Conduction and Convection Heat Transfer

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Lecture 56
Natural Convection - IV

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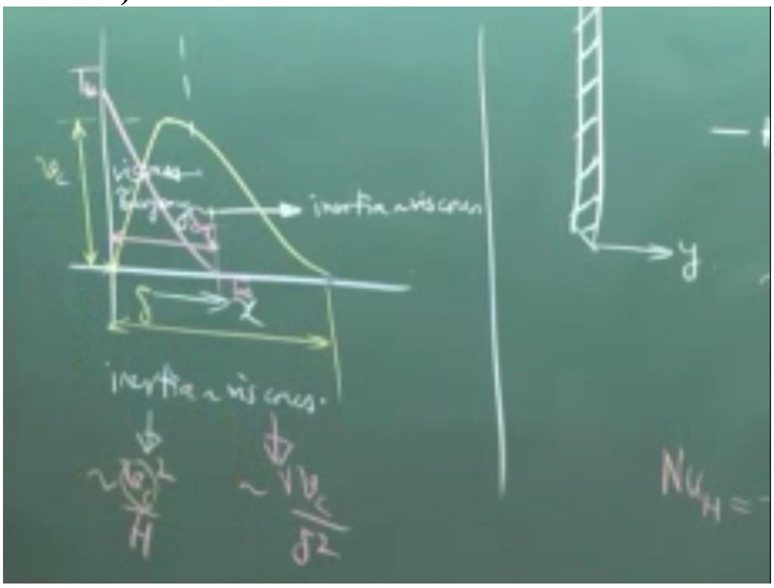

Now we have discussed about thermal boundary layer. But what happens for the hydrodynamic boundary layer. So, let us try to make a sketch of the phenomenon, the velocity and the temperature profile. So, this is x, so how will the velocity profile look? So, the velocity at the wall will be zero and then because of buoyancy effect the velocity will go up an again in the fast stream the velocity will come down to zero, seem critical.

So, this is our characteristic vc and these is your so-called delta, not delta T. So, from T wall, it will come to T infinity and this is what is your delta T. Okay? Now the question is, how do you estimate what is the thickness of, what is the order of magnitude of delta. So, for that you have to make your analysis within the hydrodynamic boundary layer but outside the thermal boundary layer.

So now if you make an analysis within this thermal boundary layer, then within this thermal boundary layer what forces are important? What forces are competing? So, on this side, viscous and buoyancy force is competing, right? What happens in this side? That is the question. Which forces are competing? So, there are 3 forces which could in general interplay with each other. Inertia viscous and buoyancy.

Outside this buoyancy has no role because the temperature has become already T infinity. So what forces have role to play? Viscous and inertia. So, in this… And this is the physical scenario. Because of inertia only, outside the thermal boundary layer still you get some velocity because the fluid has inertia, it does not suddenly come to a zero velocity when the temperature gradient is mid zero. So, it still tries to maintain its motion.

The viscous force tries to oppose its motion. So there comes a situation where the viscous force is completely successful in opposing the inertia and then the velocity comes to zero. Then you come to the age of the hydrodynamic boundary layer. So, inertia is of the order of viscous if you write. What is the order of magnitude of the inertia at all? vc square by H. What is the order of magnitude of the viscous term? Yes. Nu.

Not delta T square, delta square. Because this analysis is valid over a length scale over a region for which the length scale is delta and non-delta T. So, these are fundamental conceptual things. Concept is not algebra where some fourth power will come in the numerator or denominator all those things. It is important that, I mean, this is where if are not careful you will make mistake.

So, when you are doing the analysis where the length scale is, appropriate length scale is delta, it is that delta you should substitute. Now you tell me in place of vc should we substitute alpha H by delta T square or alpha H by delta square. Alpha H by delta T square because it is the temperature gradient within the thermal boundary layer that is responsible for this vc.

So, it is not the expression for delta, it is the expression for vc that we are substituting there.

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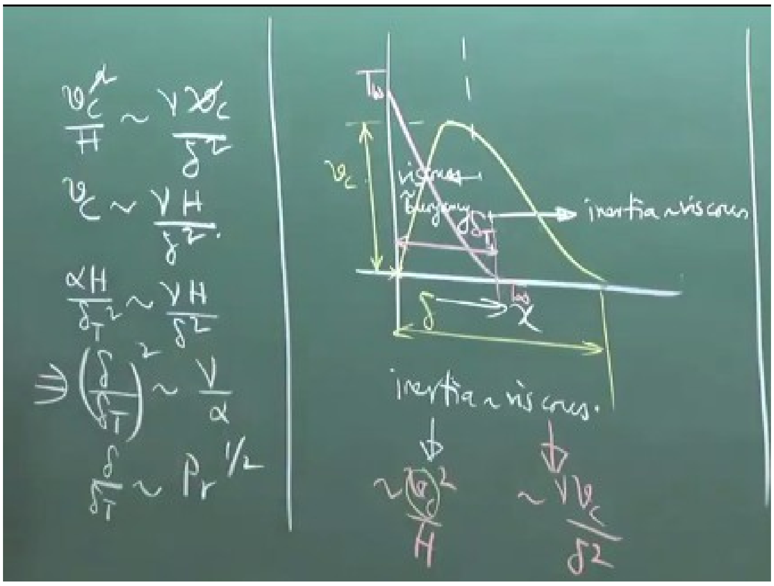

So, these two forces, now if they come of the same order of magnitude, vc square by H is of the order of new vc by delta square, so vc is of the order of new H by delta square and vc is alpha H by delta T square. So, delta by delta T square is of the order of nu by alpha. That means delta by delta T is of the order of Prandtl number to the power half, right? So, this picture diagrammatically is justified.

We have considered Prandtl number greater than one, so delta is greater than delta T and the meaning of the Prandtl number comes out to be in this example, for the case of Prandtl number much greater than one, very similar to forced convection, right? But we will see that it is dramatically different if we consider the other case Prandtl number much less than one. So, we will consider that case now.

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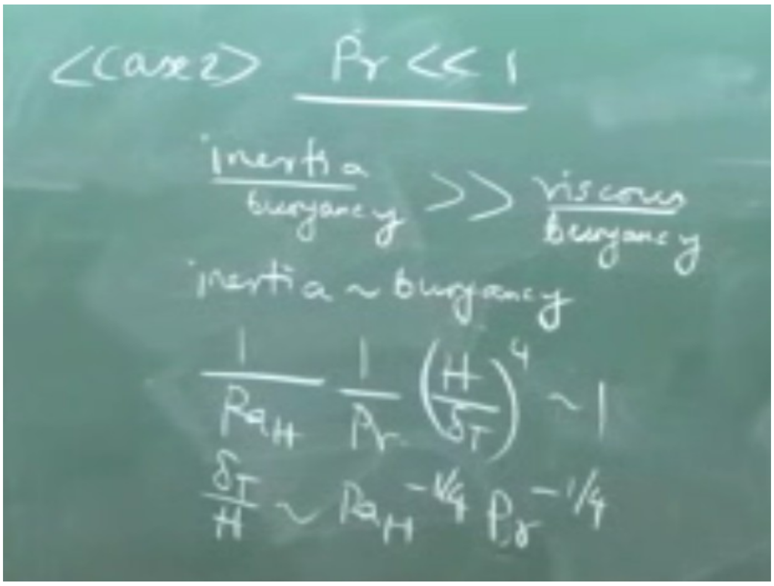

So, for Prandtl number much less than one, you have inertia by buoyancy is much greater than viscous by buoyancy, okay? So, inertia by buoyancy is much greater than viscous by buoyancy because the Prandtl number being less than one, one by Prandtl number will be much greater than one.

So that means inertia is of the order of buoyancy. So, inertia is of the order of buoyancy means you have one by Rayleigh number into one by Prandtl number into H by delta T to the power four is of the order of one. That we get from this expression. So, delta T by H is of the order of Rayleigh number to the power minus one fourth multiplied by Prandtl number to the power minus one fourth.

See the Prandtl number dependence comes into the picture, for Prandtl number much less than one but not for Prandtl number much greater than one and how do you calculate the Nusselt number?

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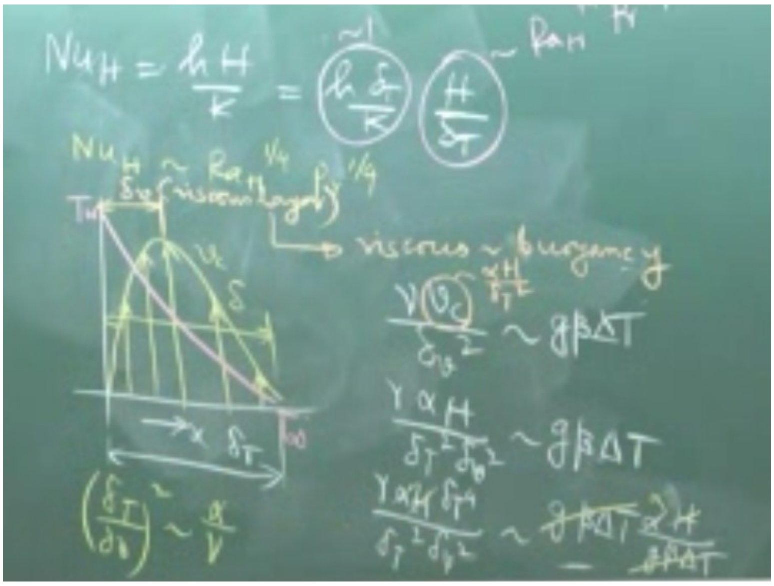

For Nusselt number is equal to hH by k which is equal to h delta T by k multiplied by H by delta T. This is of the order of one. We have shown that this of the order of one does not depend on whether its Prandtl number much greater than one or less than one and this is of the order of Rayleigh number to the power of one fourth to the Prandtl number to the power one fourth.

So Nusselt number is of the order of Rayleigh number to the power of one fourth multiplied by Prandtl number to the power one fourth. Now let us try to again qualitatively draw the velocity and the temperature profiles. So, velocity profile is just like the other case we will draw it. What I forgot to do in the last case is to put vectors for the velocity, so I am just putting the vectors for the velocity. So, this is the y component of velocity.

So, similar thing, we will do for this case. Now, I have drawn a sketch with an intention to confuse you a little bit. So, when you look into the sketch? there is a first thing that you apprehend to be incorrect, what is that? From your past experience of interpreting physically what is Prandtl number? So, your past experience of interpreting what is Prandtl number is the ratio of delta by, it is an indicator not exactly the ratio of delta by delta T.

But it is an indicator of the ratio of delta y delta T. So, if Prandtl number much greater than one, delta must be much greater than delta T. So, this sketch is fine. But if Prandtl number is much less than one, then delta should have been much less than delta T, but that I have not drawn, but this inference is based on the mental block that Prandtl number is always indicated of delta y delta T.

And we will now show that it is not necessarily true for all cases. So we will wait for a moment and our further analysis will resolve this paradox. So now can you tell that, if you now analyze the thermal boundary layer, can you recall what forces are important in the thermal boundary layer? Inertia is of the order of buoyancy, but there is always a layer within which viscous effect must be important.

Otherwise it will be like inviscid flow. It is not an inviscid flow. It is a viscous flow. So, although within the thermal boundary layer, the inertial and the buoyancy forces are competing with each other. Within that layer there should be at least a thin sub layer. It may be very thin, but it is not vanishingly zero. It will be some of some thickness where the viscous force will be important and if the viscous force is important, which layer is it?

It must be the wall adjacent layer. So, you can imagine a layer like this which we call as delta v which we all as viscous layer. This is not the delta. It is just a name that we are giving. This is defined as a layer where you have viscous force of importance, viscous forces of importance with which it will compete, inertia or buoyancy? Buoyancy because near the wall you get the maximum temperature reference.

So, near the wall it is the buoyancy that is driving the flow. So viscous will try to oppose the flow and buoyancy will try to drive the flow. So viscous is of the order of buoyancy. So, what is the viscous force in terms of order of magnitude, nu vc by what? What is the order of magnitude of this? Nu vc by, you see what?

Everything is written in the board here. It is your interpretation. Nu vc by delta v square, right? Because your layer under consideration is the delta v layer, is of the order of buoyancy. So, g beta delta T. Again, vc is of the order of, what is the vc, alpha H by delta T square. So, you can write nu alpha H by delta T square multiplied by delta v square is of the order of g beta delta T. To get a ratio of delta T by delta v you have to basically multiply it with delta T to the power four.

Then you will get delta T square by delta v square. So where from you will get delta T to the power four you will get from this. Inertia and buoyancy are of the same order that means these and these are of the same order. So, from here you will get a scale of delta T to the power four. So, what is the scale of delta T to the power four? Is of the order of alpha square H by g beta delta T, right? Because this is of the order of one.

So, delta T to the power four is of the order of alpha square H by g beta delta T. So, you multiply both sides by delta T to the power four. So, nu alpha H by delta T to the power four by delta T square delta v square is of the order of g beta delta T multiplied by alpha square H by g beta delta T, right? So, g beta delta T gets cancelled. H gets cancelled. So, what we infer from here?

Delta T square by delta v square, or delta T by delta v whole square is of the order of one, alpha also get cancelled from both sides, another alpha remains. So, one alpha remains. So, this is of the order of alpha by nu. Instead of alpha square it becomes alpha which cancel off one alpha from this side. Okay? So, delta v by delta T is of the order of Prandtl number to the power half. Okay? So Prandtl number is not a measure of delta y delta T but delta v by delta T. Okay?

So, some examples, experience with some examples prompt us to think that Prandtl number is always an indicator of delta y delta T, but that need not always be correct. This is an example where we see that Prandtl number is not an indicator of delta y delta T but it is an indicator or some other delta v by delta T. So, the figure that we have drawn with some arbitrary delta and delta T does not conflict with this, right?

So, to summarize we have done some order of magnitude analysis for natural convection across a vertical flat plate with two limiting conditions, Prandtl number much less than one and Prandtl number much greater than one. Other approaches of analyzing natural convection are the two approaches that we have discussed in the forced convection that is the similarity solution technique and the integral approach.

These two are not there in your syllabus for this particular course because, not that it is conceptually very much different from what we have discussed but mathematically it is little bit more involved because now the momentum and the energy equations are coupled. But the basic philosophy is the same for example in the integral method, you have to integrate the respective governing equations.

So, I will give you a home work that for delta equal to delta T that is a special case, very special case, you find out the Rayleigh number, Nusselt number as a function of Rayleigh number and Prandtl number using integral method. So again, I am repeating the homework that for delta equal to delta T, find the relationship between Nusselt number, Rayleigh number and Prandtl number using the integral method. Okay?

So basically, when delta is equal to delta T, I mean you can integrate either from o to delta or 0 to delta T all the time. Now the question is what velocity profile will you assume? Now can you tell that what is the difference between this velocity profile and the forced convection velocity profile? So, at the wall the velocity satisfies no slip, but at the free stream the velocity is zero. There it was u infinity, here, it is zero.

Not only that, how do you calculate the second derivative of velocity at the wall. So, if you recall, let us say that you are interested to have a velocity profile v equal to A0 plus A1 y by delta plus A2 y by delta square plus A3 y by delta cube, like that. So, to evaluate these constants you require some boundary conditions. One is velocity here equal to zero, no slip. Another is velocity here equal to zero. Third is the velocity gradient here is zero.

The fourth one, what is the fourth one? Here there are four constants. So again, the principle is the same. How did we explain the fourth boundary condition for the first convection? We applied the momentum equation at the wall. So, at the wall, both u and v were zero, so from that we got an expression for the second derivative. So here also, if you apply the momentum equation at the wall, you will get the fourth condition.

So, using this four conditions, these four constants can be evaluated. These constants you can substitute in the integral equation and then you can find out the temperature gradient at the wall and hence the Nusselt number and you will see that again the Nusselt number will scale in the same way as with Rayleigh number and Prandtl number in the way in which, in a manner in which we have seen through the order of magnitude analysis.

But you will get some fitting coefficient with that. So, order of magnitude analysis does not give you the coefficient, the constant along with the variation with Rayleigh number. That constant we will get by the integral approach and even if you do similarity solution you will get the same thing. So, let us summarize, we are almost towards the end of our discussion on convective heat transfer. So, let us summarize what we have done so far.

So, we have discussed, we started our discussion with convection with some physical explanation or physical interpretation of what is convection and how does it relate with fluid mechanics and then some basic derivatives of fluid flow equations. So, in particular we have derived the Navier-Stokes equation, then we have discussed about some exact solution of the Navier-Stokes equation.

Then we have discussed the hydrodynamic boundary layer for flow over a flat plate. Then we derived the energy equation and applied that for analyzing the thermal boundary layer over the flat plate. Then we discussed forced discussion internal flow that is forced convection through plate, pipes, channels, etc and then we discussed natural convection. So, the remaining topics in convection.

So, we have discussed so far, this where no phase change is involved. But there are many practical engineering situations when phase change is important and two such examples are condensation and boiling. So, condensation and boiling will be discussed next and then the discussion on convective heat transfer will end with one practical engineering application where all this experiences of handling the various mathematical analysis of convection will be useful, is design and analysis of heat exchanges.

So that will more or less wind up this particular book course on conduction and convection heat transfer. So, for the remaining lectures on this particular course Prof. Som will take over. So, from the next lecture he will be starting with the new topic. Thank you.