Conduction and Convection Heat Transfer
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Lecture 8

A slab is exposed on one side to a hot gas and another side cold air which is a very, very popular problem consider the wall of a furnace. It is other way in case of wall of a cold chamber inside is cold outside is hot and wall of a furnace this inside is hot gas and outside is cold air. So, what happens heat is coming from the hot gas to this surface by convection.

Then it flows to the surface through the surface by conduction and again goes to the cold air and the problem is specified by some temperature this is the scale I am just showing you in this direction the hot temperature that means the temperature of the hot gas Th and the cold temperature here, cold air temperature from the same datum, Th and Tc and you have to find out what is the value of Q.

So, in this case what happens if the problem is at steady state that means the heat which comes by convection to this surface from the hot gas is the same one that is conducted through these solid and again is convected from this surface to the cold gas and in this case, we assume that T1 which is not known, T1 is the temperature at this surface and T2 is the temperature at this surface and just by our common sense.

We understand that this T1 will be lower than Th and T2 will be lower than T1 and Tc will be lower than T2 that means Th is greater than T1, is greater than T2, greater then Tc but in this case problem is specified in terms of Th and Tc. Now what is the circumstances that here what happens if the hot gas is there hot gas may be flowing, may not be flowing. In case of a stagnant hot gas there will be a recirculate, there will be a natural circulation, flow like this which I discussed in one of the earlier classes.

This will be discussed in detail when we will take or discuss the convection, heat transfer. Now this is exposed to an (()) (28:15) cold air where this surface is hotter with respect to the cold air. So, there will be a flow natural due to buoyancy flow of air like that and this flow even in absence of any forced flow justify these different from the conduction that's why I told in a fluid medium when there is a solid adjacent to a fluid, fluid cannot be kept at stationary.

So, there will be flow that may be a forced flow or that may be a natural flow, buoyancy induced flow because of which this heat transfer will be convection heat transfer different from the conduction. Not this conviction heat transfer characteristic is that the temperature Th changes to T1 within a very short distance from this plan, convection resistance is offered like this. That means if we measure the temperature.

You will see this Th actually goes like this and within a thin film let us consider this as film the temperature drops from Th to T1 and this may not be linear. That depends upon the convection heat transfer equation the convection temperature field from here we get a linear temperature profile T1 to T2 by the similar deduction as we did for this case and then from T2 there is a thin film adhering to the surface within which the temperature change will take place

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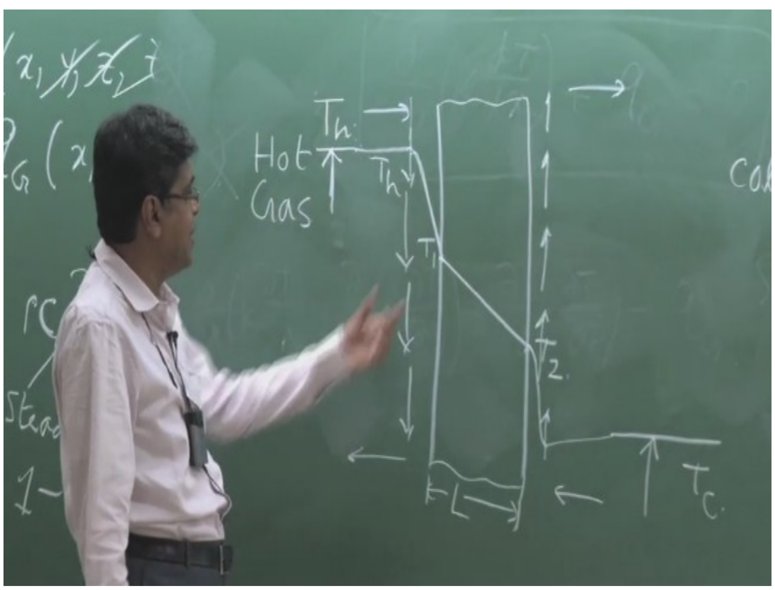

And this is defined as the thermal boundary layer which we will discuss afterward we accept this and at the same time we accept that this q which is coming from hot gas to the cold gas sorry hot gas to this left surface is given by this side q is given by heat transfer coefficient. The problem is also prescribed by a heat transfer coefficient on the hot side hh and the heat transfer coefficient at the cold side hc.

And we consider this heat transfer coefficient is constant over the area in both the sides then we know from convection heat transfer that the q can be defined as Th - T1 as I told you in one of the earlier classes that now we do not have to know anything about the mechanism of convection. How this is being written but whenever we know the heat transfer coefficient h then we can relate the heat flux into area sorry into area.

The total heat transfer is h that means heat transfer coefficient times the difference between the fluid temperature and the surface temperature. Here I am writing the positive value of the heat that means heat is flowing in this direction x so therefore temperature difference I am taking from the hot gas to the left plate times the area of the plate into hh.

Now the heat which is going out from here is again similar way hc into area into T2 - Tc, the same heat. Now if I write this here we write this Q is equal to now in sequence if I write then hh into the area, area is same is a plane are not varying also in the direction of heat flux T1. The same heat flux Q now goes from this surface to this surface flows by conduction which is nothing but T1 - T2 divided by L by KA which already we deduced here.

If the two temperatures the surface temperature T1, T2 conduction is given by this and the same heat Q because steady state, the same heat is there it is going out which is hc into A into T2 - Tc. Now if we write the temperature differences in terms of the heat flux and add it up then what we get and if we write all the equations rearrange it in the fashion as we did here that flux is equal to potential difference divided by the resistance.

Then what we get or other way if I write in this fashion. This Th - this is little rearrangement it is Q into 1 by hh A. Similarly, if I write T1 - T2 it is Q into 1 by L by KA. Oh, Q by hh not by 1 by hh. I am extremely. Sorry, Q into 1 by hh it is all right. Second one is Q into L by KA I am extremely sorry and third one is equal to T2 - TC is equal to Q into 1 by hcA and now if we add this three we can get Th - Tc is equal to Q into 1 by hhA + L by kA + 1 by hcA.

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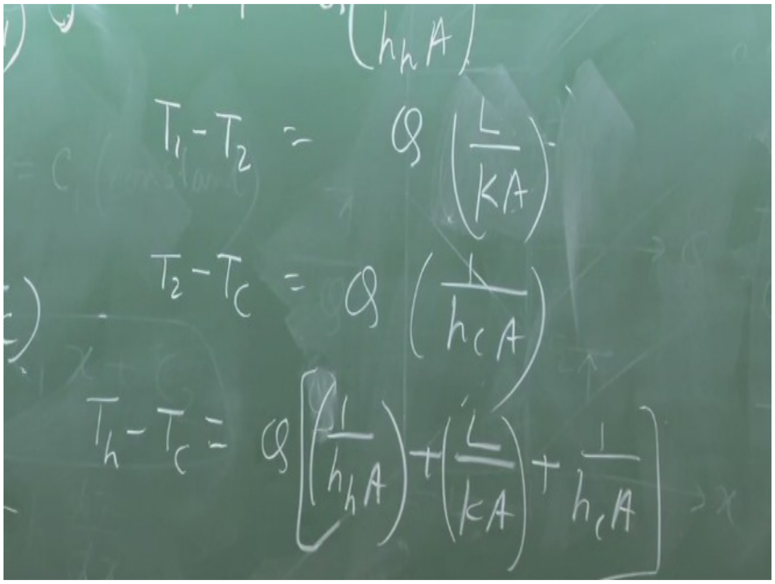

That means we can write this as Q is equal to Th - Tc divided by 1 by hhA + L by KA + 1 by hcA. That means this T1, T2 are eliminated that means if the problem is specified by the hot fluid in one side and cold fluid in one side and the heat is transferred from the hot fluid to cold fluid via the conduction due to the thickness of the wall then I can find out the heat flux knowing only hot fluid temperature and cold fluid temperature provided.

I know the information about the heat transfer coefficient on the hot fluid side and the heat transfer coefficient on the cold fluid side and this equation in the light of the electrical analogous circuit looks like a series that means a series resistances comprising the conduction resistances and conviction resistances so this can be well represented by an electrical analogous circuit like this.

There are three resistances in series and the extreme potentials are Th and Tc, hot gas temperature and cold gas temperature and the same heat that means it is a series resistance problem this is convection resistance, which is equal to 1 by hhA. Obviously heat transfer coefficient is like conductance so reciprocal of heat is the resistance. Similarly, as I already told that conduction resistance is L by KA and here is again convection resistance.

Here we can write R convection hot side and R convection cold side which is 1 by hcA. So therefore, some of the resistances will be in the dominator and in the numerator, is the overall potential drop and if we know each resistance component you can find out this intermediate voltage or intermediate potential or the temperature these are the most simple cases.

Another simple case relating to this purely one-dimensional heat flow through a slab or window type or a wall type of thing is like this instead there is a composites wall. If we have a composite wall for example like this we have a composite wall, this is one wall, this is one wall let us denote it by A and this is another wall let us denote it by B and there is a perfect matching between these two wall.

There is no gap, there is no contact resistance. There is perfect matching of this two wall first of all I will assume that and they are of different thermal conductivity KA and KB which are not equal but the heat transfer problem is steady that means that temperature is a one dimensional because of the geometry and the boundary condition.

Let us consider this face of the composite wall is T1 and this face of the composite wall is kept at T3 not T2 which I will use for the junction interface and T1 is greater than T3 and in this case because of this boundary condition and geometry one dimensional heat flow and if the temperature is independent of time that means we allow the same heat to flow out. Then how to find out the temperature distribution?

Now temperature distribution here we cannot solve the equation, thermal conductivity is same that if you write like this d by dx of K, dt by dx. No qg that means no generation of but this we cannot solve x is equal to zero and let us consider this length is L1 and this length is L2 and here x is equal to L1 + L2 that means left phase is x is equal is zero and the right phase is L1 + L2x and we cannot solve in the entire domain this equation by take K out because K is not constant.

So therefore, we have to divide this to domain like this zero less then equal to x less than is equal to L1 and another one is L1 less then equal to x less then equal to L1 + L2

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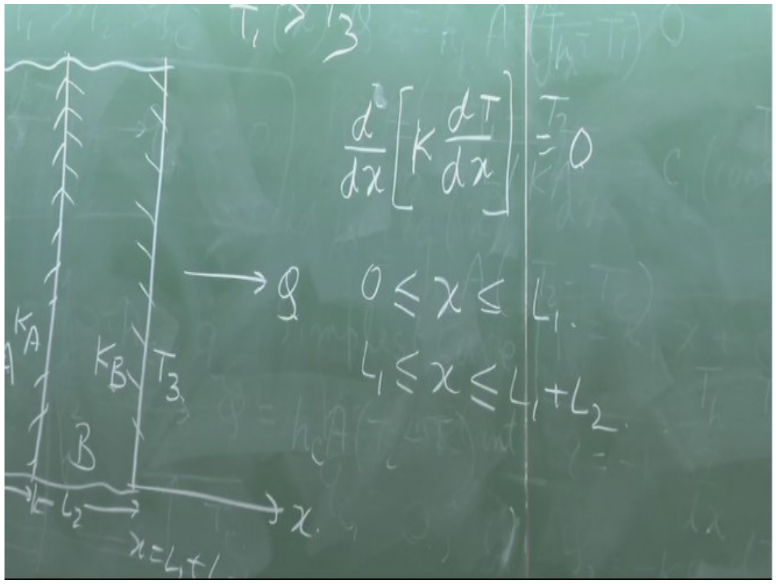

And if you solve the first one with the boundary condition that means T is equal to C1x + C2. This is the general solution for cost and thermal conductivity when we solve in these domain thermal conductivity is constant and that too that equals to Ka. Similarly, in L1 to L1 + L2 it is K constant. So therefore, T is a linear function of x C1 x + C. Now for the first one at x is equal to zero, T is equal to T1 and at x is equal to L1 if we define the interface temperature is T2 which is in between obviously by the second law of thermodynamic, T2.

So, T is equal to T2 and we immediately get an expression T is equal to T1 - T1 - T2 into x by L which is valid for zero less then equal to x less then equal to L1. So, x is equal zero T1 and x is equal to L1, T2 clear. X by L1 any problem please tell if I make inadvertent mistake while doing so please immediately detect it and tell me. So, it is L1.

Similarly, when we solve for this domain that L1 and L1 + L2 then what happens the boundary condition for this domain L1 less then equal to x less then equal to L1 + L2 you get what? at x is equal to L1, T is equal to T2, at x is equal is L1 + L2, T is equal to T3 and you get an equation T is equal to T2 - T3 divided by L2 into x - L1. Automatically it will come but more intelligently one can say that if it shifts the origin here.

And measure x from here which will be actual x because here x is measured from the origin - L1 so it will give the same equation where x is going as x - L1 is the translation of the coordinate.

So therefore, you see the two-linear temperature profile is obtained where we get the different slope and here we do not have any understanding of the qualitative picture of the slope which one is greater or which one is less that depends upon the value of T1, T2, T3 that means T1, T2, T3 for given T1, T3, T2 will be adjusted accordingly.

So that the slope will be determined and the slope will be rather found from this express

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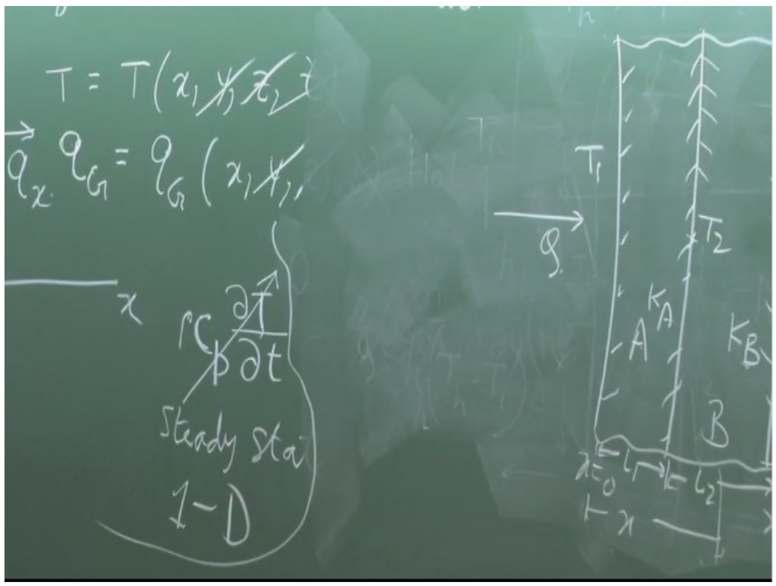

That heat which that means the qualitative picture of the slope that heat which is coming here from this side of this material A to this surface and the same thing flowing here is same Q so therefore one can write Q is equal to - k, dt by dx, KA at A. That means mathematically I write x, L1 - that means in material A that means L1 - means from this side must be equal to - KB into dt by dx at x is equal to L1 + means in material B.

Now therefore we see since the heat continuity will be there heat balance that means K times the dt by dx have to be constant which means there is a discontinuity of the slope and slope is inversely proportional to the respective thermal conductivity. So, though we see from this equation there is a continuity of the function temperature but from here also we also find out the discontinuity in the slope at x is equal to L1.

And the quantitative picture will be clear from this. Now we consider a K that Kb is greater than

Ka then what will happen this temperature gradient will be steeper, this temperature gradient will be flat because KB is higher than K, it is inversely proportional to thermal conductivity. So therefore, we can draw a picture like this I will use a color chalk T1 T2 whereas this is sorry this is relatively flat.

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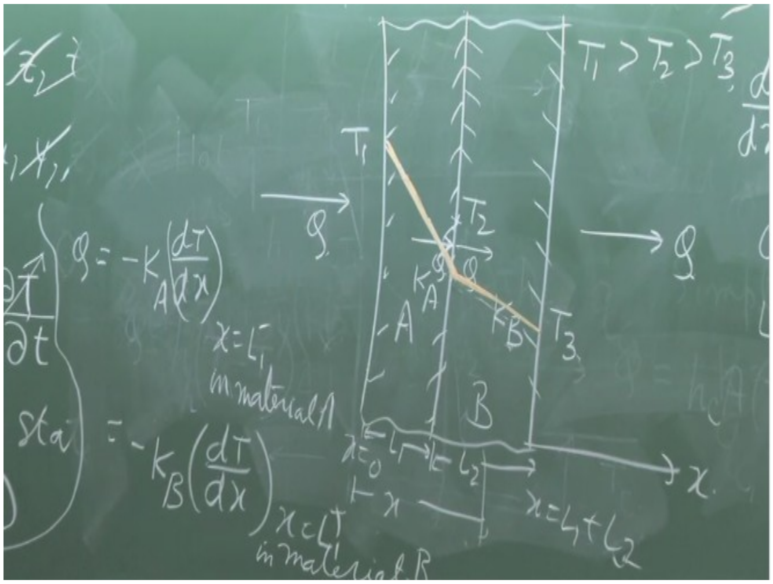

So, this will be steep this will be flat and if we apply our physical sense then also it is true that here since the thermal conductivity is low so this side at the end of this plate the temperature is less then this that means it cannot sense a temperature very close to this but since the thermal conductivity of this plate or this wall is high that means this side down stream side this outlet side or this face the temperature is close to this inlet face.

So therefore, the temperature profile is flat so more is the thermal conductivity, more is the temperature profile flat, less is the thermal conductivity, steeper is the temperature profile with a for a given heat transported. In conduction, for a given amount of heat conducted so temperature profile is always inversely proportional to thermal conductivity. So, this type of composite wall depending on the thermal conductivity.

We can draw the qualitative picture like this and here also if we also prescribe the problem with a hot gas temperature that means it is coupled with the convection boundary and the cold gas temperature then the problem becomes like this if this composite wall also be the same wall is being proposed with wall A with KA, wall B with KB and if the problem is proposed with hot gas temperature Th and the heat transfer coefficient in the hot gas Hh.

And here also cold air with Tc and with the heat transfer coefficient hc then temperature profile can be shown like this from a value of Th there is a thin film here from a value of Th it goes to T1 then there will be a steep change to T2 then there will be little flat relatively flat to T3 because KA is less then KB and then in a thin flame heat goes to Tc. This variation in the thin flame thermal boundary layer due to convection from the fee stream gas temperature to the solid surface temperature here.

Also, this nature at present I do not know until and unless I solve the convection equation, energy equation in convection. So, without that I cannot do so now if this thing is shown the composite wall with convection boundary condition in the electrical analogous circuit then what we do? We show like this. It is also a series resistance problem very simple problem that our nodal point for potential is Th, this is the T1 this is the junction of T2 this is T3 and this is TC.

And the same heat is flowing that means I am writing the electrical analogous circuit for the equation it is rub down which I wrote earlier Q and this R convection hot side which is 1 by hh A then this is R conduction material A this is LA by KAA similar way you can solve that now.

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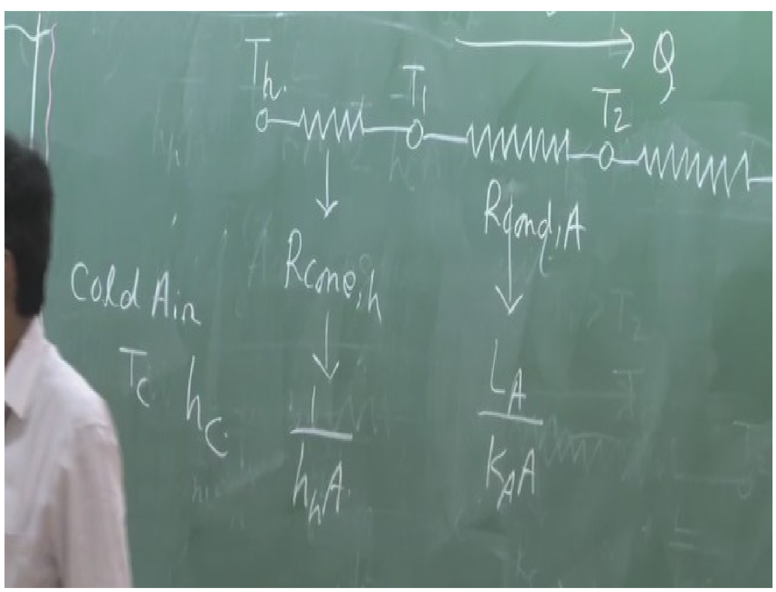

The same heat is going to that at this point there is heat continuity so this will be R conduction B. So, on has to understand these and if you can draw these thing KBA and this is R convection cold and that is 1 by hcA. So, if one can draw this electrical analogous circuit is a combination of resistances then probably may solve and we can write then Q as overall temperature difference if the problem is prescribed in terms of the overall temperature difference.

It is same at the single wall only thing is that instead there are two LA by KAA two conduction resistances, LB by KB A + 1 by hcA. So, La by KAA, LB by KBA these are the two-conduction resistance in series along with the convection. So, this is a series network problem of steady one-dimensional flow. So today I will stop here and will continue tomorrow and tomorrow, we will solve some very interesting problems. So, after continuing for a little time this conduction problem little more than we will solve interesting problems.