity $\left(\bar{V}\right)$ here is given by $\bar{V} = \frac{\int_0^R v_z \cdot 2\pi r\, dr}{\pi R^2}$ where the distribution of the velocity v_z is in the form $v_z = \frac{c}{2}\left(r^2 - R^2\right)$. The integrand $\int_0^R v_z \cdot 2\pi r\, dr$ becomes equal to $\int_0^R \frac{c}{2}\left(r^2 - R^2\right) \cdot 2\pi r\, dr = cR^4\pi\left(\frac{1}{4} - \frac{1}{2}\right) = -\frac{cR^4\pi}{4}$. The term π gets cancelled out from the numerator and the denominator and the expression of the average velocity becomes $\bar{V} = -\frac{cR^2}{4}$. Therefore, the constant c can be expressed in terms of the average velocity as $c = -\frac{4\bar{V}}{R^2}$. Then, the ratio of the velocity distribution and the average velocity is given by $\frac{v_z}{\bar{V}} = 2\left(1 - \frac{r^2}{R^2}\right)$. This expression $\frac{v_z}{\bar{V}} = 2\left(1 - \frac{r^2}{R^2}\right)$ is the velocity profile in terms of the average velocity for Hagen Poiseuille flow. Now we can calculate the pressure drop. In engineering, the velocity profile is undoubtedly important but what is fundamentally required is that if we want to drive a flow how much pumping power is required. Because it is the pumping power which is required to overcome the viscous resistance and without that the fluid cannot flow continuously through the pipe. So without that the application does not work.

Using the expression of the constant c of the velocity profile, we can write $\frac{1}{2\mu}\frac{dp}{dz} = -\frac{4\bar{V}}{R^2}$. Now the expression of the pressure gradient $\frac{dp}{dz}$ can be given by $\frac{dp}{dz} = -\frac{h_f \rho g}{L}$ where h_f is termed as the head loss due to fluid friction. This is basically a length equivalent of energy. Here $-\frac{h_f \rho g}{L}$ is the energy that is lost in order to overcome friction. Since it is a loss of energy, a minus sign is written in the beginning. Substituting the expression of $\frac{dp}{dz}$ we can write $\frac{h_f \rho g}{\mu L} = \frac{8\bar{V}}{R^2}$. One can express the average velocity in terms of the volumetric flow rate Q as $\bar{V} = \frac{Q}{\pi R^2}$. The reason behind expressing the average velocity in terms of volumetric flow rate is that it is the volumetric flow rate which is measured instead of the average velocity. Using this, the expression of h_f becomes $h_f = \frac{8\bar{V}\mu L}{\rho g R^2} = \frac{8Q\mu L}{\rho g \pi R^4}$. Normally in engineering, instead of pipe radius, pipe diameter is mainly used. If we substitute the pipe radius by the pipe diameter, i.e. $R = D/2$, we get the final expression of h_f, i.e. $h_f = \frac{128 Q\mu L}{\rho g \pi D^4}$. This is called as the Hagen Poiseuille equation where h_f is the head loss due to fluid friction in fully developed flow through circular pipe. This equation tells us that for a given flow rate, the head loss h_f is inversely proportional to the fourth power of the diameter of the pipe (*D*), i.e. $h_f \propto D^{-4}$. So if we reduce the diameter of the pipe, it requires huge pumping power to drive the flow. This is reason why in micro scale flow we have to use other forces like surface tension force, electrical force etc. to drive a flow in many practical circumstances. To maintain a particular flow rate, if we make the diameter of the pipe smaller and smaller, the head loss becomes larger and larger and huge pumping power is required. Now there are two important factors in engineering, one is the Darcy friction factor which is given by the Darcy Weisbach formula $h_f = f\frac{L}{D}\frac{\bar{V}^2}{2g}$ which is valid for both the laminar and the turbulent flow. For a fully developed laminar flow, this f can be calculated by equating the two expressions of the head loss h_f, i.e. $h_f = f\frac{L}{D}\frac{\bar{V}^2}{2g}$ and $h_f = \frac{128 Q\mu L}{\rho g \pi D^4}$ and we get $f = \frac{64}{\mathrm{Re}_D}$ where D is the diameter of the pipe. The factor f is known as the Darcy friction factor. There is another friction factor or friction coefficient which is called as fanning friction coefficient which is $C_f = \frac{\tau_w}{\frac{1}{2}\rho\bar{V}^2}$. The wall shear stress is related to the pressure gradient from which the head loss h_f has come. One friction coefficient becomes 4 times the other; one is equal to $\frac{16}{\mathrm{Re}_D}$ while the other one is equal to $\frac{64}{\mathrm{Re}_D}$. This can be easily shown and has been left out for the students as an exercise.

Example 2. Taylor Couette flow

The second problem which we will consider here is a very interesting problem which is a flow between two concentric rotating cylinders. Let us consider two cylinders of radii

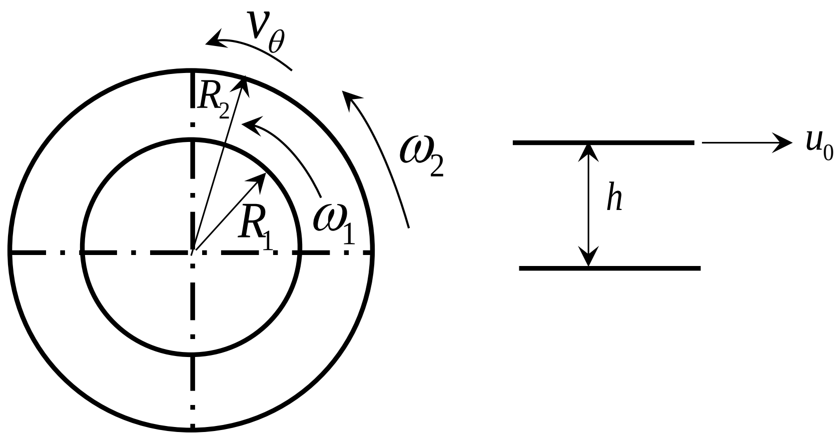

Figure 2. Schematic of the flow between two concentric cylinders. The right hand side of the figure shows the corresponding case for the plane Couette flow.

R_1 and R_2 respectively as shown in figure 2. This is called as the Taylor-Couette flow. The cylinders are rotating with angular velocities ω_1 and ω_2. The gap between the two cylinders is filled up with a fluid. If ω_1 is different from ω_2 then there is a shear which is acts on the fluid. If the gap is very small then this is equivalent to a plane Couette flow. In a plane Couette flow we have two parallel plates in which one plate is moving relative to each other. In the present case the relative velocity can be considered as $\omega_1 - \omega_2$ multiplied by the difference in the radial co-ordinate. Let us say this velocity is u_0 and the gap is *h*. If the gap *h* is very small, then the curvature of the cylinders may be neglected. This is then like a flow between two parallel plates, one plate is moving relative to the other; this is a classical shear driven flow. This scenario is actually encountered in a more realistic way. We will consider two examples. The first example is that we have a shaft (inner cylinder) and a bearing (outer cylinder) and then there is lubricating oil which separates the shaft and the bearing to avoid metal to metal contact. It is important to know the power required to rotate the shaft. On a different note this is a configuration which is commonly used to measure the viscosity of an unknown fluid which is kept in between the two concentric cylinders. This arrangement is known as rotating cylinder viscometer which is a classical way of measuring the viscosity of an unknown fluid. If we measure the torque and the power which is required to drive one cylinder relative to the other, that expression will have viscosity as a parameter from which one can measure the viscosity. With this little bit of motivation, we will use the cylindrical co-ordinate Navier Stokes equation. Since the Navier Stokes equation has already been discussed in the earlier chapters, we will directly use the different equations without referring to the physical basis. The continuity equation in the cylindrical co-ordinate system is given by

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_r\right)+\frac{\partial v_\theta}{\partial \theta}\right]+\frac{\partial v_z}{\partial z}=0 \tag{2}$$

This particular writing of the Navier Stokes equation and the continuity equation in the *r*-*θ*-*z* co-ordinate system is a little bit of a tedious job. We don't need to memorize these equations. The important objective is to see that how these equations can be applied to solve the present problem.

In the present problem, there is the v_θ component of velocity but it is not a function of the θ co-ordinate, so, $\frac{\partial v_\theta}{\partial \theta}=0$. We assume the cylinder is infinitely long so that there is negligible gradient $\frac{\partial v_z}{\partial z}=0$. So, the simplified form of the continuity equation becomes $\frac{\partial}{\partial r}\left(r\, v_r\right)=0$ which means that $r\, v_r$ is not a function of the radial co-ordinate *r*. So, from the continuity equation we get $v_r=0$ because v_r is equal to zero at radius $r=R_1$ and $r=R_2$ respectively. So, v_r has to be equal to zero at all radial locations. Now we will consider the *r*-momentum equation. Since these are long equations, we need to write these equations very carefully; otherwise we may make some mistakes. Then we will asses various terms of these equations. The *r*-momentum equation is given below

$$\begin{aligned}&\rho\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right]\\&=\rho\, b_r-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\, v_r\right)\right\}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]\end{aligned} \tag{3}$$

Here, the term $\frac{\partial v_r}{\partial t}$ is zero because the velocity component v_r is equal to zero as well as the flow is steady. Since v_r is equal to zero, all terms involving v_r becomes equal to zero.

Since there is no body force in the radial direction ρb_r is equal to zero. It is important to note that the velocity component v_θ is not equal to zero. However v_θ is not a function of θ, i.e. $\frac{\partial v_\theta}{\partial \theta} = 0$. Considering all these factors, the simplified form of the r-momentum equation can be written as

$$\frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} \tag{4}$$

Then we will focus on the other momentum equations. Let us consider the z-momentum equation which is given in the following

$$\begin{aligned} &\rho\left[\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right] \\ &= \rho b_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \end{aligned} \tag{5}$$

The z-momentum is expected to be redundant because v_θ is the predominant component of velocity. From the r-component we have got an equation for the pressure gradient. Here, in the z-momentum equation $\rho \frac{\partial v_z}{\partial t}$ is equal to zero because it is a steady flow. Since v_r is equal to zero, the term $\rho v_r \frac{\partial v_z}{\partial r}$ becomes equal to zero. Since there is no variation in the axial (z) direction, the terms $\rho v_z \frac{\partial v_z}{\partial z}$, $\mu \frac{\partial^2 v_z}{\partial z^2}$ are equal to zero. Since v_θ is the predominant component of velocity, the term $\mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$ is equal to zero. Since there is no variation in the θ-direction, the terms $\rho \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}$ and $\mu \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}$ become equal to zero. There is no body force in the axial direction, so $\rho b_z = 0$. Then the simplified form of the z-momentum equation becomes

$$\frac{\partial p}{\partial z} = 0 \tag{6}$$

The problem may be complicated by pulling one of the cylinders axially. In that case the v_z component will also exist in addition to the v_θ component. Finally we will focus on the θ-momentum equation which is given as follows

$$
\begin{aligned}
&\rho\left[\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r\,v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right]\\
&=\rho b_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_\theta\right)\right\}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]
\end{aligned}
\tag{7}
$$

Because of the steady flow assumption, the term $\rho\frac{\partial v_\theta}{\partial t}$ is equal to zero. Since there is no v_r component, the terms $\rho v_r\frac{\partial v_\theta}{\partial r}$ and $\rho\frac{v_r\,v_\theta}{r}$ are equal to zero. Because there is a symmetry with respect to the θ-direction, the terms $\rho\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}$, $\mu\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}$ and $\mu\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}$ are equal to zero. Since there is no variation in the axial (z) direction, the two terms $\rho v_z\frac{\partial v_\theta}{\partial z}$ and $\mu\frac{\partial^2 v_\theta}{\partial z^2}$ are equal to zero. There is no body force in the θ-direction, so, $\rho b_\theta=0$ and there is no pressure gradient in the θ-direction, so, $\frac{1}{r}\frac{\partial p}{\partial \theta}=0$. Then, the simplified form of the θ-momentum equation is given by

$$
\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_\theta\right)\right\}=0 \tag{8}
$$

Since v_θ becomes a function of r only, we can write this partial differential equation as an ordinary differential equation, i.e. $\frac{d}{dr}\left\{\frac{1}{r}\frac{d}{dr}\left(r\,v_\theta\right)\right\}=0$. Integrating this with respect to r we get, $\frac{1}{r}\frac{d}{dr}\left(r\,v_\theta\right)=C_1$ where C_1 is integration constant. Integrating this again with respect to r results $r\,v_\theta=C_1\frac{r^2}{2}+C_2$ with C_2 being the other integration constant. So, the velocity distribution becomes $v_\theta=C_1\frac{r}{2}+\frac{C_2}{r}$ which is of the form of $v_\theta=Ar+\frac{B}{r}$ where A and B are the two constants. To determine these constants we need to apply the boundary conditions. At $r=R_1$, the velocity v_θ is equal to $\omega_1 R_1$, i.e. $v_\theta=\omega_1 R_1$. Similarly, at $r=R_2$, the velocity v_θ is equal to $\omega_2 R_2$, i.e. $v_\theta=\omega_2 R_2$. Now if we consider about an interesting problem where there is only one cylinder rotating in an infinite fluid (Example 1) in that case there is no ω_2 and we must have finite v_θ as r is tending towards infinity, i.e. $r\to\infty$. So if there is a cylinder or sphere which is rotating in a infinite fluid, v_θ has to be finite at $r\to\infty$; this is a matching boundary condition which must be satisfied. Then the constant A must be equal to zero otherwise the velocity v_θ will be infinity as r is approaching towards infinity. So, in that case v_θ is reduced to the form $v_\theta = \frac{B}{r}$ where the constant B is obtained by applying the boundary condition, i.e. at $r = R_1, v_\theta = \omega_1 R_1$. This is the example of the classical free vortex flow. Now we can calculate the viscous stress $\tau_{r\theta}$ from the expression $\tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$. If we calculate this we will find that the viscous stress $\tau_{r\theta}$ is non-zero. Now, we can calculate the volumetric viscous force $\mu\nabla^2 v$ for which we need to use the vector identity (as used earlier) $\nabla\times(\nabla\times\vec{v}) = \nabla(\nabla\cdot\vec{v}) - \nabla^2\vec{v}$. Because it is an irrotational flow, $\nabla\times(\nabla\times\vec{v})$ is equal to null vector. For an incompressible flow, $\nabla\cdot\vec{v}$ is equal to zero (We need to remember that this condition is satisfied only for incompressible flow). Then $\nabla^2 v$ and thus the volumetric viscous force $\mu\nabla^2 v$ becomes equal to zero for an incompressible flow. So this is a non-intuitive physical situation where the shear stress or the viscous stress is non-zero but the viscous force per unit volume is zero. Local shear stress may not zero but when it is integrated over a volume the net effect is such that there is no net viscous force. We are coming to this conclusion only because of the consideration of the incompressible flow. If it is not an incompressible flow this equality does not work. So, in this chapter, we have discussed about the uses of Navier Stokes equation in cylindrical polar co-ordinate system trough two important examples. The next chapter will focus about some unsteady flow solutions based on the exact solutions of simplified Navier Stokes equation.