Conduction and Convection Heat Transfer

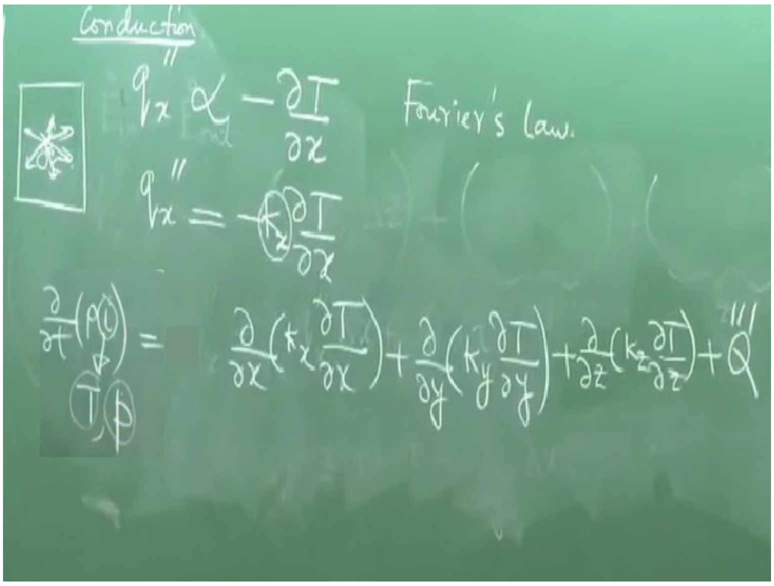
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology- Kharagpur

Lecture 5

Now this is a pretty generic equation so far, we have not yet introduced the implication of conduction in this equation. Now what can be the basic modes of this heat flux.

The basic modes of this heat flux may be one of the basic modes of heat transfer. So, we have not committed that it is conduction. Now we will commit that it is conduction. So, we will write this keeping in mind that the mode of heat transfer that we are considering is conduction that will lead to the basic heat conduction equation. So, if we write the heat flux as a function of a parameter which is directly measureable for heat conduction how can we write that.

(Refer Slide Time: 32:08)



So, let us say that heat flux along x. So, heat flux along x how do you write it in terms of a measureable parameter for conduction, so heat flux along x is proportional to the negative of the temperature radiant along x. This is nothing but Fourier's law of heat conduction. Now there are many situations in engineering when the Fourier's law is applied or is applicable but there are many situations if not many at least quite a few situations when the Fourier's law is not applicable.

So, it is very important to discuss the various considerations based on which we can use the

Fourier’s law before simplifying the equation further. So, the Fourier’s law, see one of the very, this is called as a constitutive law, it somehow relates the cause with an effect. The effect is a heat flux and what is the cause, cause is the temperature gradient because heat always gets transported from higher temperature to lower temperature.

So, the positive heat flux will be along the direction of negative temperature gradient and to make an adjustment for that you have a negative sign here. So, this is the cause effect type of relationship and this is a linear relationship as you can see that this type of relationship is a linear constitutive behaviour.

That means this is linearly related to the temperature gradient. Now this assumes that we are considering a situation when physically if you create a disturbance in temperature at a point that propagates to all possible directions at infinite speed that is the basic physical premise of the Fourier’s law of heat conduction. So that means what this is something like this. Say it is a domain.

Let us say everywhere in the domain the temperature was atmospheric temperature. Now suddenly you have make this point zero degree centigrade by bringing in contact with eyes. So, this temperature disturbance will be instantaneously propagated in all directions at infinite speed. So that all points in the domain will immediately know that there is a temperature change I have to respond to the change.

This message, so there is a messenger within the material which propagates the effect of temperature disturbance by virtue of heat conduction and that messenger is nothing but the thermal conductivity of the material. So, in our subsequent discussions we will see that a more effective parameter to describe the efficacy of this message propagation is not thermal conductivity but thermal diffusivity. We will discuss about that in a moment.

Now see if whenever we learn sine formula we also have to know that when it is applicable more importantly when it is not applicable because many times when it is applicable we apply it is fine. Many times we tend to apply it when it is actually not applicable. So, when you say that we are using this Fourier’s law there are typical situations when the Fourier’s law is not applicable.

Let me again give an example of say a treatment of a biological tissue with a femtosecond laser. See a laser with a pulse of femtoseconds, very short pulse. Now the laser pulse will be coming, it will be on and off in a very short span of time. So, let us say the medical treatment is going on with a laser, the laser is falling on the tissue for a very short period of time it is acting on the tissue then it is going off.

So, what is happening is when the laser is acting on the tissue. The tissue takes on time to adjust to the change in temperature brought in by the laser but before the tissue adjust to itself immediately the laser parts is switched off and by the time it gets readjusted another new pulse has come.

So that means that the time interval over which the change takes place is shorter than the relaxation time of the material. The material takes the time to adjust to the change. Like all of us cannot adjust to a change instantaneously materials also do like that. So, you apply a heat transfer to the material, the material cannot instantaneously adjust its temperature to take care of that. So, it will take a little bit of time.

Now if your time duration over which you impose the disturbance is still shorter than the time then what happens is that the temperature disturbance cannot propagate at an infinite speed but it propagates at a finite speed. It is very similar to the manner in which a disturbance propagates to a medium and that speed you know from basic physics is known as the sonic speed.

So there are various possibilities when the temperature disturbance within the material propagates by a finite speed and then Fourier’s law is not applicable. So those types of situations are called as non Fourier heat conduction. It is not within the scope of this particular course discussed about that, but I have just brought that situation out to give you an example that we should not take it as ritual that the Fourier’s law is always valid.

But we will proceed with the case assuming that the Fourier’s law is valid. Now the constant of proportionality is replaced by an equality where this equality is called as the thermal conductivity of the material. So, we have given a subscript x because thermal conductivity is a property which can vary with deduction. So, if you have anisotropic thermal conductivity then kx, ky, kz these are all different.

But most of the times we are dealing with materials for which kx, ky, kz all are same at some constant k. So, if that be the case then you can write. Now just out of curiosity you might observe a very interesting thing. See fluid mechanics and heat transfers of course these are two different subjects and that is why you are starting these as different courses in your curriculum.

But there is a whole lot of similarity in terms of mathematical approach of fluid mechanics and heat transfer and there are various fundamental foundations behind that. When you are talking about this Fourier's law can you think of an equivalent law in fluid mechanics. The Newton's law of viscosity right, the Newton's law of viscosity is like for our flow which is taking place along x and with velocity gradient along y, you write Tau is equal to Mu du/dy right.

So, it is like a velocity gradient related to the shear stress. Chemical engineers often call shear stress as momentum flux because the shear stress is brought about by a disturbance in momentum like let us say there is a flat plate and fluid is flowing over the flat plate because of the frictional or the physical effect of no sleep at the wall what is happening is the fluid molecules at the wall are having zero velocity if the wall is having zero velocity.

But the fluid molecules which are next to the wall will not directly feel the effect of the wall, but how it will know that there is a wall, there is an invisible messenger within the fluid that transmits the momentum flux or momentum disturbance and this invisible messenger is viscosity. So, it is very much similar to thermal conductivity.

So, this viscous effect is because of momentum diffusion and thermal conduction is because of heat diffusion, both are diffusion phenomena and you also have a similar effect in mass transfers which is called as mass diffusion and that is governed by another law called as fix law. So, we will not come into that because of mass transfer is not within the scope of our present course.

Now let us say you want to solve this equation. Now are you in a position to solve this equation. See you have temperature as one of the variables now here you have internal energy so these two variable are not the same so you must close this system with an expression that relates temperature with internal energy okay.

So how you relate temperature with internal energy, so even from practical consideration you must express it in terms of temperature or pressure whatever something which is measureable right. If you are making experiments then you have devices which can measure temperature, you have devices which can measure pressure. Now there are devices like thermometer but there is nothing called as internal energy meter.

So experimentally to measure something and to relate that measured parameters with what you predict from the theory you need to have this internal energy expressed as a function of measureable parameters. What are the measureable parameters that you require, see it depends on, first of all how many measureable parameters are required to express internal energy as a function of those parameters? How many parameters.

It depends on the physical state of the system and what kind of system it is so in thermodynamics there is something which is called as a simple compressible substance. So, a simple compressible substance is a substance for which pressure, volume, temperature changes are much more important as compared to like electrical effect, magnetic effect other normal thermal effects.

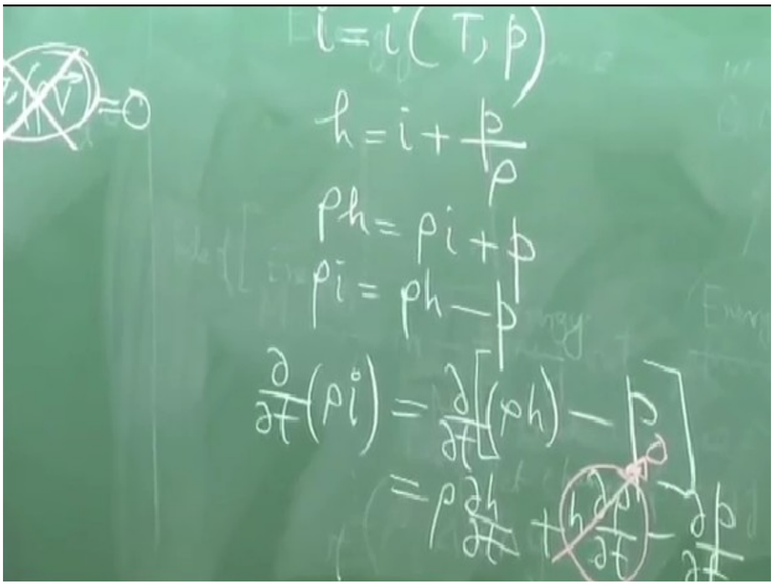
So, we are mostly concerned about simple compressible substances that is number one, number two is we are concerned about something which is a pure substance so a pure substance is a substance which is chemically homogeneous. So simple compressible substance and pure substance, these types of substances if you consider then you require two independent intensive thermodynamic properties to describe the state of the system.

This is known as state postulate in thermodynamics. So basically, you require two independent properties. So, can we write those two independent properties as temperature and pressure, so what we are trying to say is that if we have a simple compressible pure substance we can describe any intensive thermodynamic property in terms of two independent intensive thermodynamic properties.

So, can those two properties be temperature and pressure, yes or no, not always necessary why the requirement is that these two properties have to be independent. Can you describe a situation in thermodynamics when the temperature and pressure are not independent; they are dependent on each other, change of phase. Let us say that liquid water is getting converted into water vapour.

So, the phase change temperature and phase change pressure are dependent on each other. So, when we are claiming that we are writing internal energy as a function of these two, we are precluding the case of phase change. So, we are not bringing the case of phase change in the analysis.

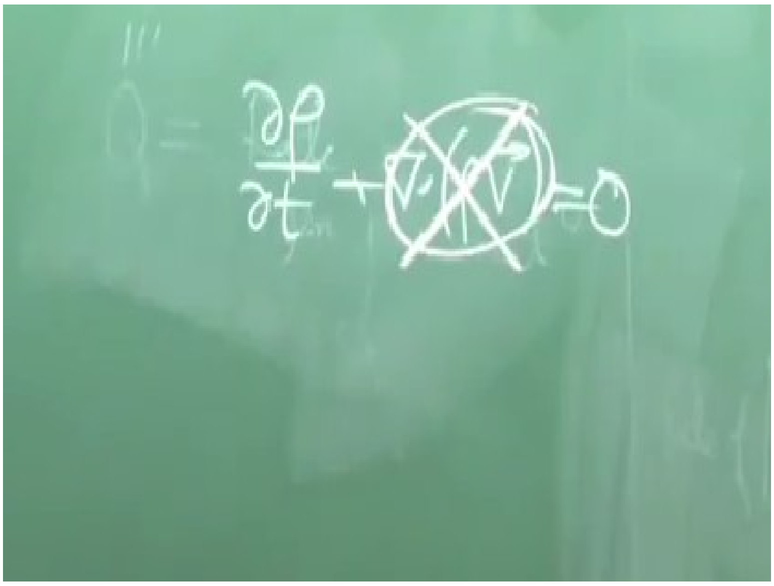
(Refer Slide Time: 47:41)



So, the internal energy we are writing as a function of temperature and pressure with an understanding that we are not addressing the problems of phase change. So sometimes, instead of internal energy we express it in terms of enthalpy so the enthalpy is, this is like h is equal to u + pv. The specific volume is one by density instead of u the symbol is i okay? So, Rho h is equal to Rho y + p.

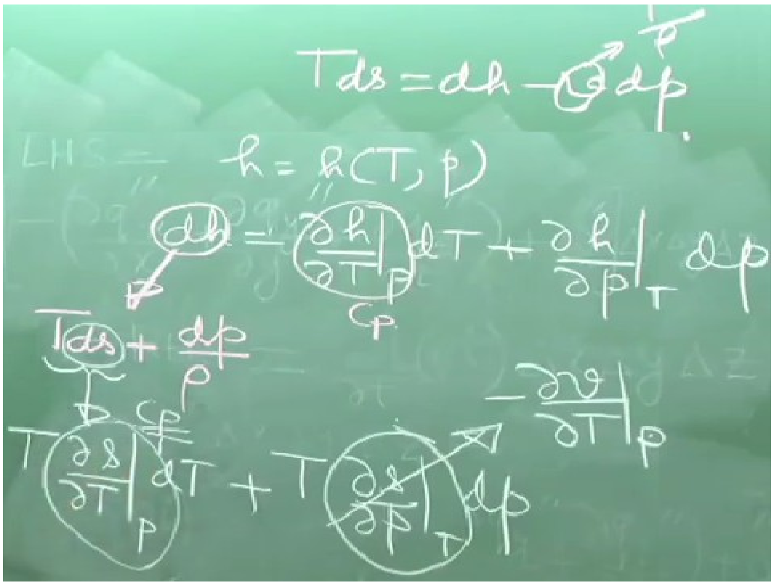
So, Rho i is equal to Rho h - p. so we are basically interested to simplify this term. So, what is this del del t of Rho i that is del del t of Rho h - p all right? Now I will ask you a very simple question, can you tell what is the value of this term, remember we are addressing the case of pure heat conduction without fluid flow.

(Refer Slide Time: 49:58)



So just think of the continuity equation in fluid mechanics. This is continuity equation in fluid mechanics right. So, we are thinking of conduction that means pure conduction. That means this term is not there. So, this must be equal to zero.

(Refer Slide Time: 51:05)



Now we will express this enthalpy as a function of pressure and temperature instead of internal energy we are writing h as a function of temperature and pressure. So, we can write dh, basic rule of partial derivatives. The total change is sum total of the partial change due to change in temperature + partial change due to change in question. By definition you know this is what cp right.

This is the definition of cp. Now we can express these by using one of the Tds relationships. So, in thermodynamics, you have encountered this tds relationship right. Tds equal to dh -

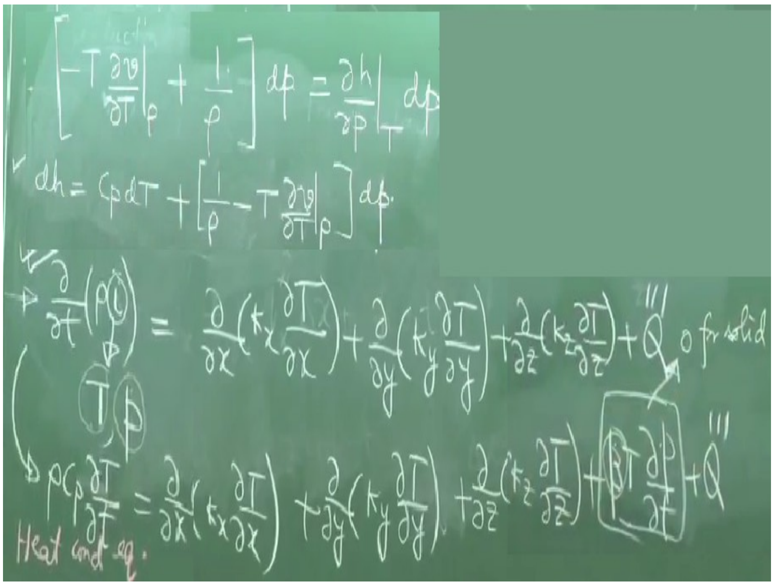
vdp. This v is one by the density. So, in place of dh you can write T ds + dp by Rho. Now you can see that right-hand side is expressed in terms of dt, dp, left hand side there is one dp and there is a ds.

So, we can express ds also in terms of dt and dp because we can write entropy as a function of two measureable parameters t and p. So, this will be, now this you can write in terms of other measureable parameters. See this is not a measureable parameter, entropy is not a measureable parameter. So, you can write it in terms of a measureable parameter using this. This is one of the four Maxwell's relationships in thermodynamics.

Tds using the Tds relationships you can derive this, you must have done this in first year physics or chemistry, then later on in thermodynamics okay. T into - del v del t at constant pressure. Now if you compare this with this, so left hand side this has dt and right-hand side this as dt then what is this, this is cp, this is t into this. So, this is cp by T right. The left-hand side and right-hand side coefficient of dT must be the same right you follow this right.

This is cp, this is T into this, so this also should be cp. So, this must cp by T okay.

(Refer Slide Time: 55:44)



$$\left[-T\frac{\partial v}{\partial T}\Big|_p + \frac{1}{\rho}\right]dp = \frac{\partial h}{\partial p}\Big|_T dp$$

$$dh = C_p\,dT + \left[\frac{1}{\rho} - T\frac{\partial v}{\partial T}\Big|_p\right]dp$$

$$\frac{\partial}{\partial t}(\rho h) = \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + \dot{Q}''' \quad \text{0 for solid}$$

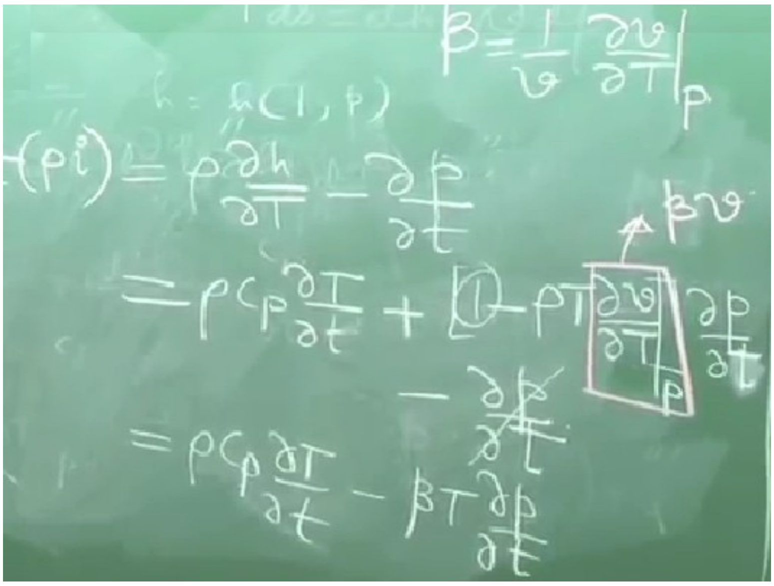
$$T \quad p$$

$$\rho C_p\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + \boxed{\beta T\frac{dp}{dt}} + \dot{Q}'''$$

Heat cond eq.

Now we can write, if you now equate the coefficients of dp, so if you equate the coefficients of dp on the left-hand side, the coefficient of dp is here - t del v del t then + one by Rho and the right-hand side, this is the coefficient of dp right. So, you can write dh equal to cp dT, this is the contribution due to pressure. This is the total change in enthalpy, this is the change in enthalpy due to temperature and this is the change due to pressure.

(Refer Slide Time: 58:08)



$$\beta = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right|_p$$

$$h = h(T, p)$$

$$(\rho i) = \rho \frac{\partial h}{\partial t} - \frac{\partial p}{\partial t}$$

$$= \rho c_p \frac{\partial T}{\partial t} + \left[1 - \rho T \boxed{\left.\frac{\partial v}{\partial T}\right|_p}\right] \frac{\partial p}{\partial t} - \cancel{\frac{\partial p}{\partial t}} \quad \beta v$$

$$= \rho c_p \frac{\partial T}{\partial t} - \beta T \frac{\partial p}{\partial t}$$

Now look into this expression, so Rho del h del t, we will use this term. So, the first term will be Rho cp. What we have done, we have basically within this as delta h is equal to cp delta T + this into delta p divided both sides by delta t and taken the limit as delta t tends to zero. So that will give rise to this equation okay. This is t time not temperature. Now this first term will get cancelled with this term right. This is del p del t, this is - del p del t.

What is the second term, this, remember, this is del v del t at constant pressure. You have a term which is volumetric expansion coefficient beta. Volumetric expansion coefficient is change in volume per unit volume for each degree change in temperature. That is what is mathematically written in this form. So, you can write this as beta into v where this beta, what is the name of this beta, volumetric expansion coefficient. So, this will be Rho cp okay.

So, this equation now we can write this as. So, what we have done, we have replaced this del del t of Rho i with this expression okay. In place of del t of Rho i we have written Rho cp del t del t - beta t del p del t and that beta t del p del t we have brought in a right-hand side. So, this equation is a general equation for heat conduction. Now typically in undergraduate texts we are interested for heat conduction in a solid and for solids the volumetric expansion coefficient this is very very small.

Not only that you do not have a pressure like quantity varying with time within a solid. So, for all practical purposes for a heat conduction within the solid this term goes away. Typically, when you see derivations of this equation in any book you will find this term is not there but we should not presume that and we should start from a consideration that yes internal energy or enthalpy could themselves vary with both temperature and pressure.

Because solids are not pressure sensitive so eventually it boils down to only temperature dependents and not pressure dependents. So, for heat conduction within a solid you are basically getting this equation without this term and this is known as the heat conduction equation. . So, this equation involves temperature as a function of position and time and this is in terms of partial differential equation theory.

This is like a initial boundary value problem where you prescribe the condition at time equal to zero and based on that initial condition the solution will evolve as a function of x, y and z depending on the boundary conditions. So, we need to discuss about what are the possible boundary condition associated with this equation and then we will work out a few very simple problems to illustrate the use of this equation.

I will not show you how to solve this equation because that will be the agenda of the subsequent lectures but I will just show you how to simply use this equation to address heat conduction problems.