Conduction and Convection Heat Transfer
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Lecture-54
Natural Convection – I

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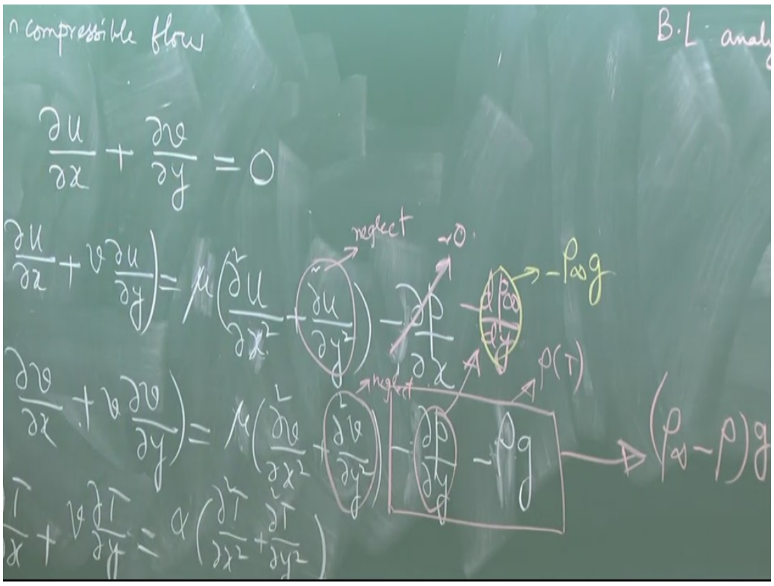

these 2 effects, which effect is primary? Which effect is secondary? right. The question is that if rho is a function of temperature for governing the physics of the problem.

Rho is the function of temperature in these term or rho is the function of temperature in these term, which is the primary effect and which is the secondary effect? Which is actually driving the flow? See, this body force that is driving the flow, so this rho being the function of temperature, is the primary effect, these rho being the function of temperature is also a physical effect but that is the secondary effect to the problem.

The primary effect in these rhos being the function of temperature. So, to simplify the problem, one important assumption was done, made by Boussinesq, what he made is an, he made an assumption that he considers the density variation with temperature to be important only for the body force term and not for the other term. So here, he considers rho= rho infinity, this is an assumption.

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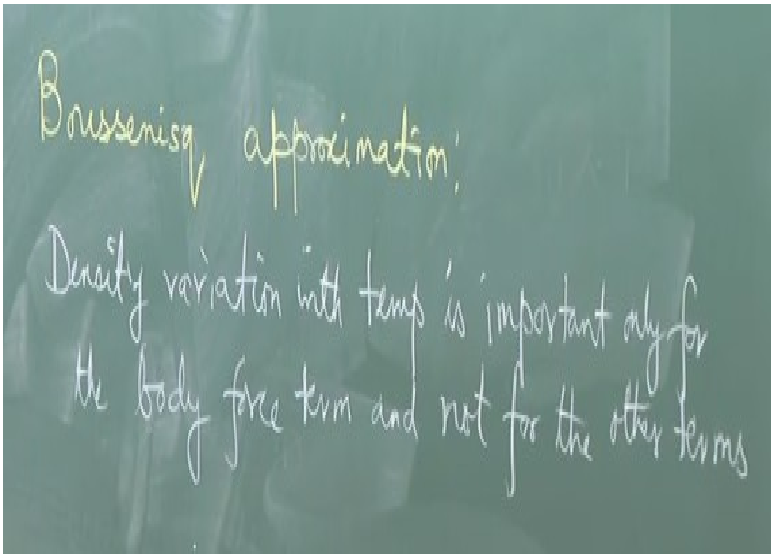

This is not the reality, here also rho will change, but you know these are certain levels of simplifications to make your analysis more and more mathematically attractable. So, let me write that, Boussinesq approximation; density variation with temperature is important only for the body force term and not for the other terms. So that means, for the other term you can write, rho as rho infinity only in these term, you do not write rho as rho infinity.

So, this is a very special term. So now how do you bring? So, you can see from the equation, you can make out that it is these term that is driving the flow. It is the difference between rho infinity and rho, right. So how do you bring the temperature dependence into account; everything is not an ideal gas so you can, so that you cannot write it simply by writing the equation of state of an ideal gas.

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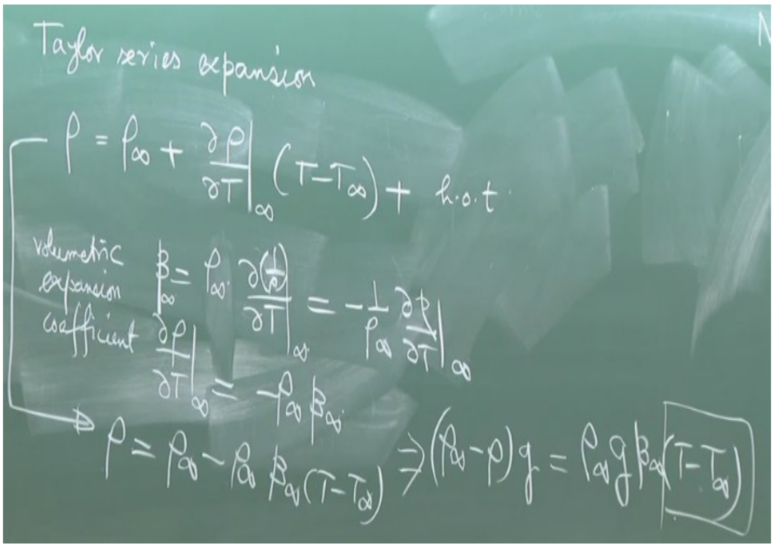

So, you have to make some mathematical analysis to tract these rho as a function of temperature. So that is often done by expanding rho, about rho infinity using a Taylor series expansion as a function of temperature. So, let us do that. So, if the temperature difference is larger and larger, the higher rod at tone terms will be more and more important. But if the temperature difference is not that large, only up to the linear term may be good enough.

Many practical problems are like that, where up to the linear term is what is good enough. Now we can write these in terms of the volumetric expansion coefficient beta. So, beta at the infinity or the fast stream condition is 1/v change in volume, per unit volume for each degree change in temperature. So typically, when you consider the partial derivatives, some other variables you keep as constant.

But, I mean depending on that you could have beta p, beta S, depending on various thermodynamic processes but we are generally calling it as the volumetric expansion coefficient without getting into that details. Now instead of specific volume, we will write 1/ density, okay. So, d of 1/rho will become -1/ rho square, so that, -1/rho will come here. So instead of del rho/del T at infinity, we can write –rho infinity beta infinity.

So, in these equation, you have rho=rho infinity- rho infinity beta infinity* T- T infinity. So, rho infinity – rho *g becomes rho infinity g beta infinity* T- T infinity. This is the body force term you can see there, so rho infinity-rho *g rho infinity- rho, you can take, these in the other side, - will become +, so rho infinity g beta infinity *T – T infinity. So, this is a sort of linearization of the problem.

So, you have a driving force, this driving force will decide that what is the strength of; related strength of various convection mechanisms. Now, if these driving force is very large, then even in a problem, where force convection is there, natural convection may be the dominant mechanism, so I will show you later on that how do you decide that given a physical problem, what is the driving mechanism?

Because a practical problem or an industrial problem. Nobody will give you a tag that this is force convection this is natural convection, like that. May be there are some effects of force convection, there are effects of natural convection, so together force convection and natural convection, if they are there, that is called as mixed convection. So, in a mixed convection, now which effect is important and which is not that you have to figure out.

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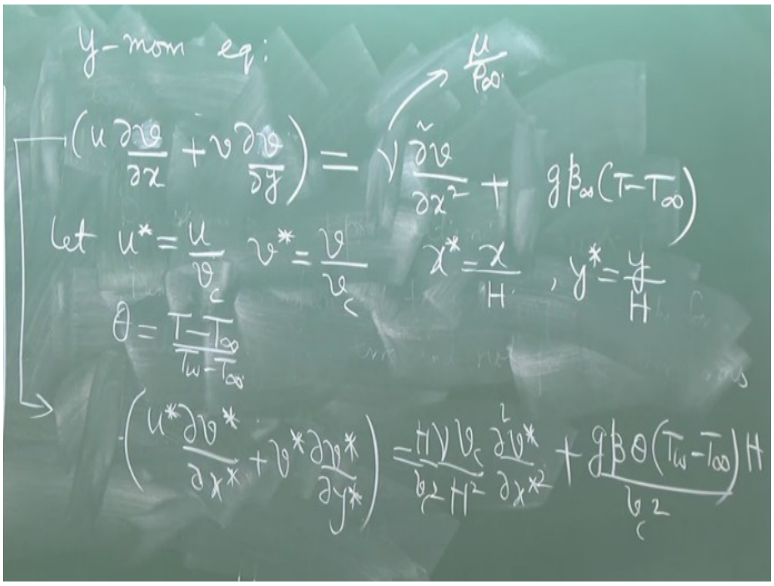

So, to do that we will first not considered the analysis of these equations but we will just nondimensionalised the equations. So, let us write the y momentum equation again. This is the equation. So, our first objective before making any analysis is to ask ourselves a question that, which mechanism of convection is important? Is it force convection that is important, is it natural convection that is important, given a practical scenario.

How will you understand that? So, we will non-dimensionalised these equations, let u star =u/ H, these v star, sorry u/ the characteristic velocity, v star = v/ the characteristic velocity, this is vc is the characteristic velocity along y. The velocity at which, a reference velocity at which the fluid is flowing, some characteristic velocity. Yes, u del v/del x. So, before that, let us divide both sides by rho infinity, so that it becomes nu at infinity.

But fundamentally it is not nu at infinity, but mu/rho at infinity, this mu can be function of temperature. But let us not complicate the problem in that way by putting everything as a function of temperature, I mean of course mu is a function of temperature but it may not be a very strong function of temperature over the range of temperature considered for the problem, but if the range of temperature considered for the problem is very large, then nu as a function of temperature must be taken into account.

Then you cannot take mu as a constant. So, all these, these are practical things like depending on the practical situation as an analyst you have to take the decision. Nobody will tell you, its not an examination problem, that somebody will tell you, assume mu= constant. Practical engineering does not talk in that way. So, given the practical situation, as an analyst you have to decide that it is judicious to take mu as a constant.

If it not judicious, then you have to take mu as a variable, there is no way out. So, let us call x star = x/H, y star=y/H and theta is T- T infinity by T wall – T infinity, non-dimensional temperature. So, what will be these term? u will be vc, vc square/H, right? This term by this term u *-. Sometimes in notation, beta infinity is written as beta, this infinity is subscript is- If you look into books, you will find that mostly, people do not use the subscript infinity, so I am omitting it here.

So now you can multiply both sides by H/vc square, right. So, H/vc square. One important thing, it is a very important conceptual thing. I have not here committed that what is the source of this vc? It may be force convection also. See is this equation valid for force convection. Yes, why not? Have you omitted any term which could be important? potentially important for force convection.

We have not omitted any term; we have added a body force term. But other than that, we have not omitted any term. So, this equation will be valid for force convection as well as natural convection. So, in general, a combination of these 2 effects, which is called as mixed convection, where both force convection and natural convection may be important. So, this vc, may be due to force convection.

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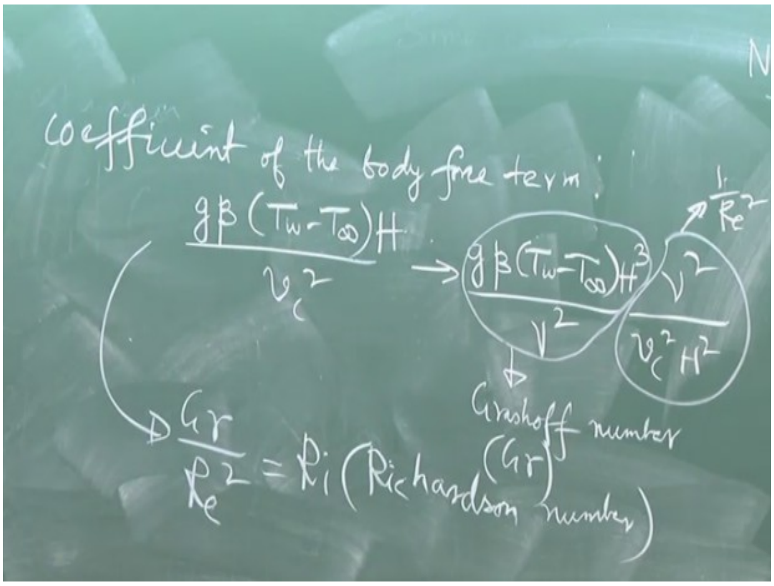

So, if, force convection effects are present, so coefficient of the body force term, let us write; coefficient of the body force term, so theta is the dimensional, sorry; dimensionless parameters, so it varies between 0 to 1. So, its coefficient is, its coefficient is the factor that decides that how strong is that term? So, g beta T wall –T infinity/; there is a H; there was a H, so we multiplied by what? There is a H, right. So, g beta T wall – T infinity/.

So, this you can write; right. So, what we have done is? Why we have made this manipulation? Because velocity characteristic velocity into the lens scale, divide by nu is a Reynolds number. So, if force convection effects are there in a flow, in a problem, then Reynolds number may be an important non-dimensional parameter. So, this is 1/ Re square. Now see by non-dimensionalising all the terms, we are made all the terms non-dimensional.

So, this being non-dimensional its coefficient, these also must be non-dimensional, right. So, this is a non-dimensional term, you can check by putting the dimensions, you will see that, it will come out to be non-dimensional, but it is very clear, so this is a non-dimensional term, so these also must be a non-dimensional term. So, this is called as Grashof number. So, this becomes Grashof number by Reynolds number square.

Again, in subjects like fluid mechanics and heat transfer so many scientist, mathematicians, they have contributed that whenever there is a non-dimensional number appearing, it is a custom to honour some of those by giving it a name as one of the; as the names of one of those scientists. So that is how, like all these scientists are honoured by giving a name to some of the non-dimensional numbers.

Again, this is called as Richardson number. So, this also to honour the contribution. Now what? What is the name; this is important but it is more important that now from here can you tell that given a physical problem how do you analyse, how do you access whether force convection effect is important? Whether natural convection is important? Or whether you have to modulate as mixed convection, where both forced and natural convection are important.

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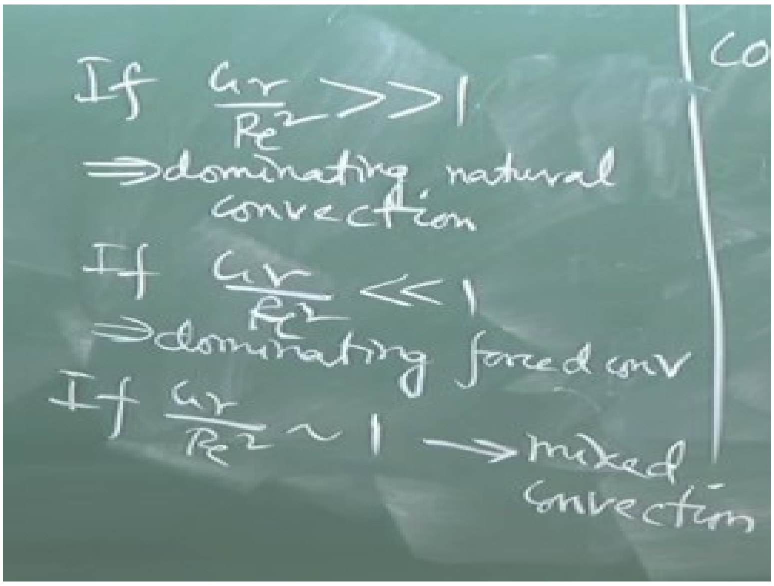

So that clue is given by these term. So, if Grashof number/Reynolds number square is much greater than 1, then what does it signify? It signifies that natural convection is much more important than force convection, dominating natural convection. If Grashof number/ Reynolds number square is much less than 1. Then that means what? Dominating force convection.

A very interesting situation if Grashof number/ Reynolds number square is of the order of 1. Then you cannot discard either of these modes and you have to consider both natural as well as force convection, that is called as mixed convection. So, from here we have seen that Grashof number is a very important non-dimensional number in natural convection, which gives as a clear indication of whether natural convection is important or not.

So, before analysing any problem, we must be convinced that yes natural convection is important, otherwise we should not take the border of, taking an additional body force modelling that and so on. We can simply neglect that. If you we find that the natural convection effect is not important, but we find the natural convection is important, then we have to model it properly.

In the next lecture, we will see that how to make an order of magnitude analysis for modelling natural convection problems. Thank you.