Conduction and Convection Heat Transfer
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Lecture - 58
Condensation - II

So up to this the concept of fluid mechanics is over. Now we will make certain mathematical arrangements, so, therefore we can write mu del square u del y square is equal to rho v minus rho l into g.

Now I integrate del u/del y, taking mu on the other side, rho v minus rho l by mu g y plus a constant C1 and finally u is equal to rho v minus rho l, this is so simple, g y square by two mu plus C1 y plus C2. Now what are the boundary conditions tell me, this is a function of y. So, what is the boundary condition, u is a function x, I am sorry, u will be a function of y, this is a function of y, I am sorry this is function of y, u is a function of y.

So, u is a function of y and this is the expression, what are the boundary conditions, please tell me, at y is equal to zero u is equal to zero. Any other boundary condition, two conditions are required, at y is equal to delta, very good, siestas at the free surface is zero. Now the first boundary condition leads to that C2 is zero, C2 has to be zero. And second boundary condition leads that C1 equals to rho l minus rho v by mu g delta. C2 is zero.

Because y zero u zero, and del u/del y is zero means C1 is equal to rho l minus rho v by mu g delta.

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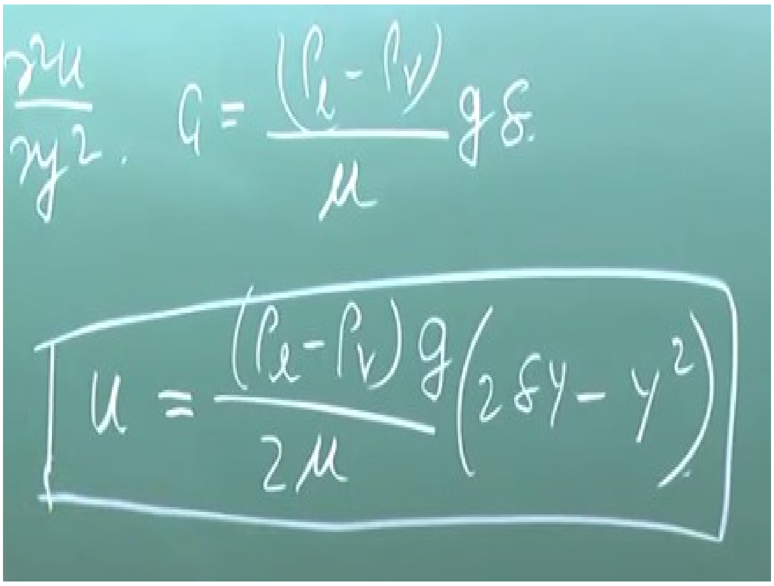

So, if you substitute this c1 here, then you get u is equal to rho l minus rho v, you take this rho l minus rho v by two mu g, then it will be two delta y minus y square. It is always positive, because rho l is greater than rho v and since y is less than delta, the quantity in the bracket is also positively proved that u is in the direction of the positive x axis, obviously, it is flow is dominated by the buoyancy, okay.

The net buoyancy force, here this is the picture where we can explain it physically that viscous force is balanced by the net buoyancy force, which we derive from the Navier-Stokes equation. Sometime we can take a control volume and can make a balance of the net buoyancy force than the viscous force, by simply explaining the inertia is zero for low velocity.

So, we neglect that and viscous force is approximated by mu del square u del y square, because of the geometry, the same thing which I derived from the basic Navier-Stokes equation, same thing, okay. So finally, this is very important thing that we have come across, u. Now what we do, okay, now what we do, we make this is the Nusselt analysis, a classical thing, I am following, make a mass balance in a fluid element of control volume, at a distance x from the leading edge, at a distance x from the leading edge, where this is a control volume.

At x, the mass flow rate is m dot x, at x plus del x let this is del x the height of this control volume or element is del x. So, at this section at x plus del x, let me write the mass flow as m dot x plus del x. And this two will not be same, because what I told for which the delta is growing, because of the m dot c, that s the mass rate of condensate. The rate of mass vapour condensed within this control volume, that due a length of delta x.

So that m x dot x plus delta x and m dot x, difference is the m dot c.

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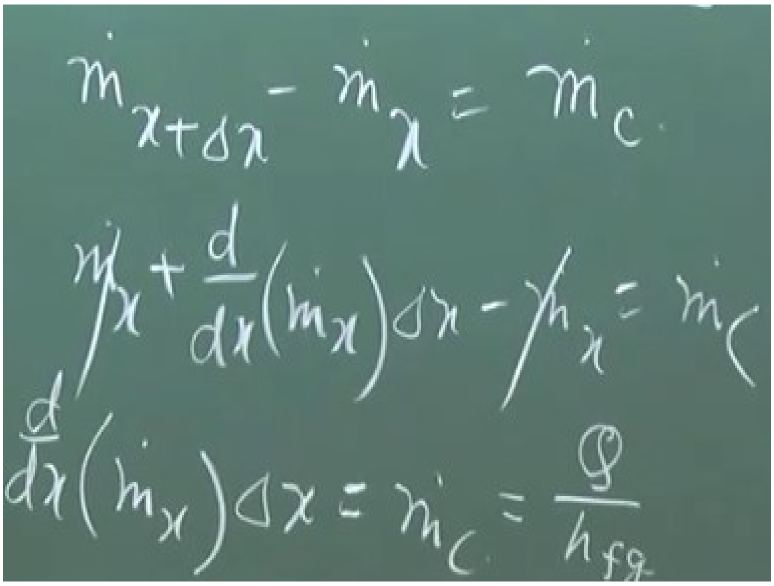

So, a simple mass balance of this control volume tell that m dot x plus delta x minus m dot x is equal to m dot c, simple. Primary school level thing that means the gross mass balance of this control volume is that m dot x plus del x is equal to m dot x plus m dot c and I am retaining that value and if I expand this in a Taylor series by neglecting the higher order term because of delta x is very small.

This we can write m dot x plus d/dx of m dot x delta x minus m dot x is equal to m dot c, so this cancels up. So therefore, I get d/dx of m dot x del x is m dot c, okay. This m dot c also can be written as the heat transfer through the control volume divided by if h, f, g is the enthalpy of vaporization.

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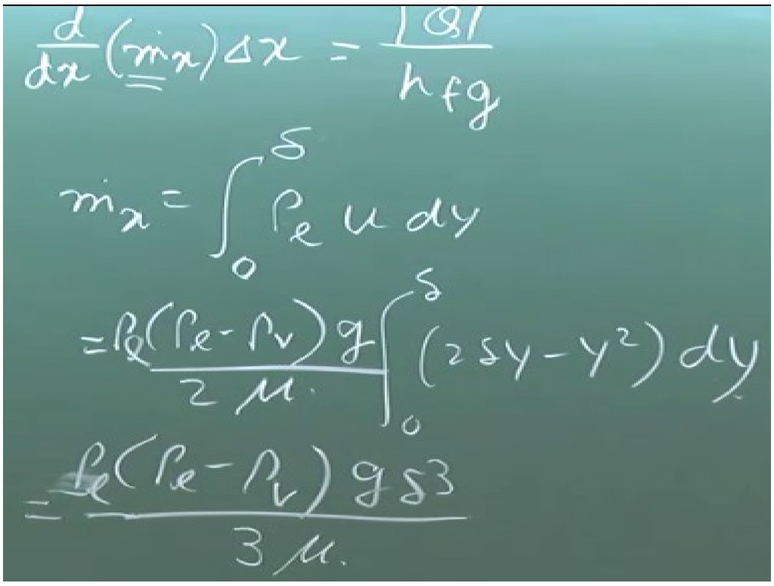

That means I can write d/dx of m dot x del x is equal to what I can write Q divided by hfg and here I deliberately put a mod value, absolute value of Q, because Q by hfg represent the rate of mass condense, which is a scalar quantity, so q may come out from some analysis with a positive and negative site, which will be so that q in the positive y direction or negative y direction of the problem.

That means of the coordinate axis taken, but I will take the absolute value to write it, because this is a scalar equation. Now, our next task is to find out the mass flow rate and heat transfer for this control volume. So now mass flow rate at any section x, m.x can be written by a simple equation as you know rho l u dy from 0 to delta at any section x. At any section x in general, at any arbitrary section the rate of mass flow is written like that we have considered a unit width of the plate in this direction, unit width of the plate.

So that, 1 into dy is the cross and u and rho l. As you know this thing by this time from your fluid mechanics knowledge rho l u dy 0 to delta is the mass flux across any section. Now, if you put this value of u, then that can be written as rho l minus rho v into g, by two mu into 0 to delta 2 delta dy minus y square dy. Now, two delta y, y square, it is y square by 2, that means 2, 2 cancels, that means this is delta Q.

You do it mentally and this is delta q by 3. 1 minus one-third is two-third, 2, 2 cancels that means this becomes rho l minus rho v g delta Q, 2 by 3 mu. This becomes d m dot x that means the mass flow rate at any section, clear. Okay rho l minus rho v g what happens? Rho l is missing, yes very good rho l, good. Rho l, rho l minus rho v g delta Q by 3 mu, okay. Now we have to find out what is Q.

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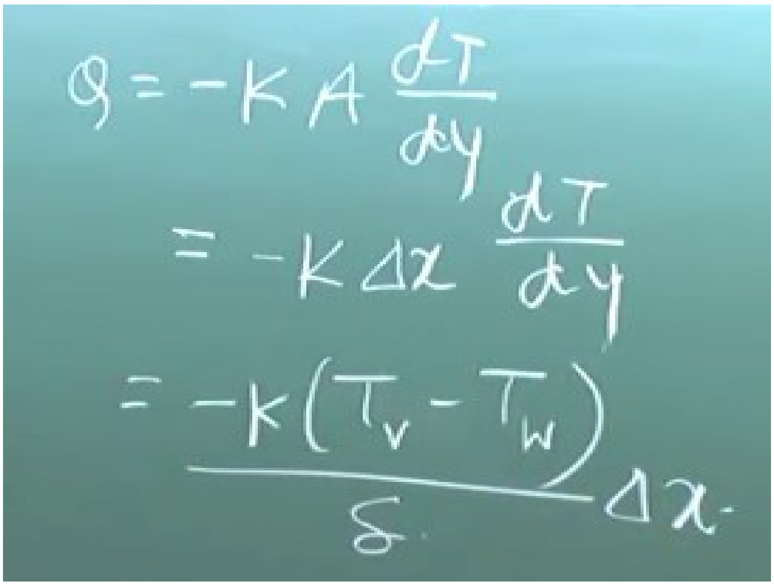

We have to find out what is Q. Q is equal to minus KA dT/dx, we know, here it is dT/dy. What is A for this control volume minus A into delta x, we have taken a unit v. What is del T/ del y at this interface? Because here is a temperature Tv vapour site and here Tw it is less than Tv that means there will be some temperature gradient from Tv to Ty, which is decreasing in this direction of the plane. Well, now there is an assumption.

Since the motion is extremely small, for which we have neglected the inertia force, we can consider that the effect of, as I have told, many times that convection is basically conduction affected by advection or flow, that is why it is convection. So, flow does not have any effect on the conduction. Heat transfer takes place, conduction as if this medium was station. This small velocity does not affect it.

That means I will assume these as a pure conduction problem in a stationary medium and that to in one direction. Again, because of the fact that delta is very, very less than l. So therefore, if we consider that this is a conduction in a stationary medium where the small flow has got no effect and one-dimensional conduction, then we can consider with a constant thermal conductivity the temperature gradient is linear.

That means this temperature gradient is therefore minus K Tv minus Tw, this is the temperature gradient divided by delta and Tv is greater than Tw, so therefore Q is negative, means it is in the opposite direction to the positive y axis. That means it is towards the plane, clear, okay, towards the plane. Now, delta x, clear, okay K delta x into dT/dy, so minus K. Now you put this Q and mx, then what you get, here. Then, we get that m.x is this one, finally this one.

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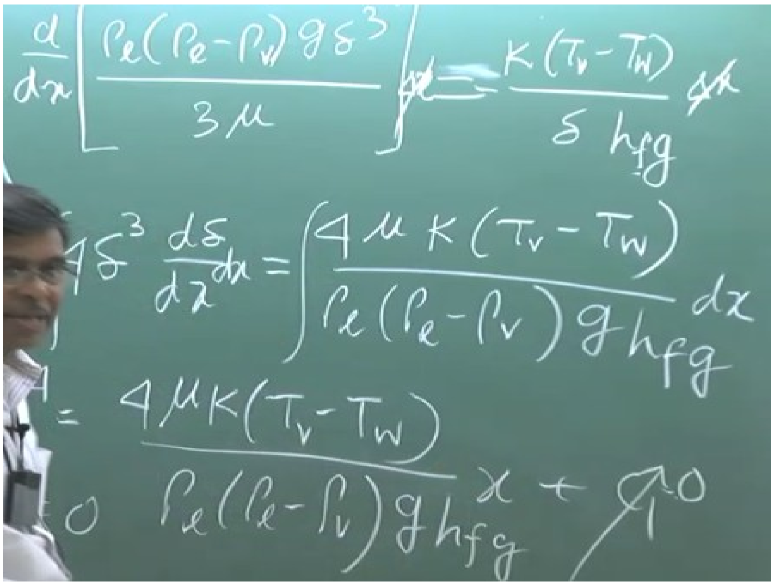

So, we get d/dx of rho l minus rho v into g delta Q divided by 3 mu is equal to, now we take the absolute value that is equal to K Tv minus Tw divided by delta. Now there is a delta, I am sorry, d/dx of m.x delta x is K tv minus Tw delta x divided by hfg, because Q by hfg. That means I am substituting this mass flow rate m.x from here and I am substituting the absolute value of Q here and I get this relationship, clear?

Now delta x is cancelled. Now, everything is constant only the delta, which is varying. Now this delta Q d/dx is three delta square d delta/dx, so 3 and 3 will cancel. So therefore, we can write delta square and this delta goes there so that I can write in delta Q d delta/dx and this 3 and 3 will cancel is equal to mu K Tv minus Tw, I am just telling you, rho l into rho l minus rho v into g h f g, am I correct, d delta/dx.

So therefore, delta square, 3 delta square d delta/dx then this delta comes, so therefore delta Q d delta/dx, 3 and 3 cancels, okay. Just you do it. I have made one step jump, which can be made mentally so that mu K Tv minus Tw divided by this d delta g h f g, clear. You do it mentally, I think you can do it, okay. I give little time, clear, okay. Now if you multiply with 4 and integrate with dx, this is school level step.

Then 4 delta Q d delta/dx is d/dx of delta 4. That means delta 4 equals to 4 mu K Tv minus Tw divided by rho l into rho l minus rho v into g h f g, x plus some constant C1. Just integrate with respect to x. Because I want to find out an expression for delta as a function of x that is growth of delta. Now, the boundary condition is that at x is equal to 0 leading to delta is 0. So that this becomes equal to zero, clear. Okay, then I can write.

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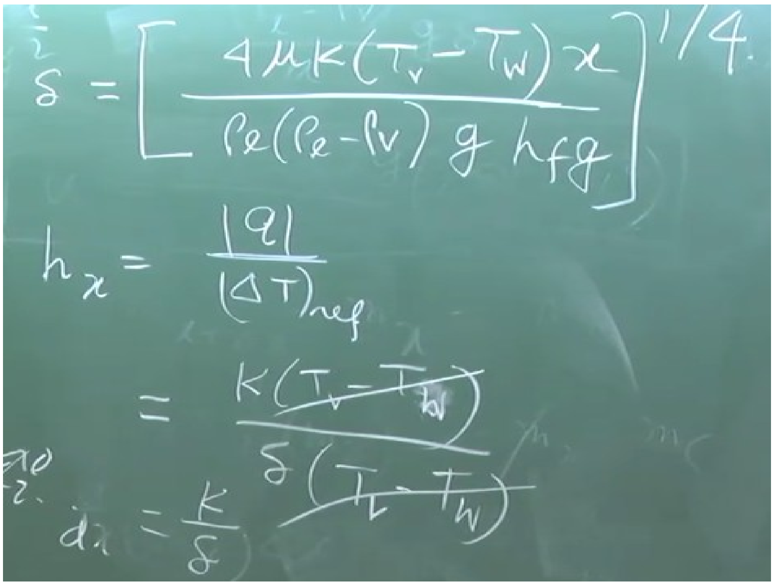

Therefore, I can write that delta is equal to 4 mu K Tv minus Tw x divided by rho l, rho l minus rho v g hfg, g is the acceleration due to gravity, hfg is the enthalpy of condensation whole to the power 1/4. This was Nusselt derivation for the growth of film, which is proportional to the fourth power of the x. If you compare it with the growth of boundary air over a flat plate in laminar flow.

The delta is proportional to the half power square root of it and it is proportional to the one-fourth of this. It grows very slowly. So, this is the basic definition with all those assumptions I have told. Now, engineers are interested more in finding out the heat transfer coefficient. As you know, the heat transfer coefficient h at any location. This already you know. I told you several times in my lecture or to the introduction to convection that it is heat flux at a location divided by the reference temperature difference.

Everything is a point function and therefore heat transfer coefficient is a point function, locally. That means local heat transfer coefficient. Yet, q is here also, I take the absolute value, because the heat transfer coefficient is a scalar quantity. So, I do not want this sign, which direction it is, negative direction of the axis or positive direction of the axis. So therefore, this is simply Q is K Tv minus Tg by delta.

And here incidentally the reference temperature difference also free steam temperature, sorry Tw, wall temperature, which is same, so therefore in a conduction dominated convection. Though it is a convection, but basically conduction, no effect of velocity. If you define a heat transfer coefficient, then it will be always K by delta. If the wall is at a constant temperature, this is a thumb rule, because the prescribed temperature difference is this.

So therefore, if you write this, then what you will get hx equals to what.

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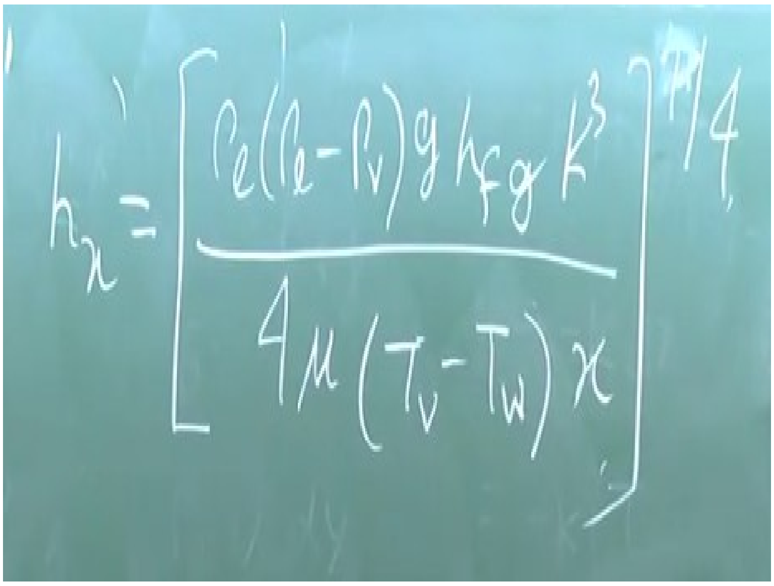

Then you get hx equals to K by delta. That means here I write hx is equal to this thing will go top, K by delta. That means K4 that means this will come, you just see rho l, rho l minus rho v into g hfg into KQ divided by this one will come 4 mu Tv minus Tw into x whole to the power 1/4. What happens K by delta, that means hx will be K by delta. This will be K inverse this and K4, I make and take it under one-fourth root, so it will be KQ, clear, Okay.

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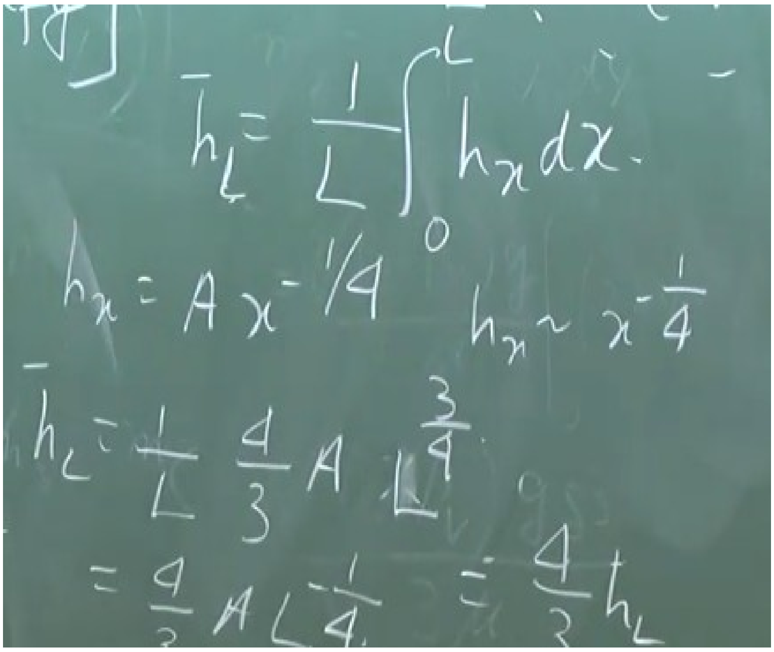

Now, if you are told to find out an average heat transfer coefficient over a length l of the plate, which is defined as this, 0 to l. This is the definition of the average heat transfer coefficient over a flat surface, what is that value in terms of hl? Can you tell me? I told several times in the convection class. You have forgot, so I tell you again. Here, you see these things are all constant.

Obviously, the character of local heat transfers with the character of x, that means if this constant I define as some A, then I can write hx is A into x to the power minus 1/4 that means hx is proportional to x to the power minus 1/4. If you put it what we will get, hL bar is 1 by L, Ax to the power minus 1/4 dx means, 4 by 3 A x to the power minus 1/4 plus 1 3/4 and that will be L, not x. If you put this simple integration will tell this thing, clear.

That means four-third of A L to the power minus 1/4 that means it is four-third of local Nusselt number at L.

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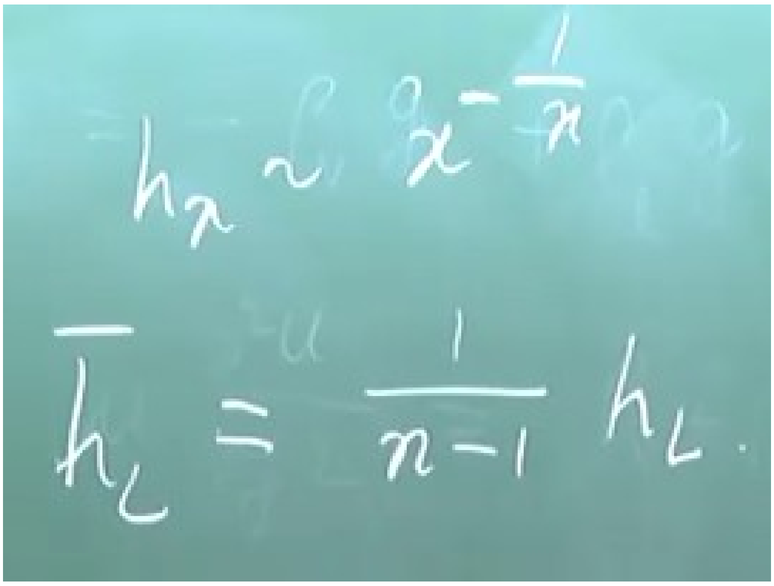

I told you if you if you remember in the class of convective heat transfer that if hx is proportional to x to the power minus upon n, then average heat transfer coefficient for a flat surface is always 1 by 1 minus n, minus 1 plus n, that is n minus 1 hL. Obviously instead of minus 1/4, you put as minus 1 by n, you get a relationship 1. Here, n is one fourth, one-fourth minus 1 is three-fourth, 4/3 where hx is proportional to x to the power minus half in case of flat plate heat transfer.

In case of flat plate, the heat transfer coefficient is proportional to x to the power the local heat transfer coefficient minus half and in that case, we have seen that average heat transfer coefficient over a length L of the flat plate is twice the local heat transfer. This is the thumb rule. So hx is proportional to minus one-fourth means it is four by three, the local heat transfer coefficient, 1/n minus 1, okay.

So, this thing sometimes works without, so therefore one can tell that hL average. I am not writing the expression. This four-third, this 4/3 into this with this x as L. So, one can write if it comes in the examination for example, this expression and immediately if you know this thumb rule, so immediately four by three times everything with x L, because this is four-third times the local heat transfer at x is equal to L, clear, Okay.

So, this is the entire derivation for condensation over a particle flat surface. Now, this thing can be little modified if you are told. This is the Nusselt classical deductions, which are there in all books. Now if this is little modified in case of that if the plate is inclined.

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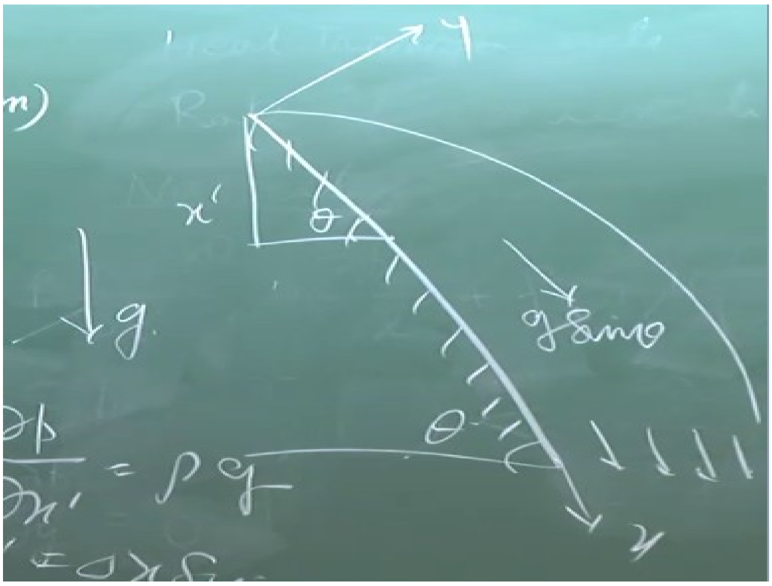

If the plate is inclined with an angle theta with the horizon, it is simply school level thing. So therefore, this is y and this is x, but g is in this direction. So entire thing will be changed by an effective g, g sine theta.

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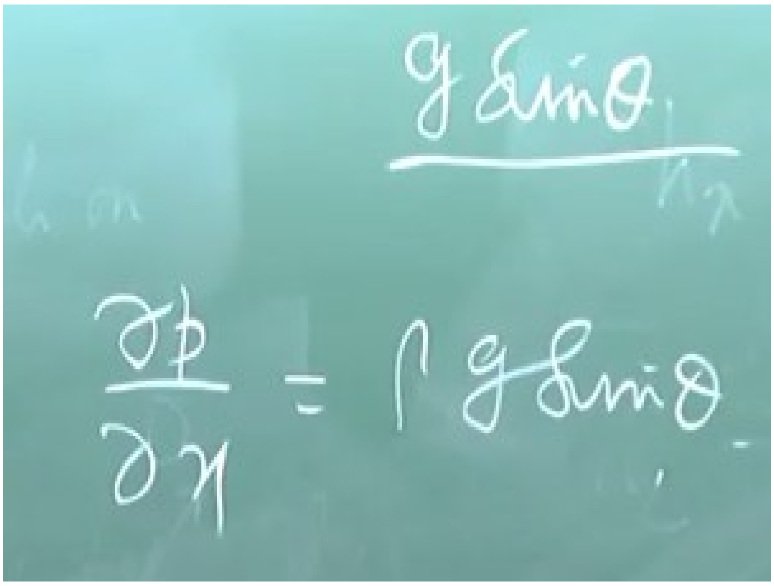

That means here the entire deduction, it will come as g sine theta. How? This is because obviously the effective g, which is working here is g sine. That means the weight, the component, the effective weight will be rho g sine theta along the direction of the flow and at the same time del p/del x. If this is x, will be rho g sine theta. So as a whole, a sine theta multiplication factor will come.

This is obvious because if this is x dash, then by hydrostatic del p/del x dash is rho g, but what x dash. If this is theta, x dash is delta x dash is delta x sine theta. This I am just telling you for your brushing up things, that is school level things. That means if there is an inclination of the plate and effective gravity g, sine theta will work, both in the hydrostatic equation pressure.

And also in the body force that means liquid, because hydrostatic pressure gradient is nothing but the weight of the vapour, so there also effective gravity comes, theta, so del p/del x dash is rho g, where x dash is this direction, vertical direction and this will be converted to rho g sine theta and the weight will be rho g sine theta. That means the effective gravity is g sine theta.

Another modification may be there. These are the things with which you should derive the Nusselt equation with these modifications, if you understand the Nusselt equation. For example, I tell the surface is made in such a way there is no slip at the surface. The fluid or the liquid slips, and I give a slip velocity, then you have to modify the distribution of velocity by taking care of the boundary conditions.

So anyway, any cosmetic change, you can do provided the basic structure of the deduction remains same. You first consider the velocity profile by neglecting the inertia term, solve the viscous terms with the buoyancy, that means pressure gradient and the body force, that is weight per unit volume and then you find out the explicit form of the velocity with the boundary conditions.

Be careful of the boundary condition, be careful of the physical situation where how the gravity is taking care of, if it is inclined plane, is it an effective gravity. So, with this, you can modify the Nusselt equation, the classical equation or classical derivation for a flat particle plate. Thank you.