Advanced Concepts In Fluid Mechanics
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Lecture – 01
Eulerian and Lagrangian Description of Fluid Motion

I. Introduction

This lecture focuses on the Kinematics of Flow. Kinematics is an important part of fluid mechanics and is a required preliminary chapter in the course of fluid mechanics.

Kinematics is the study of the motion of fluid without the consideration of the forces that cause the motion. It doesn't imply that these forces are not acting, but that we don't study these forces at this stage and give importance to the motion which is their consequence. This abstract understanding of motion is of utility because when we develop the laws of motion, we obtain a relationship between the force and acceleration as the cause and the effect respectively. Therefore, studying kinematics aids us at obtaining an expression for the acceleration, which is one part of this relationship. While fluid statics has traditionally been taught prior to fluid kinematics, it is not a strict requirement.

Fluid kinematics involves the following topics:

1. Description of fluid flow
2. Concept of steady/unsteady and uniform/non-uniform flow
3. Streamline, streakline and pathline
4. Acceleration of fluid
5. Deformation (linear, volumetric, angular)
6. Parametric representations – Stream function and Velocity potential

II. Description of fluid flow

In solids, a particular particle can be tracked with time easily as the solid deforms. In contrast, a fluid deforms continuously when in motion and therefore, it becomes very difficult to track a particular particle. Therefore, it becomes very important to have a description of fluid flow.

However, if one does track a set of fluid particles in a flow, the description of fluid motion is called 'Lagrangian Description'. In the Lagrangian description, the position vector $\vec{r}$ of any particular particle being tracked is a function of the initial position vector $\vec{r}_0$ and time t, i.e. $\vec{r} = \vec{r}(\vec{r}_0 t)$. While more difficult to handle and less commonly-used, Largrangian description is useful in certain situations, for instance, in ascertaining how good mixing is being achieved in a flow. In this description, we have specified the mass whose motion is being described. Therefore, it is also termed as 'Control Mass Approach'.

Alternatively, if one focuses on a particular region in space, which is essentially a set of positions and studies the flow at these positions, the flow description thus obtained is called 'Eulerian Description'. In this description, the velocity $\vec{v}$ at a position $\vec{r}$ is a function of $\vec{r}$ and t only, and is independent of the initial position of the particle situated there, i.e. $\vec{v} = \vec{v}(\vec{r}, t)$. Fluid enters and exits the said region in space at certain combination of its boundaries. Furthermore, this region is space could be stationary or could itself be moving. Therefore this region in space is actually a control volume, and so, Eulerian description is also termed as 'Control Volume Approach'.

These two descriptions are connected based on the understanding that the velocity at a particular point at a given instance (i.e. Eulerian description) is the velocity of the particle situated at that point at that instance (i.e. Lagrangian description).

III. Concept of steady/unsteady and uniform/non-uniform flow

We firstly define flow-field as the region in space over which you have the influence of flow acts.

If the flow velocity and other fluid properties (e.g. density, viscosity, surface tension) at a particular position in the flow field do not vary with time, the flow is called steady, otherwise, flow is called unsteady. On the other hand, if the flow velocity and other fluid properties at a given instance of time do not vary with position over the flow field, the flow is called uniform, otherwise, flow is called non-uniform.

To better elucidate these definitions, below are some examples:

(a) Steady and Non-Uniform Flow: A constant flow-rate flow through a pipe, since the flow varies from maximum at the centreline to zero at the wall, but flow at any point does not vary with time

(b) Unsteady with Uniform Flow: If the velocity at all points is 1 m/s to the east at one instance and changes to 2 m/s to the north, in such a manner that at any instance, the velocity at all points is equal

(c) Same flow is steady and unsteady in different frames: Consider a ship passing under a bridge at a constant speed. For an observer sitting on the bridge, the ship creates a disturbance as it passes under it, and the disturbance dies down with time. To this observer, the flow is unsteady. In contrast, for an observer sitting on the ship, the disturbance due to the ship carries along with the ship in such a manner that the flow is steady.

IV. Streamline, streakline and pathline

The discipline of fluid mechanics is heavily adorned with experiments of different sorts. While velocity field measurements are a key attribute of these experimental studies, another aspect is qualitative visualization of the flow. For example, smoke emitting out of the chimney of a power plant can generate vortex structures, giving a a flow visualization for the gases being emitted. Similarly, for any general fluid some traditional ways of looking into flow visualization have emerged over time, considered to be described by imaginary lines in the flow field – streamline, streakline and path line. These are defined below:

(a) Streamlines – Streamlines, at a given instance in time, are defined as the imaginary curves in the flow field, such that tangent to these lines at each point are co-directional with the velocity vector at that point. It is important to note that streamlines are curves in the flow-field at given instances of time, and can change from one moment to another depending on the nature of the flow. To obtain the expression for a streamline, the time-instance is fixed first. Now we consider a chord on the streamline extending from $\vec{r}$ to $\vec{r}+d\vec{r}$. Considering the chord to infinitesimally approach a point, $d\vec{r}$ is obtained as the tangent to the streamline at that point. By definition, this tangent should be co-incident with the velocity $\vec{v}$ at that point. Therefore, $\vec{v} \times d\vec{r} = 0$. This gives $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, which is the equation of a streamline. To obtain the streamline at a given instance of time, one needs to integrate this equation at that instance of time with the known velocity field $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$.

(b) Streaklines – Streakline is the locus of all the particles that have crossed a particular position in the flow-field at different instances of time up to the current. It can be understood as the curve created by injecting a coloured dye at a particular position in the flow-field. All the particles passing through this point get colored by the dye and at a time-instance in the future, these points joined together will give a colored appearance, which will be the streakline at that future time-instance. Therefoere, it is sometimes misconceived that the lines being visualized in a flow-field are streamlines while they are actually streaklines. However, streamlines and streaklines become identical under some special flow conditions, which will be discussed in later lectures.

(c) Pathline – Pathline is the locus in the flow-field, of an identified fluid particle, over time. In other words, pathline is essentially a Lagrangian description.

Conceptual discussion and illustrative examples on streamline, streakline and pathline are continued in the next lecture.