Conduction and Convection Heat Transfer
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Lecture - 50
Internal Forced Convection – VI

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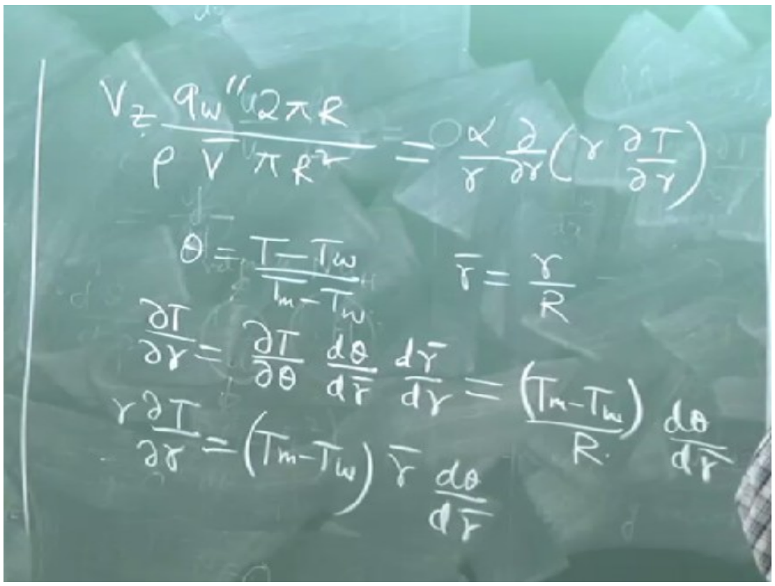
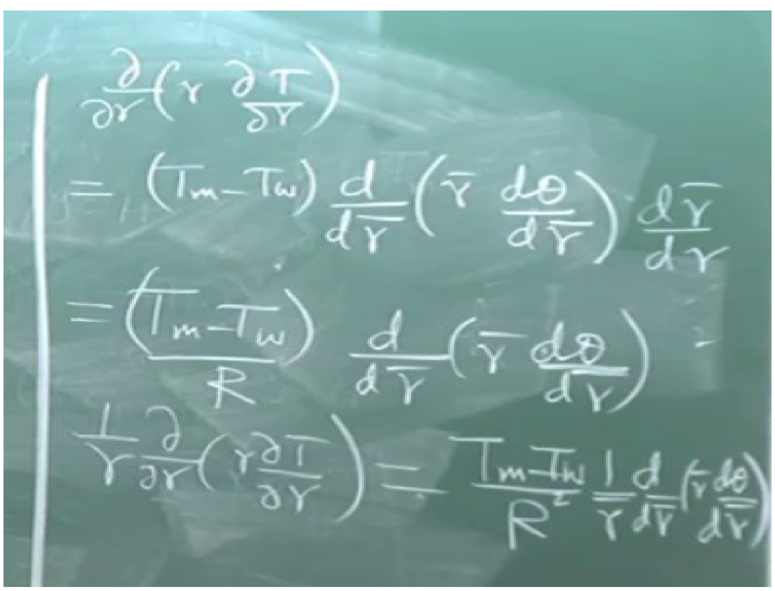


So, we can write del T del r is equal to del T del theta d theta d r bar into d r bar d r, right, just like chain rule. **(Refer Slide Time: 27:34)**



Changing the basis from T2 theta and r2 r bar. So, del T del theta is T m minus T wall and d r bar d r is 1 by R. So, what is r del T del r then what is del del r of r del T del r. So finally, 1 by r del del r of r del T del r. right. T m minus T wall by r square into because this r you can write r bar into R So that R into this R makes it R square.

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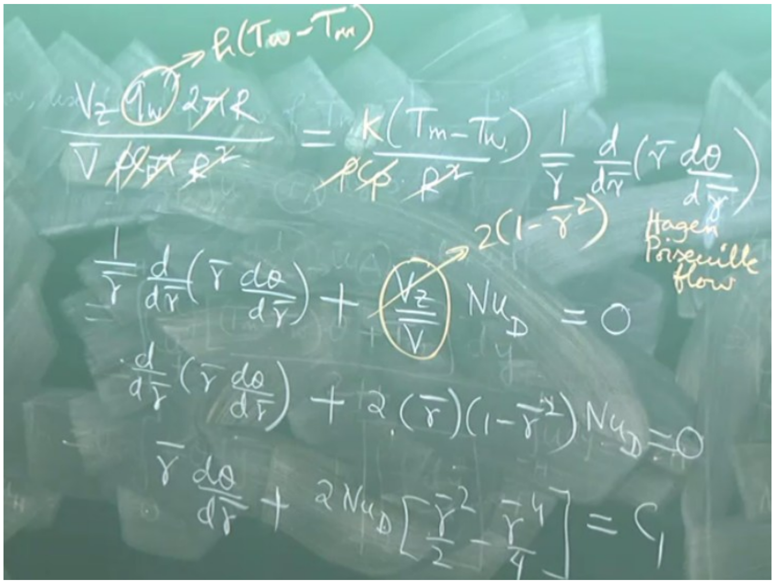

So, we can write Vz into q double prime into 2 pi R by V average C p rho pi R square is equal to alpha into T m minus. Now what is the next step in place of q all we will write h into T wall minus T m and alpha is K by rho C p. So, rho C p gets cancelled, pi gets cancelled, R square also gets cancelled so 1 by r d d r r d theta d r plus Vz by V average. So, you see here this h into 2 r by K, what is 2 r? 2 r is the diameter of the pipe.

So, it is h d by K that is Nusselt number based on the diameter d. Normally, the convention is for a pipe, the reference length scale is consider to be the diameter of the pipe, whereas for a the flat plate it is the length of the plate. So, these are physical reference links governing the physics of the problem. So, for a pipe it is never the length of the pipe that will seriously govern the physics of the problem, but it is the diameter of the pipe or radius whatever.

So, normally in engineering it is the diameter that is considered as a reference scale. If somebody takes radius also it is ok there is no problem it is just a matter of convention that take as the diameter in engineering. So, what are the boundary conditions, so let us integrate this d d r of now where does fluid mechanics come into picture here what is Vz by V, this you tell. This is the solution from Hagen-Poiseuille flow, what is that? What is the velocity profile?

This is 2 into 1 minus r square by R square this is Hagen-Poiseuille flow. Hydrodynamic fully developed flow through a circular pipe. Now, can you tell the value of c1 what is d theta d r at r equal to 0, it is 0. So, if you substitute d theta d r equal to 0 at r equal to 0 you will get C 1 is equal to 0.

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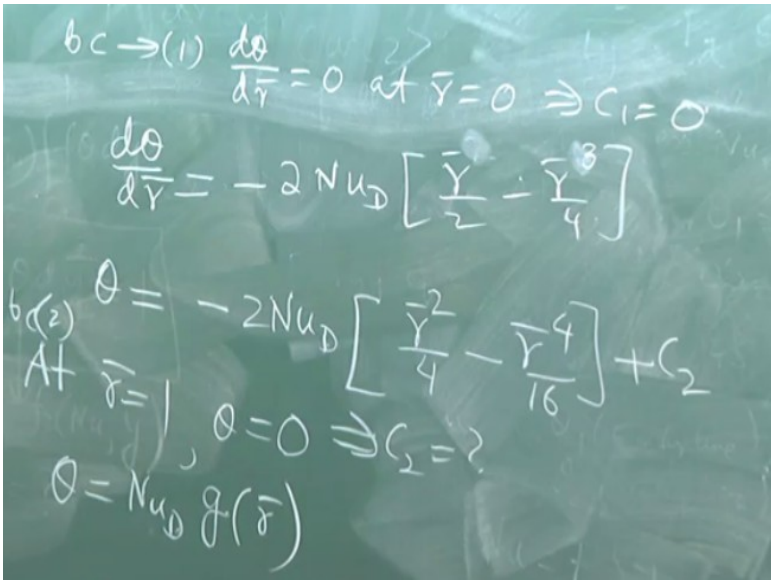

So, boundary condition 1, d theta d r equal to 0 at r equal to 0 that will tell you C 1 equal 0. So, d theta d r is equal to minus 2 Nusselt number into r square by 2 minus r 4 by 4 divided by r that is r by 2 minus r cube by 4, right. Then, we can right boundary condition 2 at r equal to 1, what is theta? Theta is T minus T wall by T m minus T wall, r equal to 1 means wal, so theta is 0. So, these will give you what is C2. So that means you can write theta is equal to Nusselt number into some function g of r, right.

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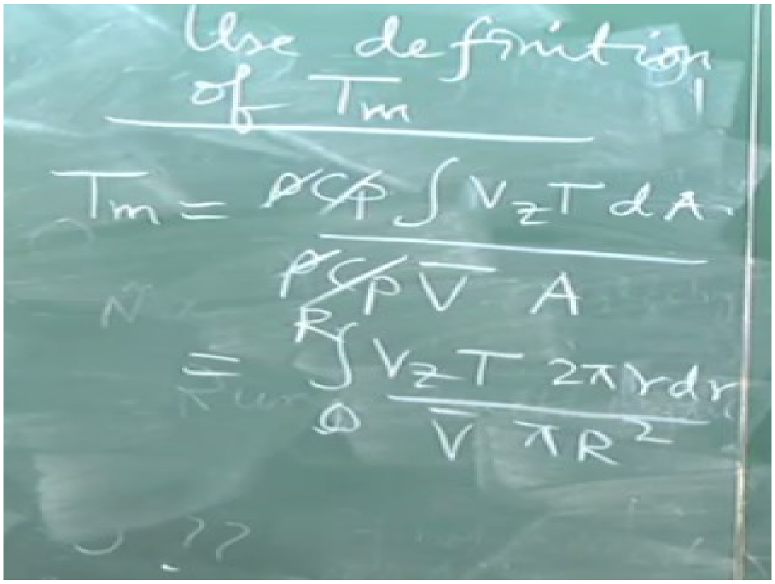

Now how do you calculate the Nusselt number just like the parallel plate channel case, we will use the definition of bulk mean temperature, so use the definition of bulk mean temperature. So, what is the definition? rho C p is constant we are assuming. So, what is d A for a circular pipe 2 pi r d r. So, integral of V z into T into 2 pi r d r by V average into pi r square 0 to r. So, you can write T m is equal to V z by V average into T into 2 r bar d r bar 0 to 1.

So, what we have done is, we absorbed one R with r and 1 R with d r r. So, it has become r bar d r bar. Now in place of T, we will use the definition of theta and write.

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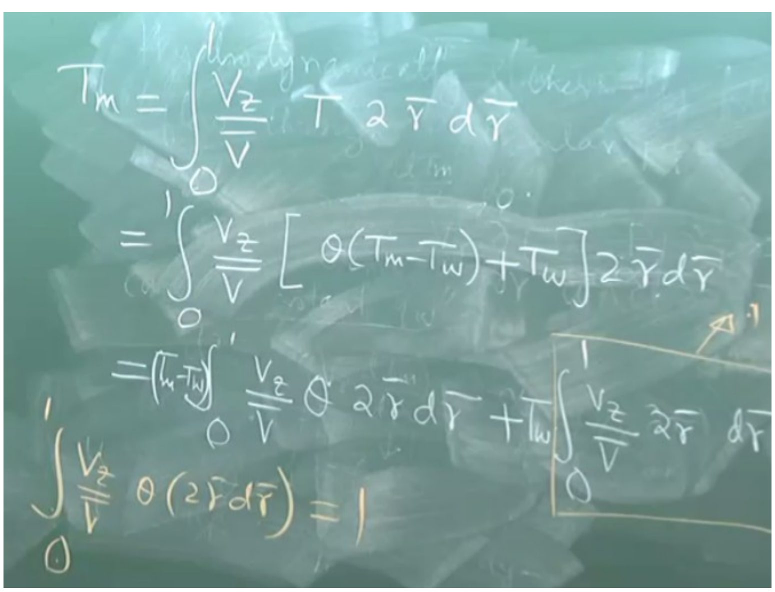

What will this last term? Remember integral of V z into 2 pi r d r is equal to V average into pi r square. So that is the nondimensional form of that ratio which becomes 1. So, that integral of Vz into 2 pi r d r by V average into pi r square that ratio is 1. So, you see I am trying to generalize it, it does not depend on what is the velocity profile, this becomes always 1. So, you can write integral of V z by V average into theta into 2 r bar d r bar 0 to 1 that is equal to 1.

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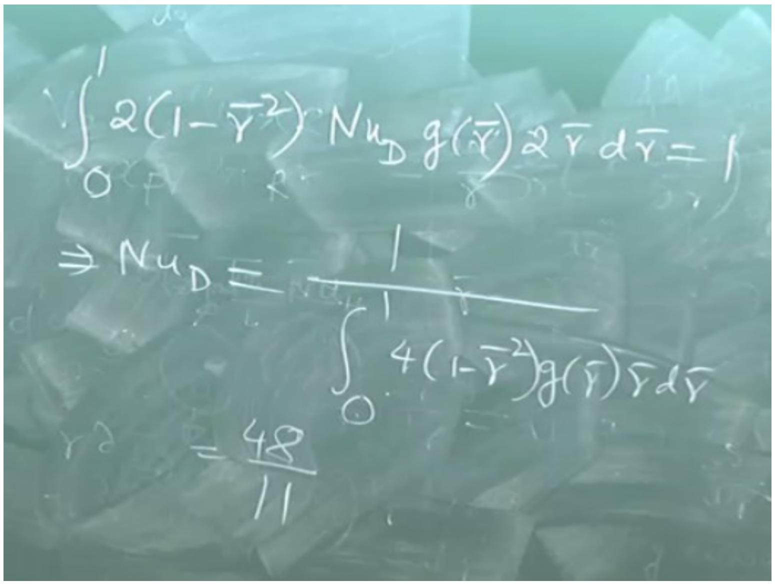

So, V z by V average is 2 into 1 minus r bar square then theta is Nusselt number into g of r bar into 2 r bar d r bar equal to 1. So, that means Nusselt number is equal 1 by integral of 0 to 1 4 into 1 minus r bar square into g r bar into r bar d r bar, right. So, this is the number once this is the simple polynomial integration, there is no trick in this integration because this is a simple polynomial.

So, once you do this integration you will get the value of the Nusselt number and the value of this Nusselt number is 48 by 11. So, I am just giving you all the answers because I think a good exercise that you complete all this things by yourself by doing the small missing algebraic calculations. Because I have given you entire framework, the entire method, but some little bit of polynomial integration and those things are not numerically evaluated.

So, you can numerically evaluate those and check this answer that will give you a good confidence of how to approach these problems. The other case is the constant wall temperature. I am giving you the answer you have to do by the shooting method, so again the important difference between the pervious case.

And this case is that when you are considering the equation what will be the change in the equation if it is constant wall temperature this will be multiplied by theta that will be the only difference just like the parallel plate channel and then you have to work it out by the shooting method and I am giving you the answer, so Nusselt number, so this case is constant wall temperature.

Nusselt number is equal to 3.66 based on the diameter. So, we have worked out 4 cases or we have discussed about 4 cases, constant wall temperature and constant wall heat flux, each for parallel plate channel and circular pipe. So, these 4 cases we have covered and velocity profile we have considered to be fully developed pressure driven velocity profile which you can derived from the Navier-Stokes equation assuming fully developed flow.

Now in practical industrial applications often the laminar flow constraint is not satisfied. So, then if the flow is turbulent flow, in turbulent flow through a circular pipe is very common in industry. So, if the pipe is either heated or cooled then what type of relationship you will have between the Nusselt number and the other parameters. So, that it is given by many correlations. For turbulent flow, you cannot work it out analytically.

So, based on experimental correlations, many popular correlations are there. I do not expect that you remember many correlations, but at least one correlation which is industrially used very extensively much be learn at this level. Because at the end we are interested to solve engineering problems and that is one of the important correlation which we used for turbulent flow through circular pipes for solving engineering problems and that is given by a correlation called as Dittus-Boelter correlation. So, let me write down this correlation.

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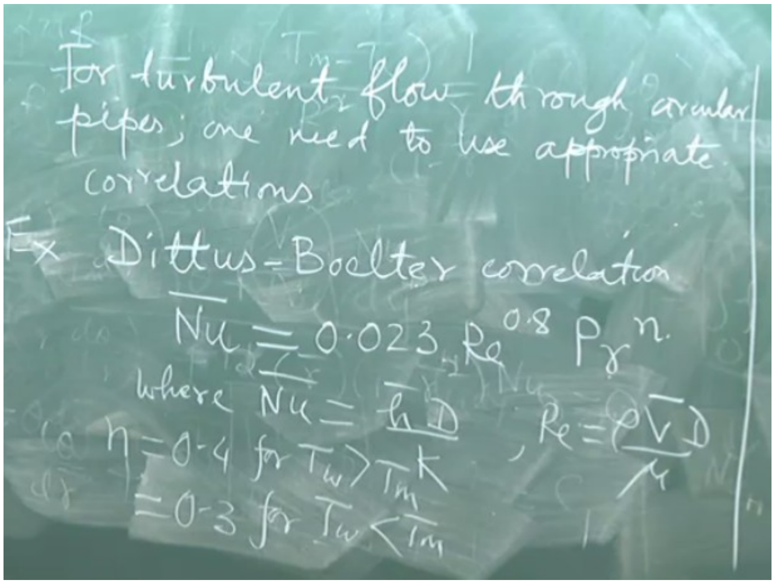

So, for turbulent flow through circular pipes; one need to use appropriate correlations. So, in this correlation the value of this, so again you see the Nusselt number is of the order of Reynolds number to the power n into Prandtl number to the power n. So, that fundamental understanding still remains. Now this Prandtl number to the power n, value of n depends on whether it is heating or cooling.

If the wall temperature is greater than the bulk mean temperature, n is point 4 and if it is less than the bulk mean temperature that is cooling that n is point 3. Remember that this is average Nusselt number because in a turbulent flow you do not achieve a situation where you have constant heat transfer co-efficient. So, you have this h varying continuously with x, so you have h average into d y K that is Nusselt number average and Reynolds number.

Of course, rho into V average V by mu. Now, a very important thing see as engineers when make calculation you have to use the properties mu, rho, K. These properties are functions of temperature. So, at what temperature you will calculate these properties. So, not at the bulk mean because close to the wall the temperature will be mostly driven by the wall temperature, mostly govern by the wall temperature.

Far away from the wall, it will mostly govern by the bulk mean temperature. So, the engineering practice is to evaluate properties at T wall plus T m by 2, ok. If it is for flow over the flat plate then instead of T m, we can use T infinity. So, T m see now you have learnt some aspects of forced convection, you tried to unify the concept. In the forced convection for flow over the plate, whatever was the role of T infinity, the same role is played by T m at the bulk mean temperature for an internal flow which is confine between boundaries, ok.

So, I do not want to bother you with many of these correlations because as I promised you that we will not discuss in the undergraduate level some formula which we cannot derive in the class, but this is a sort of an exception as an example because this is so commonly used by engineers, I believe as engineers you must know this correlation. Now, so far, we have discussed about what let us summarize.

We have discussed about forced convection with flow over the flat plate and flow in a channel or a pipe. We have derived the momentum equations, the energy equations, we have solved the fluid mechanics problem and the heat transfer problem, but in all these problems we have neglected viscous dissipation which was one of the terms that appears in the derivation of the energy equation.

Now, what happens what are practical engineering situations when the viscous dissipation term is important and how can you analyze those situations mathematically. So, you learnt that in the next lecture when we discuss about cases where viscous dissipation terms maybe important. Thank you very much.