Conduction and Convection Heat Transfer
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology- Kharagpur
Lecture 62

Good afternoon. We were discussing the heat exchanger in the last two classes. Well, we will continue the discussion. What we were discussing about the parallel and counter flow heat exchanger simple shell and tube type. Let us start with that again.

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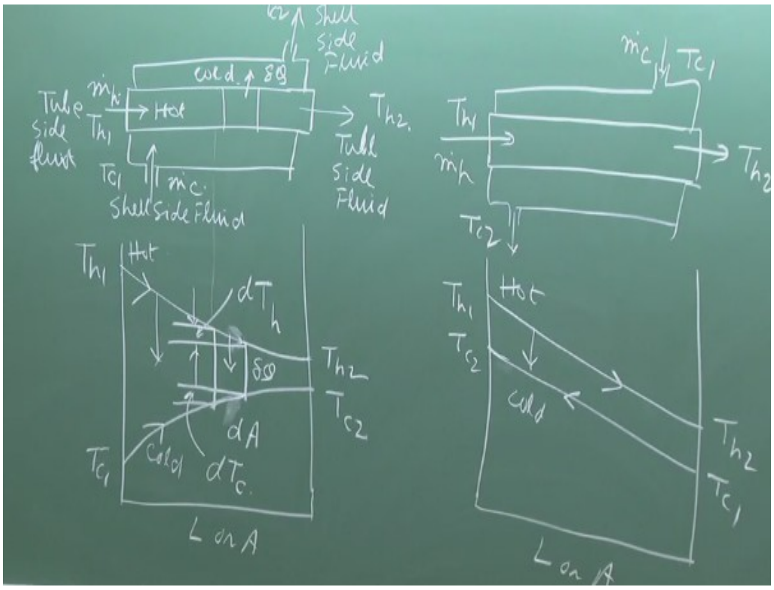

Let us have a shell and tube type, like this simple shell and tube double pipe. This is the tube side fluid in and tube side fluid out. This is the shell side fluid in and out. Let us also draw side by side, the parallel counter flow arrangement this one the parallel flow. The both shell side fluid and the tube side fluid flows in the same direction but this is the –I am not writing the tube and this thing again. This is a counter flow type shell in.

Now, the temperature diagram along the length of the tube if you see length or area whatever way you think, area is better because ultimately you will deal with the area for heat exchange L or A. Here what happens, the hot side fluid if we consider hot side tube or shell does not matter. The hot fluid not hot side, hot fluid as a decrease in temperature and the cold fluid increase the temperature in the same direction. Let us consider this is the hot and cold.

Let this temperature h is the subscript for hot, I told and one is the inlet here parallel flow T h 2 is the hot outlet. Similarly, cold inlet is Tc 1 and cold outlet is T c 2 then in this diagram we can show T h 1, T h 2, T c 1, T c 2. This is the picture. The assumption is that the fluid temperature is uniform at cross section and heat is being, this is the hot fluid sorry this arrow I should show here. It will be difficult there. I will draw something else. This is the cold fluid.

This is the hot fluid diagram temperature. This is the cold one and at any section the heat is flowing like this from hot to cold. This is the temperature. Here the situation is this. The hot fluid flows like this from T h 1 to T h 2 in a direction. Let, the similar way the hot fluid flow through the tube and the cold fluid flows in the opposite direction for which this is known as counter flow, hot and cold.

Here also the heat transfer takes place from hot to cold because of this temperature difference. Now, this type of problem, now this type of arrangement. One type of problem is like this. First of all, let us see that the energy balance or heat balance is like this if I consider m dot h as the mass flow rate of the hot fluid. And m dot c is the mass flow rate of the cold fluid in both the cases m dot h and m dot c with nomenclature the total heat.

That is transferred between the hot and cold m dot h and if c stand for the specific heat at constant pressure is T h 1 minus T h 2 that is the heat which is being given by the hot fluid equal to m dot m dot c c C c T c 2 minus T c 1.

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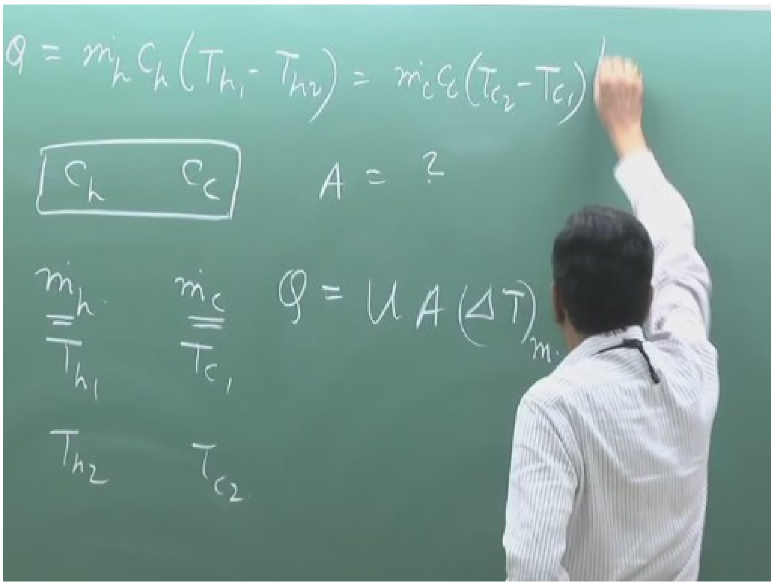

Now, this is the first line of energy balance. Now, one type of problem is this that the conditions are like this. The property, that C h and C cc are usually known thermodynamic properties, specific heat. Now, the operating parameters or conditions are back flow rate of the hot fluid. Its inlet temperature, its outlet temperature, flow rate of the cold fluid. Its inlet temperature and outlet temperature.

Now, out of this six quantities we have one equation only relating to the six quantity that means five are independent and one is dependent. Usually, the mass flow rates of both hot and cold are given in some class of problems along with three temperatures means implicitly I know the fourth temperature. That means all the six quantities are given. This type of problem means that the rating of the exchanger.

That means the operating conditions as far as the heating and cooling load. That means how much heat has to transferred from the hot fluid to be cooled or how much heat has to be gained by the cold fluid to be hot is known. Because the mass flow rate and the temperatures are all known, three temperatures are known along the mass flow rate. I can find out the fourth one. So, that this is known as in this terminology in the field of heat exchanger.

The heat exchanger is rated for its heating and cooling load. What is to be found out for this type of problem? We are interested to find out what surface area is required. If we consider a simple double pipe heat exchanger either parallel or counter. What is the surface area required? This is known as sizing problem means the heat transfer from the heat load side is rated. That means the total load is given. How much heat has to be transferred? is given.

We want to know the size. Size means the surface area required for that heat transfer. This we can find out definitely if we write this equation Q is equal to u A delta T m just I discussed in the last class that we know overall heat transfer for coefficient. The concept of overall heat transfer coefficient was told earlier which is the sum of the heat transfer coefficient of fluid sides both the side fluids plus the conductions resistance.

And if we see the resistance that some of the resistances are the equivalent resistance. If we assume that overall heat transfer coefficient is constant along the flow then we can write the rate of heat transfer in this fashion U A delta T some mean temperature difference where A is the area based on which over all heat transfer coefficient is defined. That means basically this is the definition of overall heat transfer coefficient but it is other way in practice.

The overall heat transfer coefficient is giving so that practical problem we have to find out the area based on which u is giving when we know the Q. But the only difficulty here compared to simple convection problem as we have done in the class that this temperature difference sometimes we have seen the flat plate, full stream all are constant temperature along the flow. But here this delta T responsible for heat transfer changes from one section to other section.

Here, it is inlet and outlet because both are flowing in the same direction here from section. That mean there should be some mean delta T, mean of the 2-extreme difference of delta T. This is one delta T, this is another terminal. The mean of terminal delta T in such a way that could define, this equation. So, to find out that delta T m we have to do small mathematical exercise. Let us consider the parallel flow case and an elemental area in this exchanger d A where we have a change in hot fluid temperature which is denoted by d T h.

And also, this is a decrease in the temperature and at the same time there is an increase in the cold fluid temperature through that d A area here in the exchanger through which the change is d T c and here there is an amount of heat transfer delta Q which is represented here we can represent here also delta Q. That means a small elemental area delta T A delta Q heat is transferred from the hot to cold by which the hot fluid suffers a differential decrease in temperature d T h.

And the cold fluid suffers a differential increase in temperature d T c. Then we can write that delta Q is equal to m dot h c h in to minus d T h.

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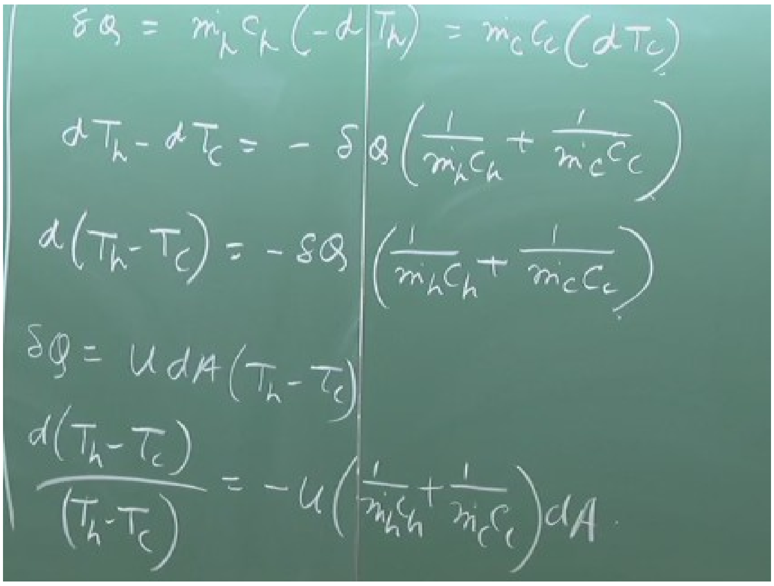

Because T h has decreased is equal to m dot c C c dTc. This decrease and increase of temperature with reference to the direction of on particular prescribed direction and here the prescribed direction in parallel fluid in the direction of flow because both the fluids are flowing in the same direction. Then we can consider that change in hot fluid temperature it is decrease and it is an increase and it is interesting to see that if you integrate this equation you get this one.

Because if you integrate this minus d T h in the direction of the flow it will be T h 2 minus T h 1 is the da so minus of that will give this. And d T c will be d c2 minus d c 1 that means simply a differential form of this equation we are writing here. Now, we make a very simple algebraic at a primary school level algebraic manipulation that d T h minus d T c equals to what? d T h minus d T c is delta Q, d T h is minus d T c is also minus that means minus delta Q 1/ m dot h c h plus 1/ m dot c C c . d T h minus d T c clear.

This can be written as d of T h minus T c this is algebraic step, minus delta Q 1/m dot h C h plus 1/ m dot c C c with the nomenclature I have told. Now from the definition of heat transfer by this equation, overall heat transfer coefficient we can write delta Q at that section is u d a into T h minus T c because T h and T c are the hot fluid and cold fluid temperature at that section. Infinite small section of infinite small area d a. So, this has to be valid.

So, therefore I can substitute delta Q here and I can write d of T h minus T c very simple, divided by T h minus T c is equal to minus u into d a I will write here m dot h c h 1/ m dot c C c into d A. You may or may not take the note this is given in any text. It will also be good if you just go on seeing you can find out inadvertent if I by chance I make it is not always necessary to take the note but you can also take the note.

Because it is given in any book you see the philosophy. So, this is all right. Now if you integrate the both the sides with respect to d A. So left side represents the temperature difference it is the way you express. Between this inlet and outlet with respect to this parallel flow heat exchanger then the left-hand side becomes

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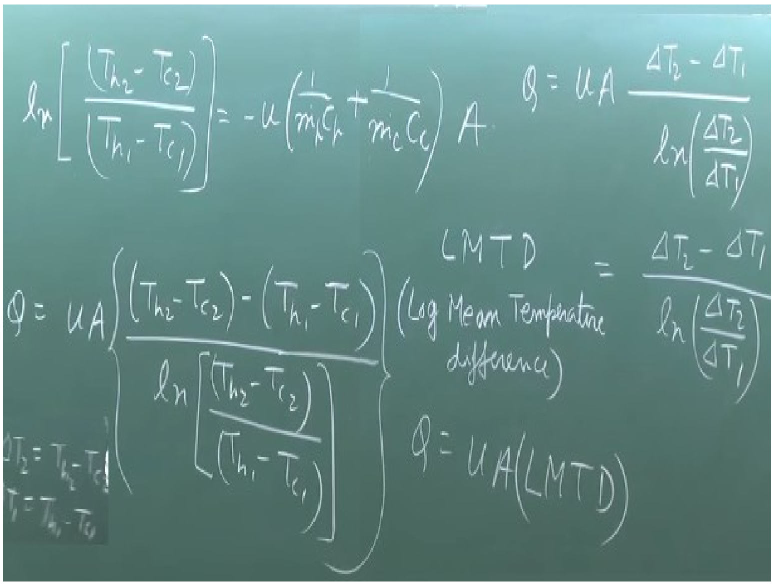

l n T h mins T c it will be T h 2 minus T c 2 divided by T h 1 minus T c 1. Any problem you can ask me. And this side will be minus u 1/m dot h C h plus 1/ m dot c C c times A. So, this is clearly thing. Now I will substitute 1/ m dot h c h and 1/ m dot C c from here there are two equations if you consider Q into picture Q is this and Q is this.

That means this quality gives two and ultimately, I can substitute 1/ m dot h C h plus 1/m what is 1/m dot h c h. It is Q T h 1 minus T h 2/Q and m dot 1/ m c C c is T c 2 minus T c 1 by Q and if you do it. It is very simple then you get this ultimately this equitation and Q will come. Q is equal to u A, T h 2 minus T c 2 minus T h 1 minus T c 1. I am not unnecessarily wasting time by doing this. The simple if you do that you will get this one.

T h 2 minus T c 2 divided by T h 1 minus T c 1 automatically you will get this. If you just substitute 1/ m dot h T c h plus 1/ m dot C c in terms of q from here. That means here the entire thing is a mean temperature difference delta T. Now if I define delta T 2 as the temperature difference at the outlet here in case of parallel flow that is equal to T h 2 minus T c 2 and delta T 1 is T h 1 minus T c 1.

Then this can be written as Q is equal to u A delta T 2 minus delta T 1 and it is will be always positive whether delta 2 and delta T 1 vary with their respective magnitudes higher and lower because plus minus will be adjusted if delta 2 is more than delta T 1 both numerator and denominator will be positive and it is other way both are negative. So therefore, we can write this equation like this and this quantity is known as LMTD that is log mean temperature difference.

Because the log is coming, log mean temperature difference. The log mean temperature difference LMTD which is defined as delta T 2 minus delta T 1 divided by that means this fashion the difference of the two delta T at two terminal divided by the l n of their ratios, this is LMTD. So, Q is equal to therefore you see that if the exchanger is rated fully by its heating and cooling load prescribing this thing so that I know Q then I can find out the surface area if I can determine LMTD.

So, if I know the definition of LMTD which satisfies this equation that means it is that average delta T between the two terminals which satisfies this equation and if I am provided with the value of overall heat transfer coefficient. I can determine the size of the exchanger. This is known as typical sizing problem. Clear, to everybody. Now there is a clue if, very important it is a counter flow heat exchanger I will derive.

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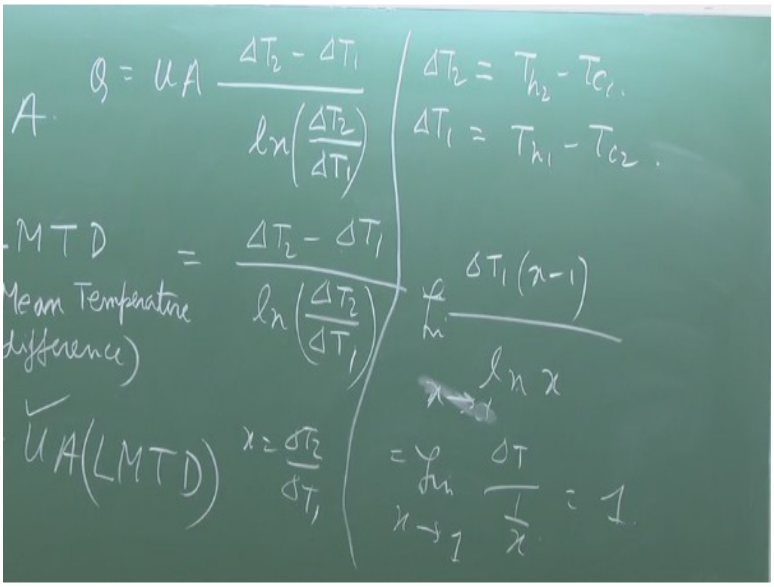

Now first of all, counter flow heat exchanger is also the same formula but what is delta T 2 for a counter flow heat exchanger the generic formula this one. Delta T 2 counter flow heat exchanger is any side you can take counter flow heat exchanger. There is no inlet, outlet reference because it is hot fluid inlet. It is cold fluid inlet. Let us make this delta to T h 2 minus T c 1 then delta T 2 will be T h 2 minus T c 1 and what will be delta T 1?

Delta T 1 will be T h 1 minus T c 2. Clear, how to derive it. I leave it to you as an exercise very simple. How to derive this one that means I have to come to this step where this will be u A into T h 2 minus T c 1 minus T h 1 minus T c 2 divided by l n T h 2 minus T c 1 divided by T h 1 minus T c 2. So that a generic formula will come out LMTD is delta T 2 minus T 1 difference of delta is divided by L n ratios of delta T. How to do it?

Very simple, I tell you the catch because I will not waste time here in this equation if you take here one prescribed direction here positive then DTH will be minus because it is decreased in this direction take the flow direction of hot fluid then DTC will be also minus because why? Because this is increasing in the negative direction. So, therefore both the things will be minus it will be minus d T h it will be d T c.

So, therefor d T h minus d t c will be changed which will be delta Q 1/ m dot c dot minus 1/m dot h c h change will be there. So, accordingly you can find out that there will be changes just only change is here. Very simple that means this equation will be minus d T h and minus d T c and think that you integrate this you will get a same result. This is clear because this is an energy balance all the scalar quantities counter flow if you integrate this minus DTH.

DTH will be this is preferred that is a positive direction DTH will be T h 2 minus T h 1 so minus DTH is T h 1 minus T h 2. Similarly, DTC will be T c 1 minus T c 2. So, minus DTC will be T c 2 minus T c 1. Therefor integration of this with minus d T h and minus d T c for counter flow will give you the same result. So, finally when you will substitute these 1/ m dot c C c minus 1/ m dot h C h because this will come then you take the same equation.

Only catches is that you have to express d T h minus d T C because d of T h minus T c, hot minus cold. So, this is an assignment everything I have told that in a counter flow also you can do the same thing. But now the most, most interesting part of this equation very, very interesting and very, very important too. I think you are intelligent you can understand. Now, the situation may come two things you must remember that a counter flow, parallel flow heat exchange.

If a heat exchanger is very infinitely long this may be equal to this. Cold fluid outlet temperature can at best become equal to the hot fluid outlet temperature I have told earlier but not the hot fluid inlet temperature. But here the cold fluid outlet temperature may be more than the hot fluid out let temperature even in a finite length because here you see this temperature there is no restriction that this will lower than that.

Because heat is transferred this way. That means at this end of the heat exchanger the temperature differential causing the heat transfer is from the inlet temperature of the hot fluid to the outlet temperature of the cold fluid. So, as because they are flowing in the opposite direction the cold fluid outlet temperature may be more than the hot fluid outlet temperature which is not the case for the parallel flow.

Another thing is there if the product of m and c, m dot n c for hot and cold fluid becomes same that means if m dot h C h is m dot c C c there will be a situation for counter flow. The two curves will be parallel to each other and ultimately delta T 2 is equal to delta T.

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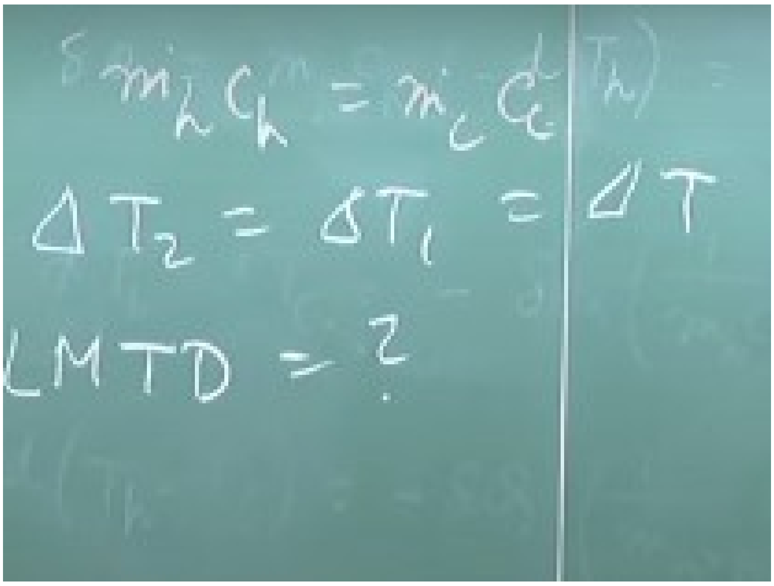

So, what is LMTD? Anybody, who told delta T? How? Zero by zero form. So, you can use the L'Hopital theorem taking the limit in many ways you can prove it that it will be delta T 1 or delta T 2 or delta T very simple to prove these things. That is also left to you we can take delta T 2/ delta T 1 as x. You can write delta T 1 into x minus 1 divided by l n x limit as x tends to 1 you will see this becomes delta T.

So therefore, we can do it in various ways x minus 1/ l n x L'Hopital theorem is a mental problem x minus one become one. And l n 1/ x and in that case here it will be delta T 1 this thing will be delta T 1 into x minus one where x is delta T 2/ delta T1 and we can write the denominator as l n x now limit of this as x tends to one, sorry, not limit of this as x tends to one. We have to make a L'Hopital theorem differentiation again it is zero/ zero form, until the zero/ zero form gone.

So, we have to go for number of steps with that. So, delta T 1/ l n x, l n x is what? 1/ x and limit x tends to one. That means there are various ways to prove it sometime you can prove that delta T 2 minus is delta T 1 plus Epsilon and put it and take Epsilon tends to zero. Denominator l n plus x you can expand in a series l n 1 plus x. x type of thing that is delta T 1/ Epsilon you can prove that limit will be delta T 1 only.

Therefore, it is very important these are school level task that when delta T 2 and delta T 1 are same. Under these circumstances mass flow rate capacity product is equal then we have to have LMTD as design one information I give you in this respect, just an information. This can be proved analytically but I am not going to prove. Just as an information for an engineer if delta T 2/ delta T 1 vary by a factor ration which one is greater, depending upon that the ratio is within two point two then we can use the arithmetic average as the LMTD with 95 percent accuracy.

Usually arithmetic average is higher than the LMTD we should not use it otherwise it will be over predicted. So, therefore the LMTD and arithmetic average will be within five percent error provided delta T 2 and delta T 1 varies, varies means the ratio is within 2.2 and when exactly this is one ratio is one the coincide with each other that means arithmetic average is equal to the LMTD because arithmetic average is delta T 1 or delta T 2 clear.