Conduction and Convection Heat Transfer
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Lecture 52
Viscous Dissipation – II

So, this phi is the viscous dissipation function which let me write the expression of this in an index notation. This we have derived earlier but let me just write it.

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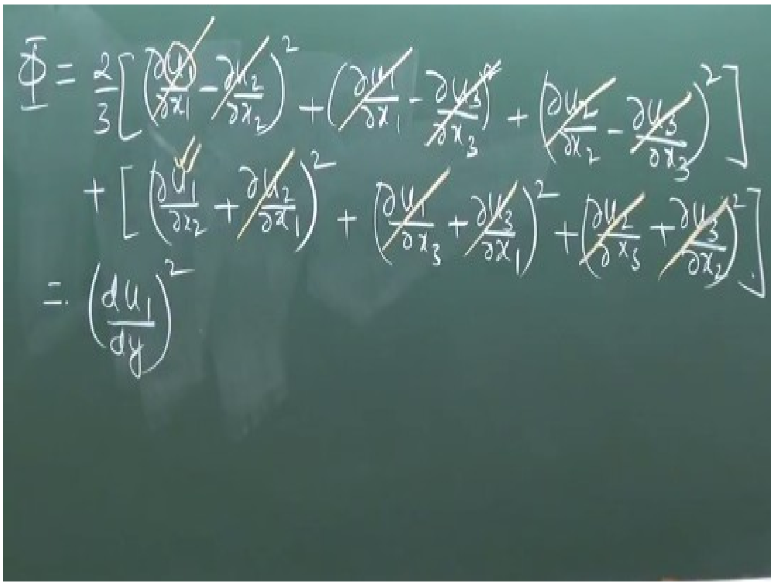

So, phi - so this we have derived earlier. So, can you recall that what are the assumptions under which this is valid? This is not a general, for any general situation this is not the expression for viscous dissipation. For what kind of fluids? Yes. Homogenous, Isotropic, Newtonian and Stokesian fluid because we had used lamda is equal to minus two third mu in this derivation.

So, Stokes hypothesis has to be valid. So, in the usual index location u1 means u, u2 means v, u3 means w, x1 means x, x2 means y, x3 means z. So now let us see, first of all u is a function of y only and there is no other velocity component. So u1, terms with u1, with derivative with respect to x2 will only be there, right? So only del u1, del x2, that term will be there. All other terms will be zero.

So, all these terms will be zero. So, this will be there. This will be zero, this will be zero and this will be zero. So, this becomes this for this particular problem. Now let us come to the energy equation. This is zero because it is steady flow T is not a function of x, so this is zero and this is zero because v is zero. So, if there is a hole in the plate, then you will have the dT dy term.

So not that the left-hand side always become zero. You have to be careful about the situation and then put that understanding into the mathematical terms because T is not a function of x, this is zero. So, you are left with k d2T dy2 plus mu phi is equal to zero. What is this phi? Phi is du…du dy square, so this is u1 square by H square. So d2T dy2 plus mu u1 square by k H square is equal to zero.

This is our governing differential equation. Now for the boundary conditions we have to assume one of the plates to be hotter than the other. Let us take the example that T1 is greater than T0. If the two plates are at the same temperature, then it is not an interesting problem for heat transfer. But even then, we will see that if the two plates are at the same temperature there is some interest in the heat transfer scenario because of the existence of this term.

So, there is a heat source. Now, let us first complete the solution of this problem.

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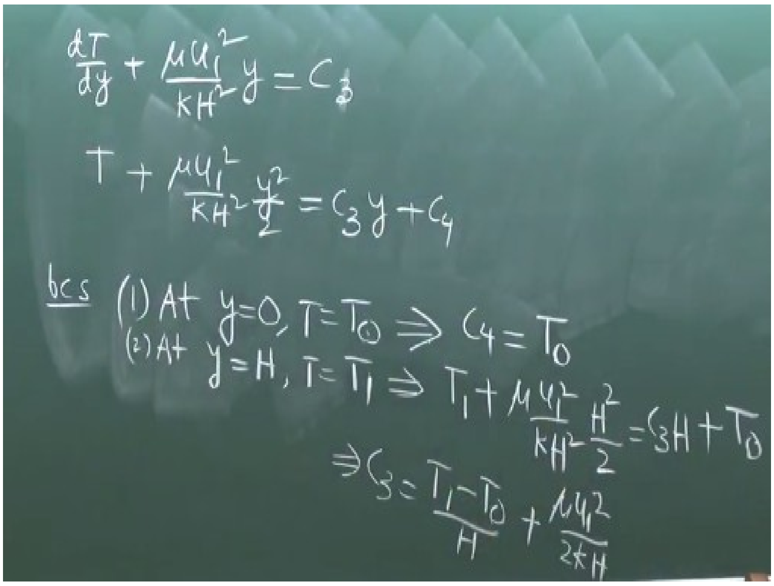

So, let us integrate it once, dT dy plus mu u1 square by kH square y is equal to c1, c1 c2 we have used, so c3, T plus mu u1 square by K H square, y square by 2 is equal to c3, y plus c4. What are the boundary conditions? At y is equal to zero, T equal to T0 and at y equal to H, T equal to T1. So, the first boundary condition tells, that c4 is equal to T0. Right, at y equal to zero, T is equal to T0, so c4 is equal to T0. At y equal to H, T equal to T1.

So, at y equal to H, you have T1 plus mu, mu one square by k H square into H square by 2 is equal to c3H, plus c4 is T0. So, you have c3 is equal to T1-T0 by H plus mu u1 square by 2 k H. So, the solution is T plus.

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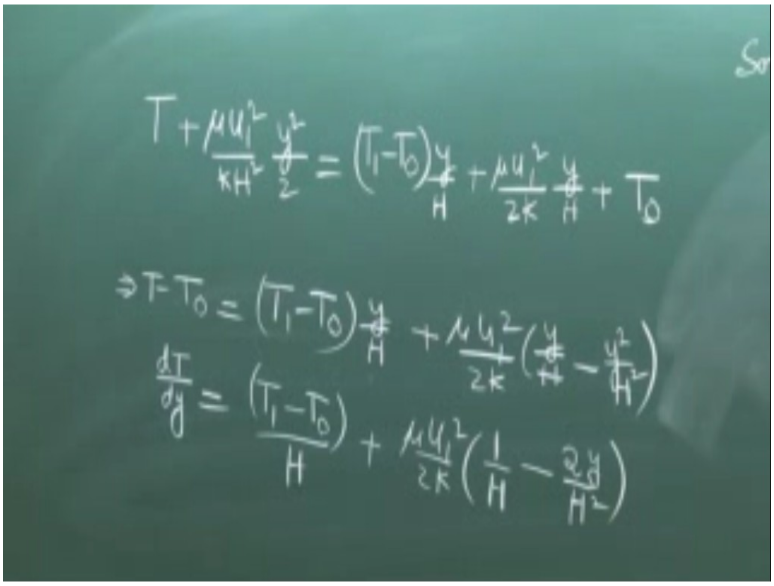

So, the solution is T plus mu u1 square by k H square, y square by 2 is equal to c3y plus c4. So, we can write, T minus T0 is equal to T1 minus T0 multiplied by y by H plus mu u1 square by 2k multiplied by y by H minus y square by H square. So, you can see that the temperature distribution is because of two components. This component is because of what? This is because of heat conduction. You can see this is a linear temperature profile.

This is because of heat conduction and this is because of viscous dissipation. So, the net change in temperature is due to a combination and here a linear combination because the governing equation is linear, is because of a combination of heat transfer due to conduction plus heat transfer due to viscous dissipation. Now what is the matter of our interest is to calculate the heat flux.

So, what is our intuition. Intuition is that this top plate is at a higher temperature than the bottom plate, so heat has a natural tendency to flow from the top plate to the bottom plate. That is the common intuition. Now let us see, let us find out what is the heat flux, so dT dy. dT dy is T1 minus T0 by H plus mu u1 square by 2k into one by H minus 2y by H square.

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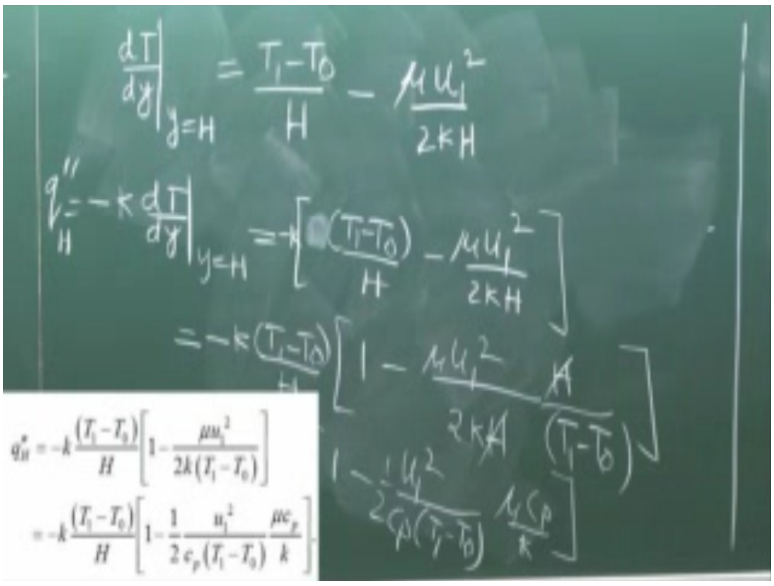

So, let us find out what is dT dy at the top wall. So, what is dT dy at y equal to H? First term is due to conduction and the second term arises due to viscous dissipation present within the fluid. One very important thing to note from this equation is that when the two plates are at same temperature or T1 is equal to T0, then also one can obtain a non-zero temperature gradient at the top wall.

So, the viscous dissipation which acts as a source of thermal energy causes this temperature gradient. Now, the heat flux at the top wall can be obtained by using the above temperature gradient in the following way. qH is equal to minus k dT by dy at y is equal to H is equal to minus k multiplied by T1 minus T0 by H minus mu ul square divided by 2kH. Rearranging different terms.

We obtain the heat flux at the top wall as qH is equal to minus k multiplied by T1 minus T0 divided by H multiplied by one minus mu multiplied by M1 square divided by 2k multiplied b T1 minus T0 is equal to minus k multiplied by T1 minus T0 by H multiplied by one minus cp multiplied by T1 minus T0 multiplied by mu cp by k.

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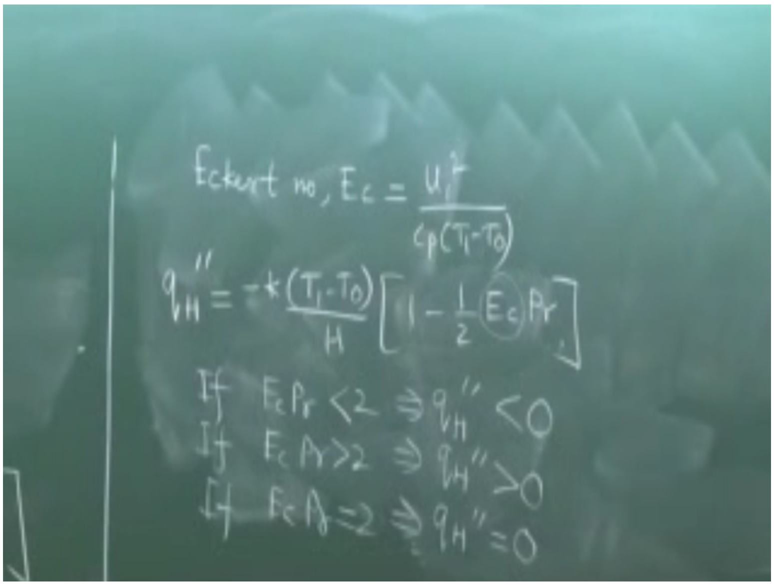

The above representation is of prime importance from the view point of understanding the physical process in terms of non-dimensional numbers which are the combination of different physical parameters, combined effect of which govern the heat transfer process. The right-hand side of this equation can be represented in terms Prandtl number defined as Pr is equal t mu cp by k and Eckert number defined as Ec is equal to u1 square by cp multiplied by T1 minus T0.

Prandtl number, Pr represents the ratio of momentum diffusion and thermal diffusion, while the Eckert number (Ec) is basically the ratio of kinetic energy associated with the fluid velocity and the thermal energy associated with the temperature difference. So, the heat flux at the top wall can be written as qH is equal to minus k multiplied by T1 minus T0 by H multiplied by one minus Ec multiplied by Pr divided by 2.

Now we have expressed the heat flux in terms of temperatures at the two walls and the two important non-dimensional numbers which are Ec and Pr. A closer look into this equation reveals that the direction of heat flux is solely governed by the following two terms T1 minus T0 and one minus Ec Pr divided by 2. Relative sign of these two terms decides the direction of heat flux.

So, for the case T1 greater than T0, the direction in which the heat through the top plate will depend on the magnitude of the dimensionless group Ec Pr that is a, if EcPr less than two implies qH less than zero and the heat will flow from the upper to the lower plate as per intuition. If on the other hand Ec Pr greater than two implies qH greater than zero then interestingly the heat flow from the lower plate to the upper plate even though the upper plate is at a higher temperature as compared to the lower plate.

So, in this particular case there is no heat transfer between the top plate and the bottom plate because that will violate the second law of thermodynamics. So, in this case due to viscous dissipation, locally, the temperature near the upper plate becomes more than T1 and hence heat transfer takes place from the lower plate to the upper plate.

Here the viscous dissipation basically acts as a heat source. So as if there is a heater sitting in lower plate which is making the local temperature more than T1. This is something which is non-intuitive. Finally, if EcPr equal to two implies qH is equal to zero and the upper plates acts as an insulator. Here despite a temperature gradient being present between the two plates, there is no heat transfer.

This is because the temperature difference created due to the viscous dissipation is exactly nullified by the conduction between the two plates. So, there is zero net heat flux. So, the product (and not the Eckert number alone) determines the strength of viscous dissipation in heat transfer. Based on the value of this product, the direction of heat transfer between two plates will change.

This kind of analysis is very important in determining the heat transfer in bearings. In bearings, the temperature of the shaft and its outer casing is very important as the viscosity of the lube oil is a function of temperature (although we have considered the viscosity to be a constant for the present case). Top wall towards the bottom wall, at the top wall, that is something very little because of the natural temperature difference between the top plate and the bottom plate.

However, when Eckert and Prandtl number becomes greater than two, then despite the top plane greater than, I think the temperature greater than that of the bottom plate there is actually the heat transfer of the bottom to the top. This is something which is non-infinitive. So actually, there is no direct heat transfer between the bottom plate and the top plate because that will violate the same wall boundary linings.

So, what is happening is that because of viscous dissipation locally here the temperature is becoming greater than T1. Viscous dissipation essentially is in line with heat source. So, I think there is a heater setting here that heater is making the local temperature greater than T1. So, there is a heat transfer from T1 to T0. But this is something which is not included. The third case is interesting that despite our temperature gradient being created by a natural boundary condition there is no heat transfer.

So why it is happening like that? It is happening like that because this heat transfer because of the temperature difference created by viscous dissipation is exactly nullified by the heat transfer because of the heat conduction between these two. So, there is zero break heat flux. So that you can also see mathematically. This is the heat transfer; this term is equationally what the heat transfer through conduction.

This term is the representation of the heat transfer in viscous dissipation and you can see that when they are exactly nullified each other the heat will pass this. That is what is the physically founded. So, the very important message is that, the Eckert number into Prandtl number is done by the Eckert number, but the Eckert number into Prandtl number that decides the strength of viscous dissipation in heat transfer.

And based on the value one can have interesting upper edge inversions of the direction of the flux. So (()) 48:36) what is happening is, there is local rise in temperature because of viscosity and that creates the (()) (48:46). So, this kind of analysis is very important for analyzing heat transfer in bearings. So, one can understand the temperature distribution, the heat flux and so on in bearings.

And why temperature bearings temperature in the gap between the shaft and the outer casing is important. It is important because the entire functionality of the oil, which is say a lube oil, it depends strongly on the viscosity of the oil. And the viscosity of the oil is a strong function of temperature. So here, for simplicity we have assumed that mu is a constant. But in practice mu is not a constant.

Viscosity of the oil, if it varies the temperature of the oil, it can vary thus dramatically and that will dramatically alter the performance of the bearing. So, in the practical engineering scenario also this appears to be very important. So, to summarize the discussion so far, in convection, we started with derivation of the energy equations. Of course, before that we had a preliminary discussion on fluid mechanics and then we had derivation of energy equation.

And we analyzed cases of flow over flat plate for different Prandtl numbers and internal flow that is flow through parallel plate channel and circular pipe. In boundary conditions like constant volume and constant volume plus and we also have considered the examples where viscous dissipation is important as has been discussed today. So, we will be having some (()) (50:34) problems for this portion of the scenes of nature that we had, these projects will be uploaded.

And this will be discussed extensively in the tutorial sessions. As I told you the type of problems that you will face for this part of the discussion is not dramatically different from a derivation that we have done. These derivations all are actually problems. So, in some cases boundary conditions may be changing, some cases one additional term may come in, some case velocity profile will change.

Please try to (()) (51:15) but the entire (()) (51:18) these problems is what exactly what we have discussed in the class for this portion. So, we will stop this lecture now.