Conduction and Convection Heat Transfer
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Lecture 55
Natural Convection - III

In our previous lecture, we were discussing about some fundamental, physical and mathematical considerations for natural convection. Now today what we will do is we will perform some order of magnitude or scaling analysis to unveil some important physics associated with natural convection.

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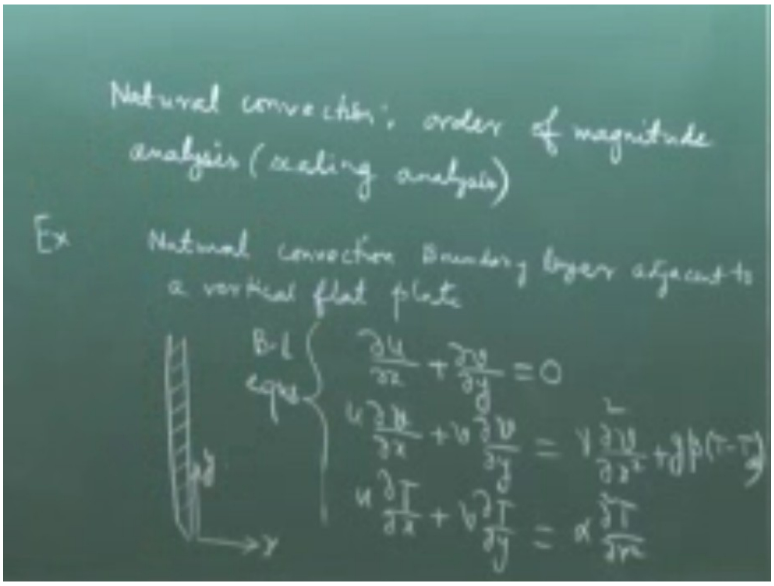

So, we will as a model problem consider the natural convection, order of magnitude analysis or scaling analysis with an example of natural convection boundary layer adjacent to a vertical flat plate. So, this is exactly the situation that we discussed in the previous lecture, but we will now try to get some deeper insight. In the previous lecture we had a broad understanding and now we will try to get a deeper insight.

So, this is the vertical flat plate and we set up the coordinate axis, so that this is x axis and this is y axis. So, what are the boundary layer equations for the continuity equation? Sorry, it is the y momentum equation that is important, so u del v del x. These are the boundary layer equations. And these equations are coupled because you have to know the temperature field to solve the velocity field. This sense of equations we discussed in our previous lecture.

Now we will take it up from there and try to make an assessment of the various parameters.

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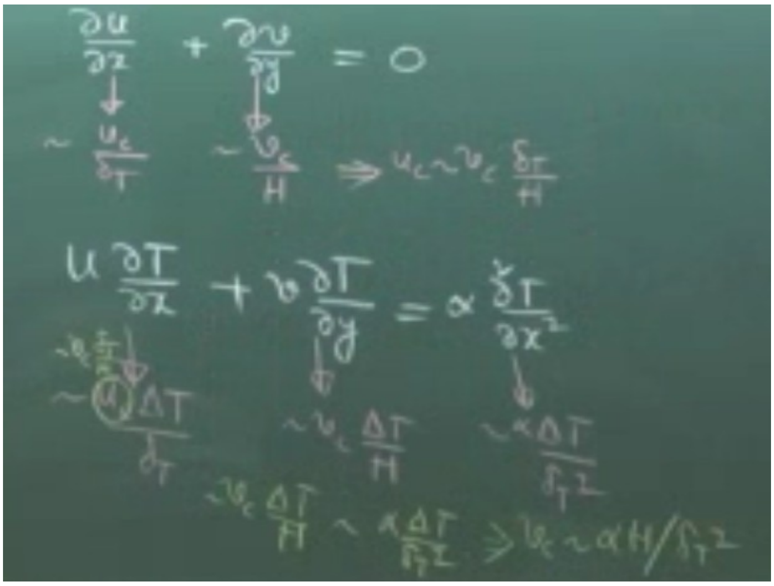

Now let us say that we make a scaling analysis of the continuity equation first. So, what is the order of magnitude of del u del x. Let us say you see is a characteristic velocity along x. Let us say that the height of the plate is H, so del u del x what is this? See you are analyzing the natural convection, that means where you are analyzing it? Within what boundary layer? When we say boundary layer equations, there is a little bit of sense of vagueness about it.

Because it does not tell whether it is hydrodynamic or thermal, I mean one obvious reason is they are coupled, but when you are writing these equations or analyzing these equations, what is your domain of interest, is it hydrodynamic boundary layer or thermal boundary layer. It is the thermal boundary layer; the reason is the temperature gradients are existing only within the thermal boundary layer and that temperature gradient is actually driving the flow.

It is T minus T infinity. So, if you look at the source down, this is T minus T infinity. So, T is different from T infinity only within the thermal boundary layer. So, the important body force in the momentum equation comes into picture only within the thermal boundary layer. That does not mean that the hydrodynamic boundary layer and thermal boundary layer thing is the same because the fluid also has its inertia.

So, when it attains its peak velocity not that just outside the thermal boundary layer it suddenly comes down to zero. So, it takes a bit of a distance to come down to zero, so that is because the fluid has its inertia. So even when the heating or cooling effect is stopped still there is some motion. Therefore, hydrodynamic and thermal boundary layers are not of the same thickness but what is important is that the hydrodynamics is not the continuity equation in the hydrodynamic boundary layer that is of importance for this boundary layer analysis

But it is the continuity equation analyzed within the thermal boundary layer. So uc divided by delta T, right? Not delta. This is, what is the order of magnitude of this? Let us say vc is the characteristic velocity along y, vc divided by H. So, as we have discussed earlier if these two terms are of the same order of magnitude then what it means is that uc is of the order of vc multiplied by delta T by H. So, if delta T is much less than H.

Then the velocity scale along x is much less than velocity scale along y, but it is still not zero. Then we will analyze the energy equation before entering into the momentum equation. What is the order of magnitude of this? Uc, this is some characteristic temperature difference by some Characteristic length. What is the characteristic temperature difference? T wall minus T infinity. That is the triggering temperature difference for heat transfer, right?

So, in short hand notation we write it as delta T. So, delta T is nothing but T wall minus T infinity. So uc multiplied by delta T by delta T, this is of the order of vc multiplied by delta T by H and this is of the order of alpha delta T by delta T square. And uc in place of uc we can write vc delta T by H. So, you can see that this two terms are of the same order of magnitude. You can see that this is philosophically such an important inference that from the momentum equation.

From the energy equation, from any physical scenario, we have seen that when we come to the boundary layer analysis, in the advection terms no term is dominating with respect to the other. Until and unless one is identically zero. Like for internal flow, the situation is different because for fully developed flow v is identically equal to zero. But if v is not identically equal to zero, then no matter how small the, u or v whatever, how small is it?

You will see that these two terms are of equal order of magnitude. This is something which is very intuitive. So, these two terms together are of the order of vc delta T by H, that is of the order of alpha delta T by delta T square. So, delta T square or you can write vc is of the order of alpha H by delta T square. See, why we have come up with an expression for vc is because this is what is our matter of interest. That is see, try to understand it practically.

You are investing by hitting a plate, hitting a vertical plate. So always just like any business you want a return of investment. So, in heat transfer business, in natural convection, your return of investment is what flow you are getting out of this temperature gradient that you are creating. So, this is a scale of velocity that you are getting by creating a temperature difference delta T.

And interestingly that temperature difference delta T does not appear anywhere in this expression. That is again something which is not very intuitive.

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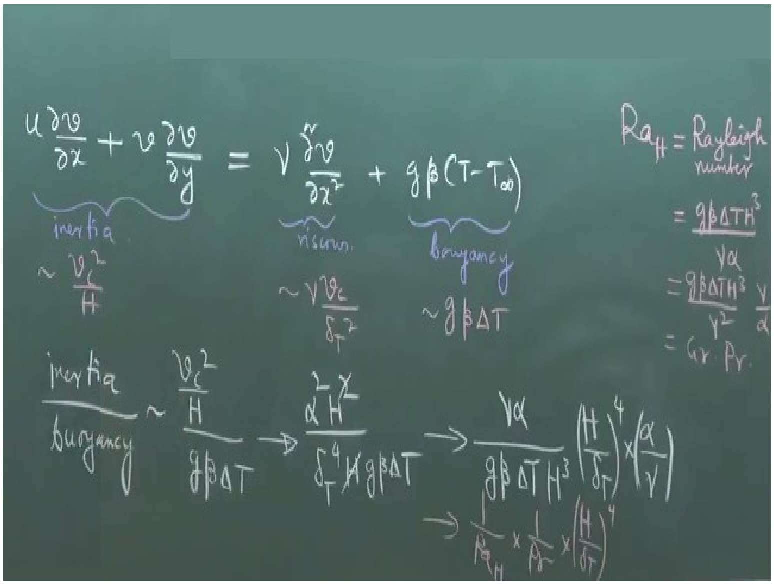

So, we will keep this expression in consideration, now we will come into the momentum equation. So just to give you a perspective, this term physically is what? This is inertia. This is viscous. And this is buoyancy. So, let us try to get order of magnitudes of these three terms. So, what is the order of magnitude of the inertia term? So, you can calculate any one of these because we have already seen that they will be eventually the same order of magnitude.

So, it is more straight forward because this does not involve delta T. So vc square by H. This, what is the order of magnitude of this? Nu. This is a very important thing. See, we are making our analysis within the thermal boundary layer. So, it is delta T and not delta, although this is momentum equation. When we will be analyzing this momentum equation, for the hydrodynamic boundary layer then this will be replaced by delta.

But we are using the momentum equation for analyzing the physical situation within the thermal boundary layer and that is why this is delta T and not delta. And this is of the order of g beta…Now can you tell out of these 3 forces there is at least one force which you expect to be important in natural convection, what is that? Buoyancy, because this force is the driving force for natural convection.

Now given that this force is an important parameter for natural convection, the question is out of the other two forces which one is dominating because out of the other two forces whatever is dominating that will compete with this, right? So, we will find out inertia by buoyancy and viscous by buoyancy and see which is more and which is less. So, inertia by buoyancy is of the order of, now in place of vc you can write alpha H by delta T square.

So, this is alpha square H square by delta T to the power 4 by H g beta delta T. So, one H gets cancelled. Because here you have delta T to the power 4, so to make it compatible and non dimensionalized, the numerator should have H to the power 4, then you will get H by delta T whole to the power 4. So, for that you have to multiply both numerator and denominator by H cube.

So, this becomes g beta delta T H cube by mu alpha multiplied by H by delta T to the power 4 multiplied by mu by alpha. Why we have done this is because again this is our by the time so familiar, non-dimensional number, the Prandtl number. This is non-dimensional. So, this is another non-dimensional number. A very similar non-dimensional number we discussed in the previous lecture.

That was g beta delta T H cube by mu square, Grashof number and this is g b delta T H cube by mu alpha. This is called as Rayleigh number. So, this you can write as g beta delta T H cube by nu square multiplied by nu by alpha. So, Rayleigh number is nothing but Grashof number multiplied by Prandtl number.

“Professor - student conversation starts” No. That is alright. I am just writing this one. g beta delta T H cube by nu alpha. Oh, these term you were talking about. So alpha by nu, sorry, not nu by alpha. But I am just talking about this Rayleigh number. This term. “Professor - student conversation ends" So Rayleigh number is, again you can't see, this is Grashof number multiplied by Prandtl number.

So anyway, this we can write g beta delta T H cube by nu alpha, so one by Rayleigh number multiplied by one by Prandtl number, this is one by Prandtl number multiplied by H by delta T to the power 4, right? So, this is inertia by buoyancy. Let us calculate viscous by buoyancy.

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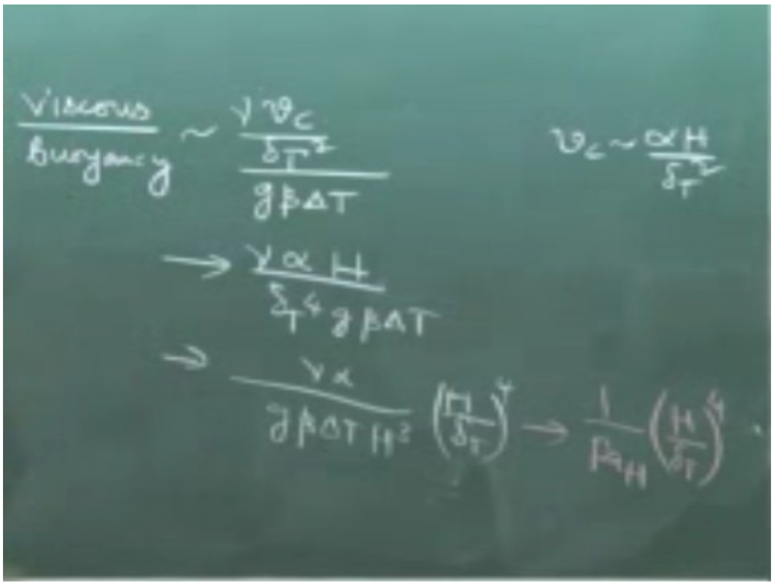

Viscous by buoyancy, viscous force nu vc by delta T square divided by g beta delta T. What was vc? Alpha H by delta T square, right? So, this is nu multiplied by alpha multiplied by H by delta T to the power 4 g beta delta T. So, this you can write it as g beta delta T, H cube by nu alpha multiplied by H by delta T to the power 4, right? So, this is of the order of one by Rayleigh number multiplied by H by delta T to the power.

Now can you tell that by this analysis how can we figure out that which force will compete with buoyancy in the thermal boundary layer, is it inertia force or the viscous force, depends on Prandtl number, right? Because you see the difference between inertia by buoyancy and viscous by buoyancy is the factor one by Prandtl number in the inertia by buoyancy. The remaining factors are the same.

That is one by Rayleigh number into H by delta T to the power 4, they are the same. Only difference is one by Prandtl number. So again, the situation boils down to whether Prandtl umber is much greater than one or Prandtl number much less than one.

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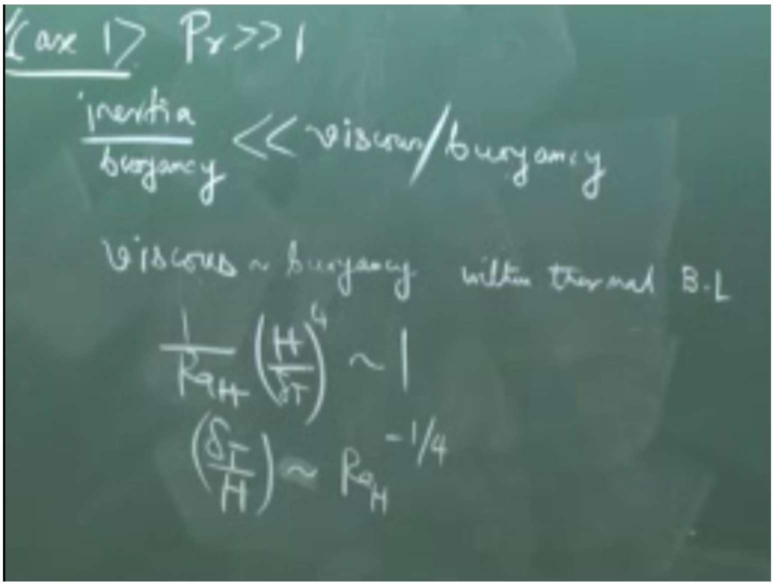

So, we will consider the first case when Prandtl number much greater than one. If Prandtl number is much greater than one then inertia by buoyancy is much less than viscous by buoyancy, right? Because other factors are the same. That means which forces are competing? Viscous and buoyancy within which boundary layer, thermal boundary layer.

So viscous is of the order of buoyancy means one by Rayleigh number multiplied by H by delta T to the power 4 is of the order of one, right? These two are of the same order. That means, delta T by H is of the order of Rayleigh number to the power minus one fourth, right? From this, delta T by H to the power 4 is of the order of Rayleigh number to the power minus one. So, delta T by H is of the order of Rayleigh number to the power minus one fourth.

So just like the assumption the boundary layer assumption that in the boundary layer theory for forced convection what was the assumption? For hydrodynamic boundary layer, delta much less than L whatever we consider the length of the plate. Delta much less than that. For thermal boundary layer, delta T much less than that. For thermal boundary layer delta T, much less than L.

So here also for natural convection, delta T much less than 8 will be our assumption, why? Because if you recall, the energy equation we have neglected del square T, del y square as compared to del x square. That assumption is based on the fact that delta T is much less than 8. So, delta T is much less than 8 only when the Rayleigh number is large. Just like for the hydrodynamic layer analysis for flow over a flat plate is valid for large Reynolds number, similarly here the situation is coming for large Rayleigh number.

For thermal boundary layer, in forced convection, it is Reynolds number multiplied by Prandtl number, that. So here- but it is basically related to Reynolds number to the power n multiplied by Prandtl number to the power n, where n and n maybe two different coefficients. So here also it may depend on Rayleigh number to the power n multiplied by Prandtl number to the power n. Here Prandtl number is not coming into the picture.

But for the other case where Prandtl number much less than one, Prandtl number will come into the picture because you can see the existence of Prandtl number in the denominator. We will come into that subsequently but before that we will try to find out what is the Nusselt number. Again, in any problem of convection, it is Nusselt number that is our matter of interest.

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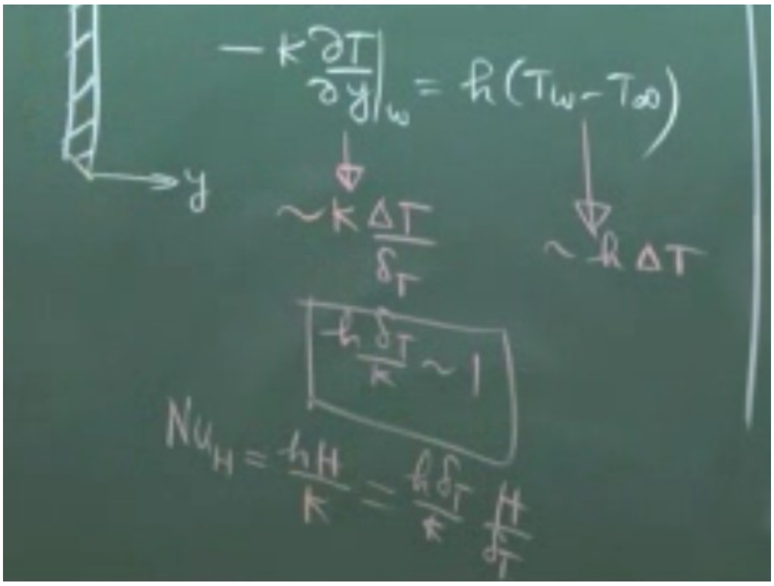

So, if this is the plate and this is the y axis, at the wall we can write minus k del T del y at wall is equal to H multiplied by T1 minus T infinity. Nusselt number is this H expressed in a non-dimensional form. So, what is the order of magnitude of this? Yes, what is the order of magnitude of this? K. The characteristic temperature gradient takes place within the thermal boundary layer.

And that is T wall minus T infinity by the thickness of the thermal boundary layer and this is h delta T. This is exactly equal to h delta T, you need not put as order because this is h and this is delta T. So, this two terms must balance each other, so that you can write h delta T by k is of the order of one. This inference does not depend on the value of Prandtl number. So now the Nusselt number which is hH by k based on the length h is equal to h delta T by k multiplied by H by delta T.

So, this is h delta T is of the order of one and H by delta T is of the order of Rayleigh number to the power one fourth. That means, Nusselt number is of the order of Rayleigh number to the power one fourth. If you do very rigorous mathematical analysis and what new thing you will get is, you will still get this with the constant c which you are not able to capture through this order of magnitude analysis.

So, you will get Nusselt number is equal to some constant multiplied by Rayleigh number to the power one fourth, that constant's value you can get by more detailed analysis. But entire physics you can capture through this except for the value of that constant.

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