Conduction and Convection Heat Transfer

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Lecture - 49
Internal Force Convection – V

In the previous lecture we were discussing about the case of thermally fully developed flow in a parallel plate channel with constant wall heat flux. Now, we will see the consequence of constant wall temperature boundary condition for that problem.

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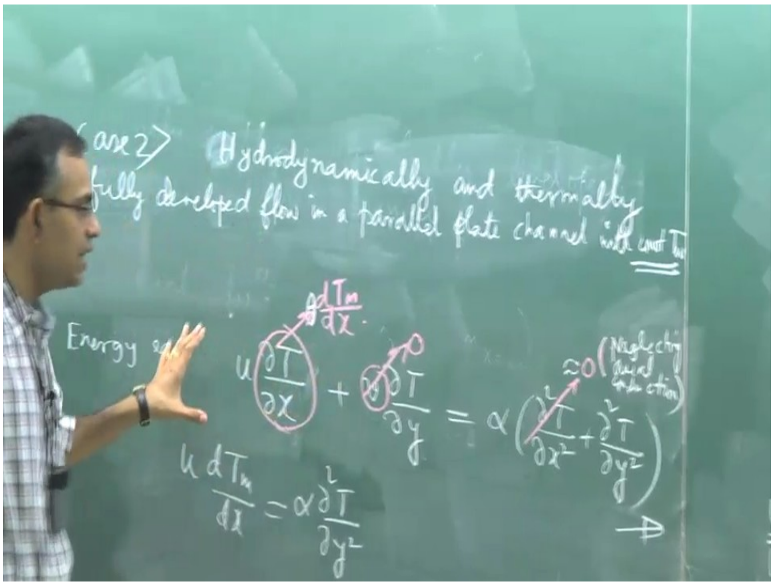

So, case 2, hydrodynamically and thermally fully developed flow in a parallel plate channel with constant T wall. So, what I have done is, I have kept in the board the derivation for the constant wall heat flux because I want to show you the contrast between that derivation and this derivation. So, wherever the things are the same as the previous case I will not erase and wherever of the things which are different from the previous case of constant wall heat flux, we will erase that.

So, we will start with the energy equation u del T del x. What is del T del x? for constant wall temperature we derive in one of our previous lectures. What is that? Theta into d T m dx for constant wall heat flux, it is d T m dx; for constant wall temperature, it is theta into d T m dx, ok. So, this, here you have for this, this is 0. What about this one del 2 T del x 2. For constant wall heat flux, thermally fully developed flow del T del x is a constant, but for constant wall temperature, del T del x is not a constant.

Therefore, this is not identically 0 for constant wall temperature. We can at the most make an approximation that this is approximately 0. When we make this us approximately 0 what is the physics that we are considering what does this term represent, this represent axial conduction, conduction along x. So, this means we can assume this to be approximately 0 neglecting axial conduction.

So, this is an assumption for constant wall heat flux that was identically 0. For constant wall temperature, this is not identically 0, but this may be approximated to be 0 by neglecting axial conduction in comparison to axial advection. So, you can see that in the governing equation the sole difference coming out is with d T m d x, there is a multiplier of theta. ok.

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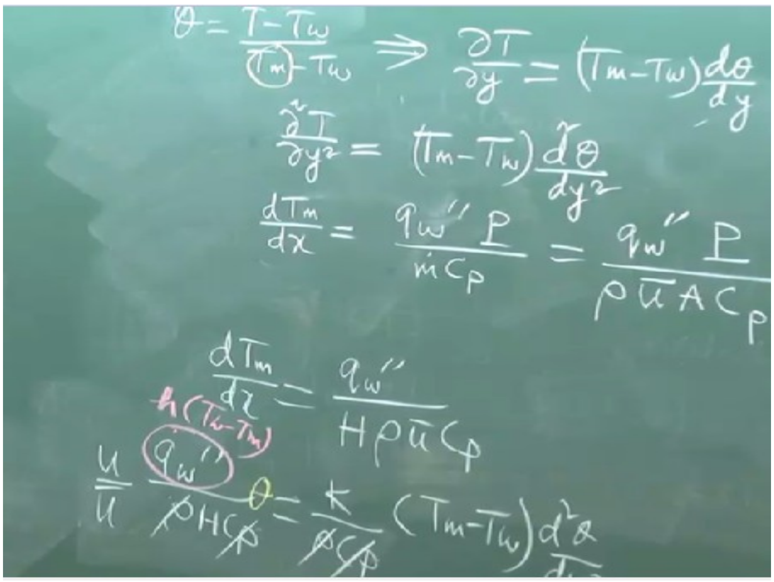

So, now let us look in to this derivation, so this does not depend on whether it is constant wall heat flux or constant wall temperature, d T m d x expression this also does not depend on whether it is constant wall heat flux or constant wall temperature. So, in the governing equation when we have substituted d T m d x. Now in this governing equation you have an extra theta multiplier with d T m d x. So, with the previous term now we will have a multiplier of theta, right.

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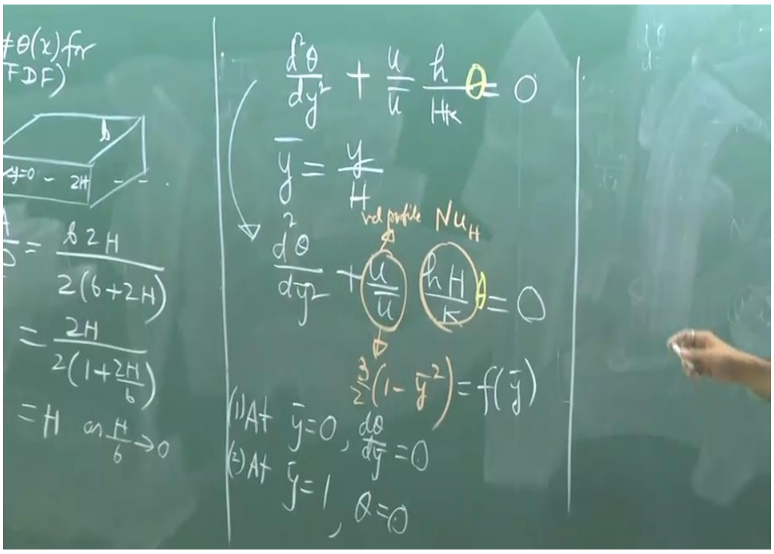

Whatever was in the previous case that was d T m d x, now it will be theta d T m d x. So, here it will be theta, so this will be theta. So, the governing equation instead of d 2 theta d y 2 plus Nusselt number into u by u average, it will be d 2 theta d y 2 plus Nusselt number into theta into u by u average. So, when this theta multiplier has come but this theta multiplier will spoil the ability of this problem to yield an analytical solution.

So, this problem can no more be addressed analytically until (()) (06:05) you have a very special case where u by u average is 1 that is plug flow. But for a general flow, you cannot solve this problem analytically in fact this kind of problem is an Eigenvalue differential equation, Eigenvalue problem in ordinary differential equation. So, you cannot solve this analytically, so I will give you an outline of how to solve this numerically.

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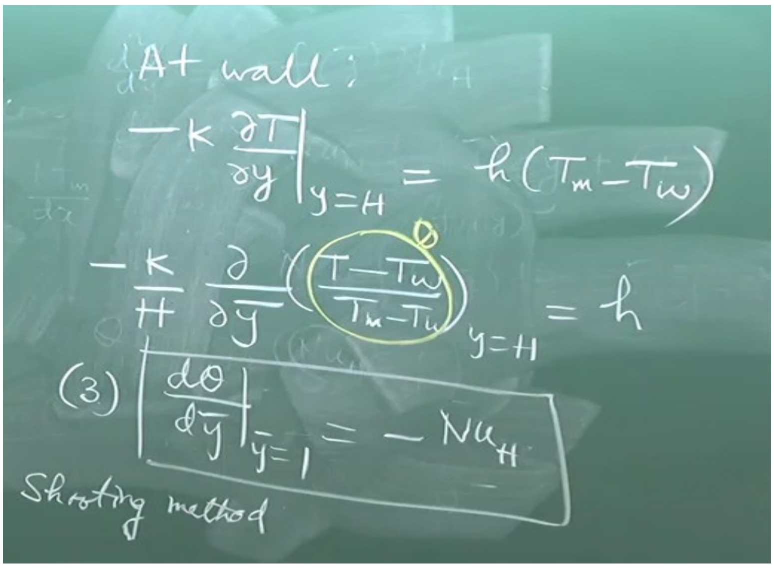

So, you have these 2 boundary conditions on the top of that at the wall, so you have this is the boundary condition at the wall, so you can write minus K del del y bar by H. So, what is this term in the bracket? This is theta, right. Can we write instead of partial derivative and ordinary derivative here? Yes, we can write because for thermally fully developed flow, theta is not a function of x. Theta is a function of y only.

So, we can write d theta d y nondimensional at y equal to 1 is equal to minus Nusselt number based on H. This is an additional constant. So, we have a constant one, we have a constant two, this a third constant. So, we can solve this problem by using a method numerical method called as shooting method. So, shooting method is a method were what you do is, so from the name it suggests that you have a target.

You shoot the target, you try to hit the target and based on your error in hitting the target, you make a revised calculation. So, what you do is that, you convert this second order ODE into a coupled system of two first orders ODE, ok. So, I am giving you the outline and your job will be to implement this in Metlab, this will be your one of the home works. See all problems cannot be analytically solved.

So, you must learn to write small programs at least to solve problems numerically. That will give you a good grasp on basic numerical method for solving the heat transfer problem. So, I will give you the outline, I will tell you what to do, but you have to implement it in the computer. So, what you do is that you write this equation as a coupled system of two first order ODEs. What are will be the variables? Theta and d theta d y, ok.

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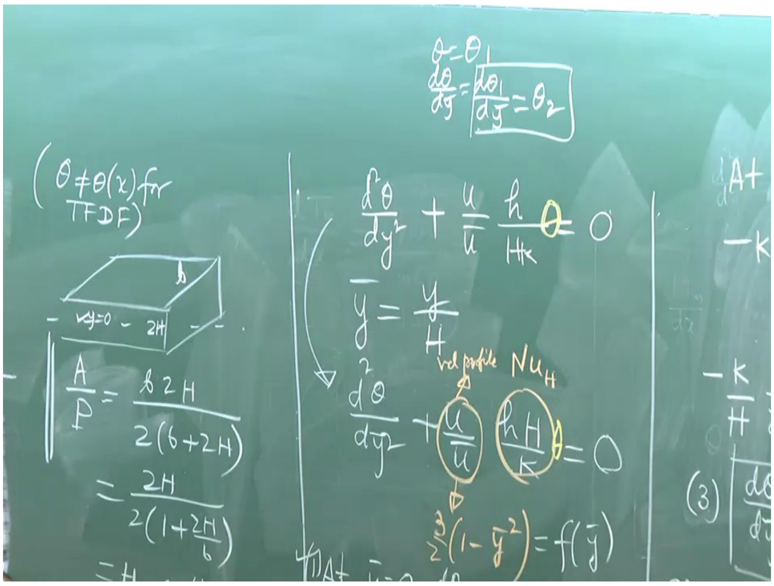

So, you can assume that theta is equal to theta 1 and d theta d y that is d theta 1 d y is equal to theta 2. So, you will get theta 2 as a function of theta 1 and another equation this one. So, these are the two coupled first order. One is d theta 2 d y plus f into Nusselt number into theta 1 equal to 0, right and the other equation is d theta 1 d y is equal to theta 2.

So, you will get a coupled equation 2 equations with theta 1 and theta 2, but two first order equations. So, this is a trick of reducing (()) (10:36) order equation into coupled n number of first order equation. Here, we are reducing the second order equation to coupled system of two first order equation. Then, what are the boundary conditions at y equal to 0. See, if we have both the conditions for theta 1 and theta 2 at y equal to 0, then we call it an initial value problem.

That is, we know at the start of the domain, what is theta 1 and what is theta 2. So, at the start of the domain, we can use the start of the domain at y equal to 1 also. So, at the start of the domain at y equal to 1. Let us say y equal to 1 that is the wall. We considered the start of the domain, so theta 1 is 0 and at y equal to 1, this is theta 2 at y equal to 1 is minus Nusselt number, but you do not know Nusselt number.

So, you can give the boundary condition, if you guess the Nusselt number, right. So, the first step is so I am writing the broad steps.

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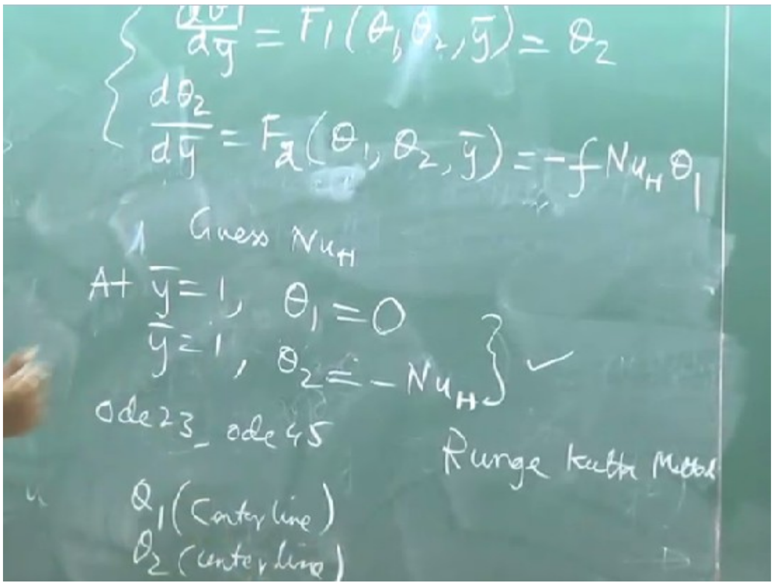

So, you have d theta 2 d y bar as the function of theta 1 theta 2, y bar and d theta. What is this f 1? This is theta 2 and what is this? Minus f into Nusselt number into theta 1, right. So, these two equations. Now you use the boundary condition, these are the two coupled equations. Now guess Nusselt number. Once you guess the Nusselt number then what you will get, at y equal to 1.

You have boundary condition theta 1 equal to 0 and at y equal to 1 theta 2 is d theta d y is minus Nusselt number. So, these 2 now will become known if you guess Nusselt number. So, with this you can use the initial value problem solver. In Matlab, you have some built in functions like ODE23, ODE45, this kind of functions, built in functions in Matlab, you can use to solve for theta 1 and theta 2.

Then, using these so these are basically using a method known as Runge–Kutta method. This is the method of numerical solution of initial value problems. ok. So, once you calculate this, then you can calculate what is theta 1 at center line and theta 2 at center line this you can calculate from wall, you come to the center line and you calculate theta 1 at center line and theta 2 at center line.

So, these initial value problems are marching problems that is in a particular direction you march. The direction may be in time, the direction may also be physical direction I mean, x or y direction. So, here in the direction from the wall to the center line we are moving, so at the center line you can calculate theta 1 and theta 2 starting from the wall you march. Now, theta 2 at the center line you calculate.

And what do you expect, see these boundary conditions is that at the center line you expect theta 2 to be 0. So, you ask now the question is it 0. If it is 0, then you have guess the Nusselt number correctly, but if it is not 0 you have to do a define guess based on what deviation from 0 you have got from this. So, that is what is shooting method that is your target is to hit the bull's eye. The bull's eye is 0.

So, if you do not hit the bull's eye you look into the deviation from the bull's eye and then retake your shooting so then if not correct the Nusselt number guess and (()) (0:16:49). So, in this way you go on correcting the Nusselt number guess till you (()) (16:58) to a convert solution that whatever guess of the Nusselt number gives theta 2 equal to 0 at the center line that is the convert value of the Nusselt number. So, that is the procedure for solving this problem.

There is no short cut way of doing this you have to write the program by yourself you have to solve and I give you the answer. You have to check the answer. So, let be give you the answer for this. So, the Nusselt number is 7.54. That is the answer for this problem. So, you write a problem and then get then value and check whether you get this. Of course, now you know the answer, so you can play a trick with me by giving these are initial guess.

And then in one step without iteration, you can get the answer. But I mean all of us understand that life is not as simple as this.

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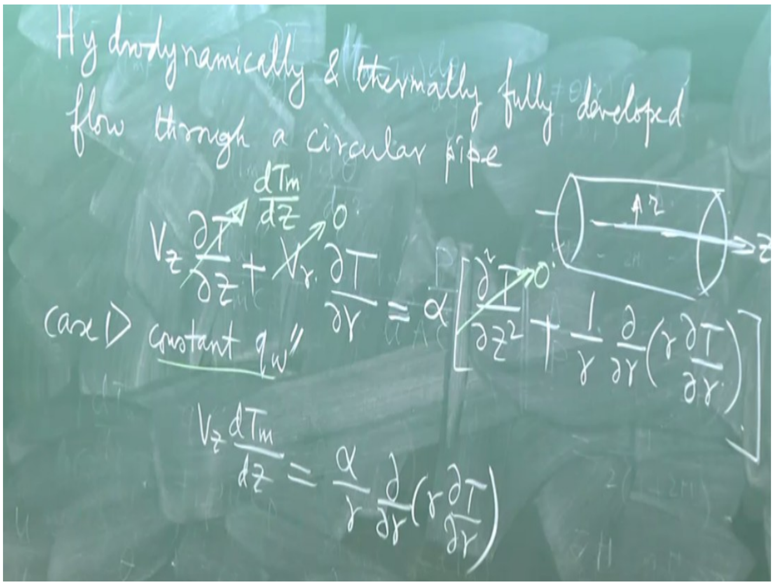

So, anyway now move onto the other geometric which is the circular pipe. Hydrodynamically and thermally fully developed flow through a circular pipe. So, we have now seen the parallel plate channel situation just now we have seen, now will see the circular pipe. So, the concept wise, there is absolutely no difference only the mathematics wise, it will be little bit different which we will see that what different it is

And what is the consequences in the solution but physical concept wise that is why first I started with the parallel plate channel that it maybe a very simple physical situation, but it gives you the physical understanding of the problem completely. Now for flow through a circular pipe, let us start writing the governing equation. So, u del T del x in place of u, what will be this? So, the circular pipe, let us say this is the pipe with this as the z axis and this as the r axis, ok.

So, in place u, it will be V z del T del x will be del T del z plus in place V, it will be V r. So, the only difference is that in the cylindrical coordinate system, Laplacian will take these form that is when you consider the derivative with respect to r, so this is 1 by r del del r of r del T del r that del square term will involve this. If it was a spherical system, it would have been r square. So where from this come r, r square all these?

Because in the cylindrical system the elemental area involves 2 pi r d r and in the spherical system, it is proportional to r square 4 pi r square. So, the elemental volume is 4 pi r square dr for the spherical system and here 2 pi r d r into L. So, when you divide the all the terms per unit volume that 2 pi r becomes 1 pi r and 4 pi r square becomes 1 pi r square that is all this terms come.

So, it just a matter of changing from one coordinate system to the other, it has I mean no special significant. So, case 1, constant wall heat flux which will work out and constant wall temperature I give you the as the homework. Just like the previous case you have to use the shooting method to solve this problem, but I will work out the constant wall heat flux case. So, constant wall heat flux del T del z will become d T m d z, right.

What will be V r, V r will be 0 for fully developed flow. What will be this just like del T del x is equal constant for thermally fully developed flow with constant wall heat flux in parallel plate channels. Similarly, del T del z equal to constant. So, this will be 0. So, you have left with z d T m d z what is d T m d z. We have derived it for a general cross section, so we can use it for a circular pipe. So, what was that q double dash p by m dot C p.

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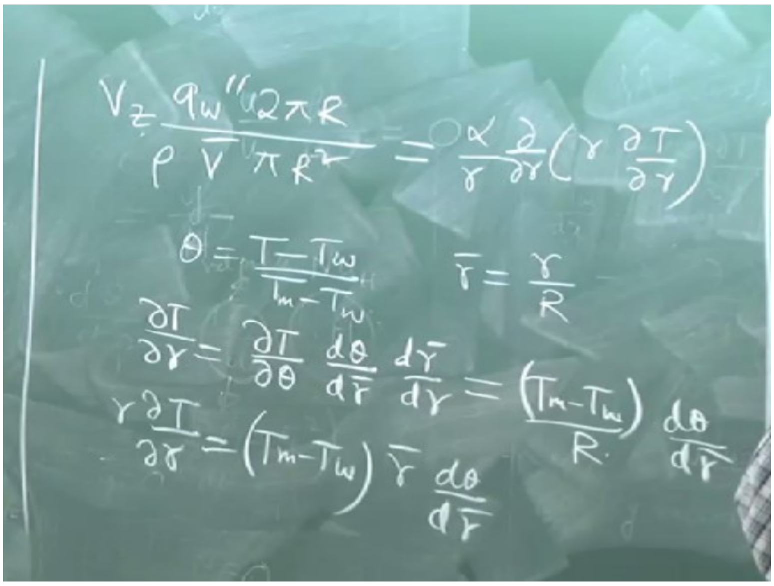

So, you can write this as V z into q double dash. P is 2 pi r perimeter of a circle for is 2 pi r divided by m not is rho into V average into A pi r square is equal to alpha into 1 by r del del r of r del T del r. Now, we will define theta is equal to T minus T wall by T m minus T wall and nondimensional r is equal to r by R. Where R is the radius of the pipe.