Conduction and Convection Heat Transfer

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Lecture 63

Now, another interesting thing comes now into picture that in this rating even sizing problem is very simple but if it is other way. That means, I tell only m dot h, m dot c, T h 1 and T c 1, neither T h 2 nor T c 2 is given that means this is not written. But I know A this is known as rating problem two type of problem sizing and rating. Sizing problem is the earlier one the entire heat load is given, q is known.

That means explicitly m dot h, m dot c T h 1, T h 2, T c 1, T c 2 of course this four are not required T 1 is given T c 2 all known. Then we find out the area we know the LMTD, so using LMTD method we can find out but if it is other way area is given. Now this delta T m is now delta A LMTD and LMTD is what? LMTD is delta T 2 minus delta T 1 by l n delta T 2 by delta T 1. Now, if I know only this two T h 1 T c 1 not T h 2 T h 2.

This equation cannot tell anything so how to find out this we cannot know LMTD. So, this type of problem, where the sizing is given but we have to rate the exchanger means a particular exchanger with a given surface area is suitable or not then what we have to do? We have to use an Iterative method that means the area is given we have to rate the exchanger whether it will be suitable.

What is this (()) (31:25) we have to assume I will solve a problem today it will be more clear but still I am telling now either T h 2 or T c 2 any one we assume first. If you assume T h 2 T c 2 will be found out from this equation both are assumed helm. then you find our LMTD obviously with this assume helm then with this LMTD since A is known we find out. Now with that Q, we iterative that means we define the value of D h 2, clear.

So, this way we do both T h 2 and T c 2 until they converge with each other that means, the new value is close to the earlier value, very simple. So, today it is nothing but when it was discovered long, long back there was no MATLAB. No program not even a calculator even. So, therefor people thought that it is tremendous tough and iteration method by hand even without a calculator whereas the sizing problem is explicitly found out with the slide rule even which is not done.

So, therefore a difficulty was faced in this case that LMTD method is not that suitable because it requires tedious iterations and you gaze value is not very close you have to go for number of iterations to confront the solution. Today, it is a laughing stock but it was there, when it was discovered. Then two scientist Kays and London. They are two stalwarts in the field of heat transfer they found out another suitable method for this type of problem where seizing is given that means surface area is given.

We have to rate the heat transfer in terms his heating and cooling load that means we have to tell what are the outlet temperatures of the hot and cold fluid and this method which was found by Kays and London is known as NTU Epsilon method. So LMTD is one method another method into an employer in your placement interview we will ask this thing only. That when NTU Epsilon method, when LMT?

What is the difficulty of LMTD in rating problem and why NTU Epsilon method is difficult for sizing problem that also I will tell. Both the methods have relative merit and demerits. Now, NTU Epsilon method. Now what is NTU? NTU I will tell you afterword when the course of deduction it will come. But now you know the meaning of NTU it is an abbreviation, Number of Transfer Unit.

And I will come to this afterward because now if I write the NTU expression like Nusselt number h d/k you will know, why unnecessarily this because automatically in the way of deduction a term will come which is a none dimensional term know as number of transfer unit.

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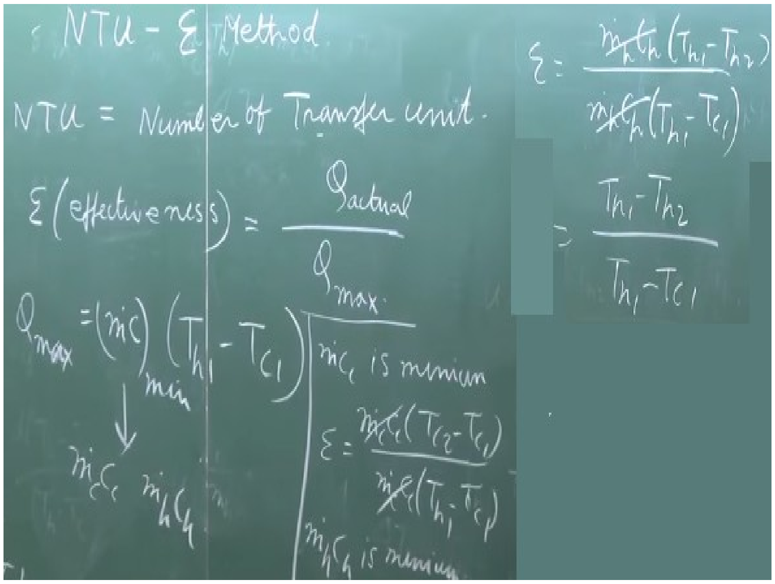

NTU - ε Method

NTU = Number of Transfer unit.

$$\varepsilon\,(\text{effectiveness}) = \frac{Q_{actual}}{Q_{max}}$$

$$Q_{max} = (\dot{m}c)_{min}\,(T_{h_1} - T_{c_1})$$

$$\downarrow$$

$$\dot{m}_c C_c \quad \dot{m}_h C_h$$

$\dot{m}C_c$ is minium

$$\varepsilon = \frac{\cancel{\dot{m}_c C_c}(T_{c_2} - T_{c_1})}{\cancel{\dot{m}_c C_c}(T_{h_1} - T_{c_1})}$$

$\dot{m}_h C_h$ is minium

$$\varepsilon = \frac{\cancel{\dot{m}_h C_h}(T_{h_1} - T_{h_2})}{\cancel{\dot{m}_h C_h}(T_{h_1} - T_{c_1})} = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_1}}$$

Epsilon is known as effectiveness which is also a non-dimensional term, effectiveness and this effectiveness of an heat exchanger Epsilon is defined as Q that is the heat transfer actual/ Q maximum, means ideal that is the ideal that is the maximum actual discharge by the ideal discharge is the coefficient of this type of thing, the maximum and actual. What is actual heat transfer? This is this. M dot h c h, the temperature change this the actual heat transfer.

But what is the maximum heat transfer which is possible. The maximum heat transfer if you write Q max try to understand this again there is a concept many people you can make people nervous by asking one question I will tell you that maximum heat transfer will be associated with the maximum change in the temperature. What is maximum change that is T h 1 minus T c 1 that means let us consider the counter flow.

I will tell the maximum heat exchange will be possible when the cold fluid will attain the inlet temperature of the hot fluid that is the maximum temperature available in the exchanger. Or other way, the hot fluid will be cooled to the inlet temperature of the cold fluid which the minimum temperature of the heat exchanger which means that Qmax will be associated with the maximum temperature difference T h 1 minus T c 1 of the exchanger. But there is a clue hot and cold fluid physically will not attain simultaneously the T h 1 minus T c 1.

It depends upon their capacity, that means the product of m dot n C c. The fluid for which this m dot c is minimum that will catch it first afterward the heat exchanger operation will stop. So therefore, it is written in a generic way m dot c minimum which may be either m dot c C c that I have to calculate from the value. I have to calculate from the value specific value then the mass flow rate value.

That means that ideal or the maximum heat transfer is defined as minimum capacity rate or minimum capacity that is mass flow rate time this specific heat capacity times the maximum temperature change one thing this is never achievable in a parallel in a counter flow like a reversible process. It can be achievable if we make an infinitely long exchanger in a limiting case like a quasi-static process that in a limiting place.

This c 2 will reach the h 1 or T h 2 with T c 1 if you keep an infinitely long. But here even if you keep an infinitely long exchanger it will never reach, delta one is universe it can be zero in an ideal situation of reversible process, a quasi-static process but can never become less than 0. It is impossible. So therefore, in a parallel flow heat exchanger it is never possible not feasible at all by the physics. Sad then what happen?

This happen like that this definition of effectiveness can be understood concept with respect to a counter flow heat exchanger nevertheless as a criteria of comparing the exchanger with this effectiveness we will use the same for parallel flow also. Though it does not have any physical meaning like the ratio of actual to maximum heat transfer in that case. Is it clear to everybody? Good students.

Now if I consider the cold fluid is the minimum fluid then what is Epsilon. Now if cold fluid if m dot C c is minimum then what is this Epsilon, it is what is Epsilon you tell me if this is minimum then m dot c C into T h 1 minus T c 1 and in that I will prefer to write this Q this equation. Why? Because this thing will cancel C c into T c 2 minus T c 1 cancels out. Similarly, if m dot h C h is minimum then Epsilon is m dot h C h into T h 1 minus T c 1.

And I will write then here this one m dot C h as T h 1 minus T h 2. And ultimately this will cancel T h 1 minus T h 2 divided by T h 1 minus T c 1. So, from here I can make another definition in terms of temperature change which will be more convenient and parallel flow extension will be little comfortable with this definition. It is the ratio of the temperature difference or temperature change of the minimum fluid.

Minimum fluid means minimum capacity fluid divided by the maximum temperature change of the exchanger. I think parallel flow heat exchange will be much comfortable that the definition via heat transfer is gone that means it is the change of temperature for the minimum capacity fluid divided by the maximum temperature change available in the exchanger.

Now with this I will for a simple algebraic deduction. Let us go back to our earlier deductions where we came here l n T h 2 minus now we consider the parallel flow arrangement T h 2 minus T c 2 divided by T h 1 minus T c 1 is equal to I remember correctly one minus u by A just one step ahead m dot c C c plus 1/ m dot h C h where after this, this was replaced from there and I got expression for LMT but if I stop here in that deduction I just change the root.

Let us write this T h 2 minus T c 2 divided by T h 1 minus T c one you just follow you may not have to note because it is given in the text book you may or you may take but follow it. Because deduction if you logically follow afterward also taking notes you can follow then you will be able to deduce for a new condition is equal to just I write this thing in the exponential function nothing great minus u A.

And here I will make sum assumption before doing because it is case by case in a parallel flow I first assume the cold fluid is the minimum fluid that means m dot C c C is the minimum the catch you will know afterwards first accept it. So here (()) (43:13) this is minimum the product is less than this product then I take this as common m dot c C c when I will tell the case for which m dot h c h is minimum then we will take m dot h c h as common.

I will tell you then it will be one plus m dot c C c divided by m dot x. Now we have to play with this. Now here this equation is always used here what we will do we will express T h 2 in terms of T h one what is that write that T h 2 is equal to from here T h 2 will be T h 1 minus m dot C c divided by m dot h c h, T c 2 minus T c 1. I think I have done correctly.

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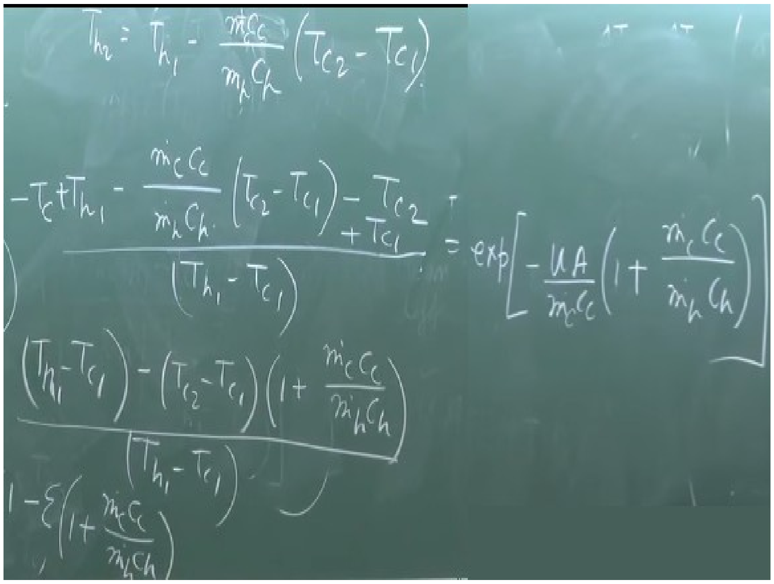

$$T_{h_2} = T_{h_1} - \frac{\dot m_c C_c}{\dot m_h C_h}(T_{c_2} - T_{c_1})$$

$$\frac{-T_c+T_{h_1} - \frac{\dot m_c C_c}{\dot m_h C_h}(T_{c_2}-T_{c_1}) - T_{c_2} + T_{c_1}}{(T_{h_1}-T_{c_1})} = \exp\left[-\frac{UA}{\dot m_c C_c}\left(1+\frac{\dot m_c C_c}{\dot m_h C_h}\right)\right]$$

$$\frac{(T_{h_1}-T_{c_1}) - (T_{c_2}-T_{c_1})\left(1+\frac{\dot m_c C_c}{\dot m_h C_h}\right)}{(T_{h_1}-T_{c_1})}$$

$$1-\varepsilon\left(1+\frac{\dot m_c C_c}{\dot m_h C_h}\right)$$

That means the trick is that T h 2 is substituted in terms of D h 1 and T c 2 T c 1, T h 2 will not work here. You will understand why. All this tricks will be understood at the end now if you substitute these at the present moment you accept that this show it has to be done only follow the algebraic continuity T h 2 then I can write here T h 1 minus m dot c C c divided by m dot h C h, T c 2 minus T c 1 minus T c 2 so T h 2 minus T c 2 minus divided by T h 1 minus T c 1 is equal to exponential. This space is not utilized.

u A/ m dot c C c one plus this one m dot c C c divided by m dot h C h. So, what I have done. The left hand side T h 2 minus T c 2 T h 2 I have substituted by this. Now what I do minus T c 1, here I add that is why I am writing plus because there is no place to write that. Some teacher intelligently, tactfully, give a space to minus T c 1 and plus T c 1. So, what does it look T h 1 minus T c 1 minus 1 plus this into T c 2 minus T c 1.

So if you divided by this let me write that T h 1 minus T c 1 minus T c 2 minus T c 1 into one plus m dot c C c / m dot h c h and divided by T h 1 minus T c 1 just follow it. Right hand side I am not writing. So this becomes equal to one minus since I have considered from this step after approaching further here approaching to this step that the cold fluid is the minimum fluid. That means T 2 minus T c 1 by T h 1 minus T c 1 is the affective place, Epsilon.

Because change in temperature of the minimum fluid divided by the maximum change in temperature, clear. So therefore, this will be one minus Epsilon into one plus m dot c C c divided by m dot h C h. This equals to this exponential. So, if we go at this and write this Epsilon then you will get this expression here I will write here if you now equate this one minus Epsilon, one plus this by this.

With this you get finally an expression that Epsilon is equal to one minus exponential minus u A/ m dot c C c into one plus m dot c C c divided by – No, sorry okay. m dot h divided by in terms of this quantity clear.

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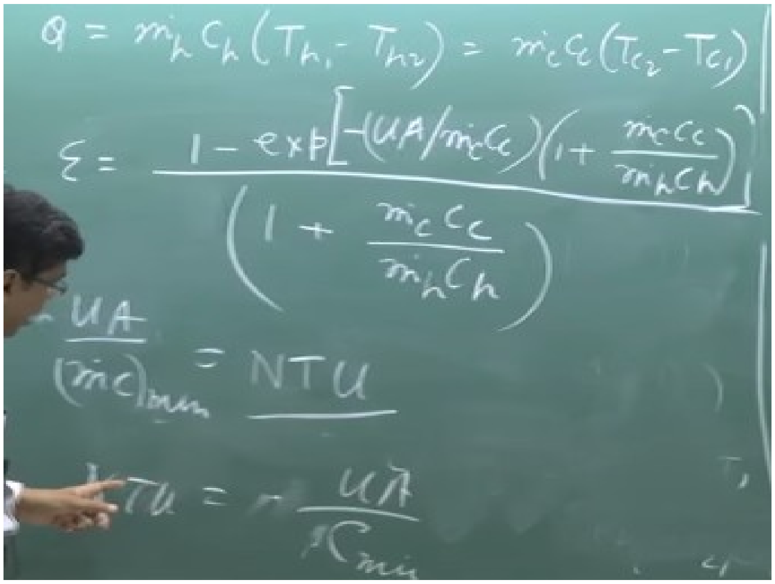

$$Q = \dot{m}_h C_h (T_{h_1} - T_{h_2}) = \dot{m}_c C_c (T_{c_2} - T_{c_1})$$

$$\varepsilon = \frac{1 - \exp\left[-(UA/\dot{m}_c C_c)\left(1 + \frac{\dot{m}_c C_c}{\dot{m}_h C_h}\right)\right]}{\left(1 + \frac{\dot{m}_c C_c}{\dot{m}_h C_h}\right)}$$

$$\frac{UA}{(\dot{m}c)_{min}} = NTU$$

$$NTU = \frac{UA}{C_{min}}$$

Now here m dot c C c is the minimum heat capacity. Now this term minus u A/ m dot c minimum is known as number of transfer unit NTU which is a dimensionless term overall heat transfer coefficient, surface area of the heat exchanger divided by m dot into c minimum. Now if you can ask me the question you should ask this question to me. Without gossiping or chatting with your friend at the back sir, if the hot fluid is the minimum fluid what could you have done?

Now if the hot fluid is the minimum fluid first I tell you this expression will be same just m dot c and m dot h will be interchange with each other so that in a generic formula I can write in terms m dot c minimum and m dot c, generic formula. So this will interchange if m dot h c h is the minimum then it will be m dot h c h NTU is minus u A/ m dot c minimum. This will be NTU and this will be one plus m dot C minimum/ m dot c.

Similarly, it will be one plus m dot c minimum/ m dot c but how to do it just interchanged. Here I was start again I started with cold fluid as a minimum fluid hot fluid as the minimum fluid I will m dot h c h. This is not all I will first take secondly, I will not T h to that is the second catch here T h 2 will not be substituted from this equation T c 2 will be substituted and in that case T c 1 will not we added subtracted T h 1 will be added and subtracted.

This is having a rhythm; Good boys can understand this thing better. Immediately by intuition here, we will take m dot h c h as common then it will be one plus m plus m dot h c h/ m dot c C c. Here T h 2 will remain as it is T c 2 will be replaced in terms of other temperatures from here T h 2 T h 1 and T c 1 and after that I will add and subtract T h 1 and in that case, I will use the definition of Epsilon as T h 1 minus T h 2 divided by T h 1 minus T c 1.

Because the definition of Epsilon is the change of temperature for the minimum fluid and the change of temperature maximum change of temperature available in heat exchange. Now if you write in a very generic fashion, a generic expression is like this if I make a very shortcut compact form by defining a parameter –now if I define it like this m dot c as the capital C

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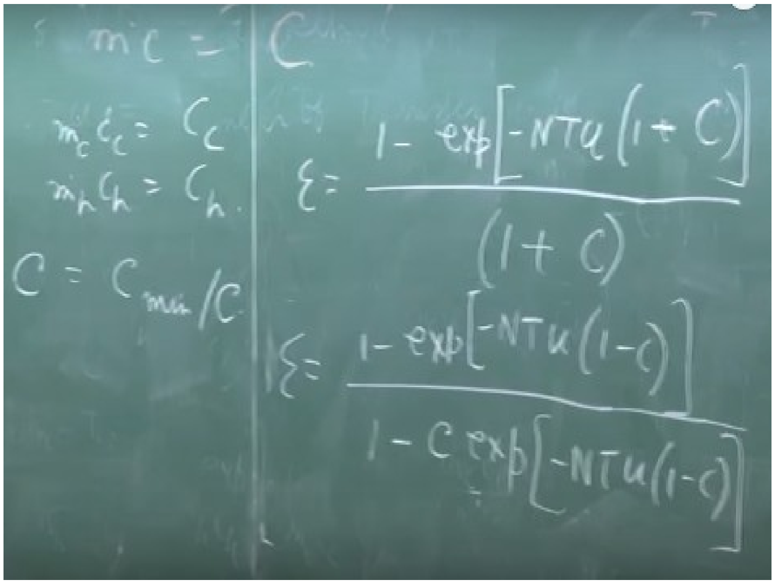

$\dot{m}c = C$

$\dot{m}_c c_c = C_c$

$\dot{m}_h c_h = C_h.$

$C = C_{min}/C$

$$\varepsilon = \frac{1 - \exp\left[-NTU(1+C)\right]}{(1+C)}$$

$$\varepsilon = \frac{1 - \exp\left[-NTU(1-C)\right]}{1 - C\exp\left[-NTU(1-C)\right]}$$

That means the capacity rate or simply the capacity and m dot c m dot c C c is the C c and m dot h c h is equal to C h and if I define C as the ratio of C min/ c which may be either this by this that means this by this or this by this depending upon the case. In that case the expression will be very simple it looks like simple expression remain same 1- exponential minus NTU where NTU is minus uA. Sorry, not minus, sorry.

NTU is a scale quantity dimensionless m dot C minimum that means C minimum with this nomenclature where NTU, is NTU into 1 plus capital C and therefore here it is 1 plus capital C this is the expression. Where capital C is the ration of the total capacity that means m dot c specifically ratio, ratio of the total capacity. What is the ratio minimum to the other one? That means C min by C so this is the expression and this expression is for parallel flow.

Counter flow expression will be again a tricky and that I leave to you to do the counter flow expression in both the cases that is left and an exercise and the expression will be Epsilon is equal to one minus exponential function of minus NTU into here it will be 1 minus C and here it will be 1 minus C into again exponential minus NTU into one minus C. Now, you see that in condensation and boiling I told one of the fluids will be parallel in case of condensation the hot fluid is a parallel line.

In case of boiling the cold fluid is parallel line and parallel line means temperature does not change and I told qualitatively that counter flow and the parallel flow remains same but now mathematically you see that for a condensation and boiling problem this one of the fluid specific heat tends to infinity. The fluid which is boiling, and the fluid which is condensing C infinity. I showed you earlier in the figure which means that C is equal to C min/ C that means

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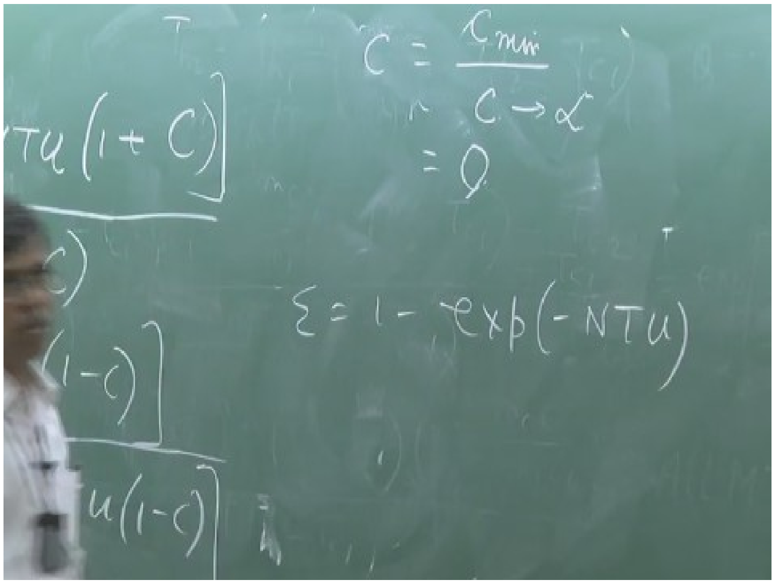

This higher C tends to infinity because which tends to infinity that will always be higher no question of calculation. That means an intelligent boy will tell that for boiling condensation the ratio which you denote as C will be always zero and if you put C zero in both the expression will yield the same value Epsilon is one minus exponential minus NTU this is the argument. If you put C zero because the ration is always zero.

Because the higher heat capacity rate whatever we got the product of m dot C is tending to infinity. Which is not a matter of hot fluid or cold fluid, which one is higher and which one is lower. But here one thing is very much open that if I do not know the temperatures explicitly then what we can do when we know the NTU because area you know overall heat transfer coefficient we know and the mass flow rate and specific heat.

We know, we can straight away find out the effectiveness and when the effectiveness is known, I can find out the temperature. So therefor this is a case where the rating will be done explicitly no iterations rating means the size is given we have to find out the heat transfer that means all four temperatures are not know explicitly. So, we can find out the Epsilon.

And whenever we find out the Epsilon. T h 1 and T c 1 all right I will work then from the definition of Epsilon I can find out the temperature difference of the minimum fluid that means heat load will come in the temperature difference of the other fluid will also come. So therefore, where rating heat load and cooling load is not done but size is given NTU Epsilon method is an explicit method to find out effectiveness, very quickly temperatures are found.

But where the temperatures are given sizing is to be made LMTD is useful because from here knowing Epsilon NTU as a function of Epsilon will be little difficult you understand this may require tedious calculation or iterations.