Conduction and Convection Heat Transfer
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Lecture - 48
Internal Force Convection – IV

So, we have discussed about the situations of the constant wall heat flux and constant wall temperature somewhat qualitatively that is how the wall temperature and how the bulk mean temperature varies with x, but question is what is the rate of heat transfer that is what is the Nusselt number. So, we need to discuss about what is Nusselt number for thermally fully developed flow.

So, there may be situations when the flow is hydrodynamically fully developed, but it is not thermally fully developed like let me give you an example.

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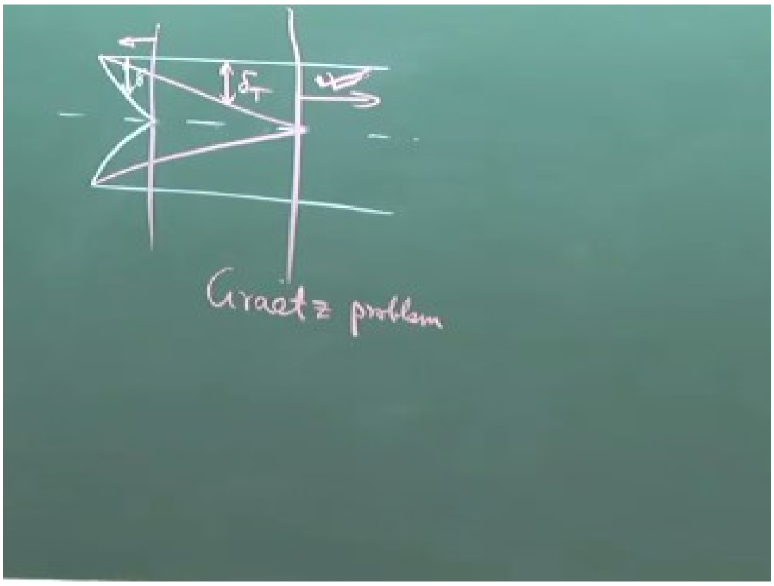

So, this is hydrodynamic boundary length and this is thermal boundary length. So, if you have a fluid say of value of high Prandtl number, then what will happen, this delta will be significantly more as compared to delta T, so the hydrodynamic boundary layer will grow very fast. So, the hydrodynamic boundary layers will merge here and the flow will become hydrodynamically fully developed.

So, from here to here, there is a region when the flow is hydrodynamically fully developed, but not thermally fully developed, right. So, that kind of problem is known as Graetz problem. This kind of problem is actually in the (()) (33:32) advanced level of convective heat transfer. So, we will not come into that here. There is another type of problem where the flow may be both hydrodynamically and thermally developing like this region.

But in this particular course, we will be concentrating on this region where the flow is both hydrodynamically and thermally fully developed and we will evaluate the Nussselt number for that cases.

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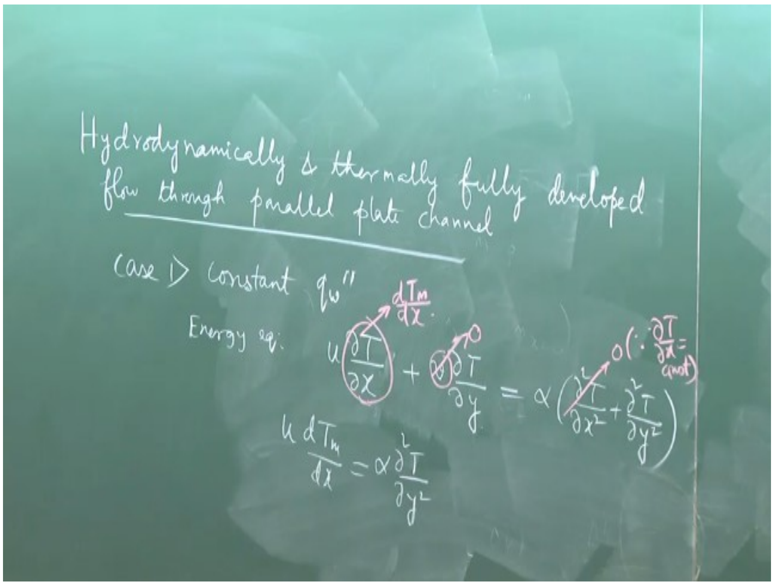

So, hydrodynamically and thermally fully developed flow through parallel plate channel. Case 1, constant wall heat flux. So, what answer can we expect for these types of problems. What are you assured of when you are solving a problem of thermally fully developed flow. We are interested about the Nusselt number. The Nusselt number will be a constant that is what is expected.

So, if we do not come up with the constant Nusselt number that means something is wrong in our analysis or approach. So, the Nusselt number will be a constant and our objective will to evaluate the constant value. So, we have to keep in mind that see what earlier cases we have studied for evaluation of Nusselt number flow over flat plate. The Nusselt number was of the form of Reynolds number to the power something say half into Prandtl number to the power something say half or one third.

This kind of expressions we have seen. So, Reynolds number to the power m into Prandtl number to the power n. In general, for force convection, Nusselt number is actually of the form Reynolds number to the power m into Prandtl number to the power n, but for hydrodynamically and thermally fully developed flow for internal force convection that is the special case for Nusselt number becomes a constant.

But in general, if you have a situation, even the situation changes if the flow from laminar becomes turbulent, so then I mean you cannot workout those problems analytically you have to deal with sudden correlations based on experimental studies which engineers used for designing a system with the turbulent flow. I will come to 1 or 2 such expressions, but before that remember here we are assuming laminar flow, so constant wall heat flux.

Now let us write the governing equation, the energy equation. This is our governing equation. Now tell what will be the simplification. So, let us write it with different color may be, so in place of delta T delta x for constant wall heat flux, we can write d T m d x, which is the constant, whatever this term, v is equal to 0 for hydrodynamically fully developed flow, so this will be 0. What about this term in the right-hand side, delta T delta x, delta T delta x is what?

Delta T delta x is equal to constant for thermally fully developed flow with constant wall heat flux. Because delta T delta x is constant, delta 2 T delta x 2 is 0. So, you can write u d T m d x is equal to alpha delta 2 T delta y 2. Now, we will change these variables from T to theta, the nondimensional temperature which does not vary with x anymore for thermally fully developed flow.

Why we want to change the variable, because T is the function of both x and y, but for thermally fully developed flow theta is only a function of y, but not a function of x. So, theta is T minus T wall by T m minus T wall. So, you can write delta T delta y, remember T wall minus T m is the functions of x, so with respect to y derivative those are like constants.

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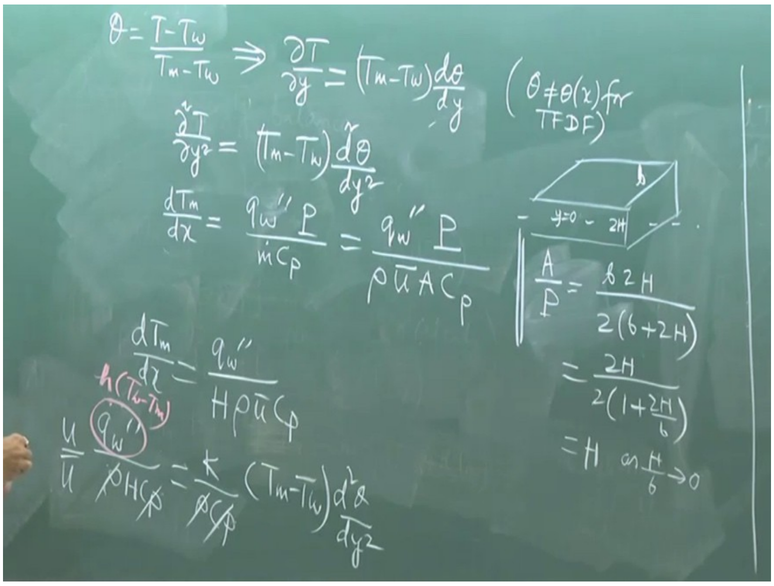

So, similarly the second derivative and what is d T m d x, d T m d x is what is that expression q double dash P by m dot C p, so q double dash in place of P, let us draw a parallel plate channel. Let us say that the width is b and height is 2 H with the center line is y equal to 0. So, what is the perimeter, so before that let us write another step. What is m dot, m dot is rho u average into A into C p by perimeter.

So basically, we have to calculate perimeter by area, so perimeter by area what is perimeter or let us calculate area by perimeter whatever. We will invert that area by perimeter what is that b into 2 H by 2 into b plus 2 H, right. Now, we have to consider a limit value calculating the area by perimeter what is that limit, limit is that h by b tends to 0 because by definition the parallel plate channel the width is infinitely large.

So, what we will do is we will divide both numerator and denominator by b, so this becomes 2 H by 2 into 1 plus 2 H by b, so this will become h as h by b tends to 0, right. So, when h by b tends to 0, this term is zero, this term is 1 and this 2 get cancelled, so it becomes H. So, you can write d T m d x is equal to q double prime into perimeter by area is 1 by h rho u average C p. So, u d T m d x is u by u average.

So, we are writing this equation, this differential equation u d T m d x is u by u average into q double prime by rho H C p is equal to alpha, alpha is K by rho C p into T m minus T wall into d 2 theta d y 2, ok. So, then rho C p gets cancelled. In place of Q wall what we can write Q wall is h into what, h into T wall minus T m.

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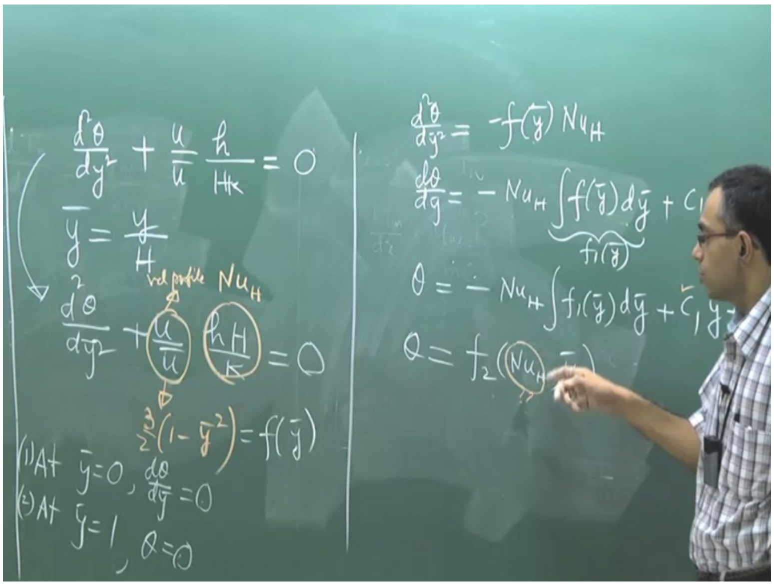

So, we can write d 2 theta d y 2 plus u by u average into h by H K equal to 0, right. Now you can non-dimensionalize like this, right. You can define a new parameter y bar as y by H, ok. So, if you define a new parameter y bar as y by H then this will be d 2 theta d y 2 plus u by u bar h H by K is equal to 0. So, there will be a 1 by H square here that H square coming here will make it h H by K.

So, in a non-dimensional form this becomes what. This is a Nusselt number based on H and what is this, this is the velocity profile. This is where fluid mechanics comes into the heat transfer calculation. So, the answer how theta will vary with y will depend on what will depend on the velocity profile. Now there are many types of velocity profiles which are possible like there may be a case when molten metal is flowing very slowly.

So, in that case the entire velocity profile may be almost uniform and that is called as the plug flow or slug flow. So, if have a plug flow or a slug flow that means uniform velocity profile then what is u by u average 1, right. So that is a very special case, very simple algebra to deal with. I am dot working it out here, but please take it as a homework and complete it by yourself that consider a plug flow with u by u average is equal to1, then complete the remaining derivation.

I will do the derivation for a more involved scenario when u by u average is not equal to 1, but it is a fully developed pressure driven flow that is Poiseuille flow. So, for a plane Poiseuille flow what is the velocity profile u by u average is equal to 3 by 2 into 1 minus y square by 8 square this was derived by Prof. Som when he was discussing about the exact solution of Navier-Stokes equation.

So, you substitute that here, so this is a like from the fully developed flow Navier-Stokes equation you can derive. There is no problem associated with this. So, now what are the boundary conditions, this is y bar, this is non-dimensional y. What are the boundary conditions at y bar equal to 0, what is the boundary condition? Y bar is equal to 0 is the center line, what is the boundary condition for theta at the center line.

Top wall and bottom wall are having symmetric boundary condition, so d theta d y equal to 0 at y equal to 0, it is the center line symmetry and at nondimensional y is equal to 1 that is the wall, what is theta. Remember the definition of theta, T minus T wall by T m minus T wall, so theta is 0. So, with this you have d 2 theta d y 2 is equal to some function of y. Let us say this is f y bar into Nusselt number with minus sign.

So, d theta d y is equal to minus Nusselt number, integral of f y d y bar plus some constant C 1, so when you integrate you will get theta is equal to, so this is let us say f 1 y, right. I have just written it generically. This integration is very easy because this is a simple polynomial, I do not waste time by doing the integration. I want to give you the frame work in which you can just put the numbers and values.

So, you can evaluate the constants C1 and C2 by using this 2 boundary conditions, but does this solve the problem. You have an expression for theta is equal to a function of some function f 2 Nusselt number and y. These are all nondimensional y, so make this as y bar, but this does not tell you exquisitely theta as a function of y because you do not still no what is Nusselt number, what is a value of the Nusselt number?

In fact, obtaining that is the objective and remember that although I have written it as a function of Nusselt number and y, actually Nusselt number is separated from the function of y. It is Nusselt number into some function of y. I mean it is not mixed with y. Nusselt number is outside this integration. So, it will come out to be Nusselt number into some function of y. Now you might argue that yes, we have used 2 boundary conditions for a second order differential equation.

So, what makes the situation that still the problem is not completely solved. One of the reasons is that actually this boundary condition theta equal to 0 at wall is not actually giving you any new information. Because theta by definition is T minus T wall by T m minus T wall, so at wall theta will always be 0 that is the definition of theta, but it does not depend on whether it is constant wall temperature or constant wall heat flux whatever.

It is just a simple English sentence T equal to T wall at wall, so nothing more than that. So, that is what that has been utilized here. So, we need to close this problem by putting some additional constraints what is that additional constraint that additional constraint is the definition of bulk mean temperature which we have not yet used. We have not yet used the definition of the bulk mean temperature.

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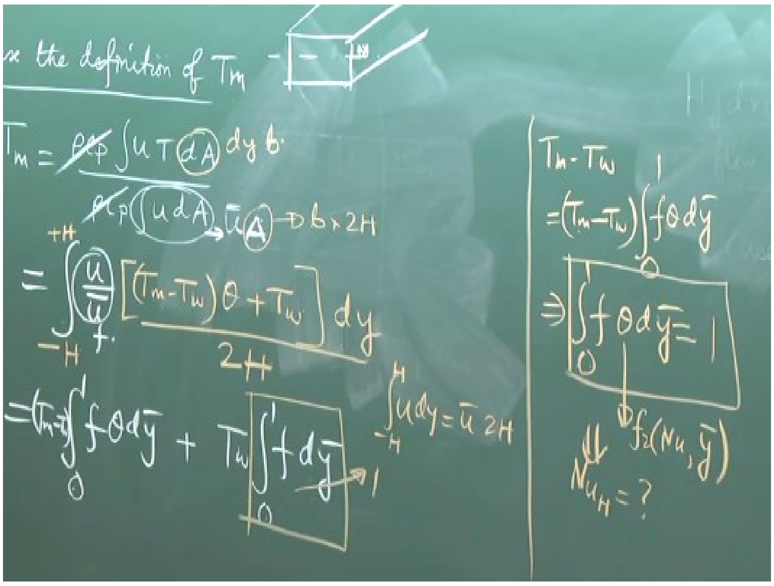

So, now use the definition of T m. The rho C p is constant for this problem. So, we cancel the rho C p part. So, we can write what is integral of u d A that is u average into A. So, we have integral of u by u average what is d A, d A is so if we have a parallel plate channel like this so what is d A, so at a distance y you take a small strip of a width d y. D A is d y into b, so d A is d y into b and A is b into 2 h. So, integral u by u average in place of p you can write T m minus T wall into theta plus T wall.

You can change the variable from y to y bar by putting this H within the derivative. So, it will become d y bar. This is the function f and integral from minus H to H is 2 into integral of 0 to H, because it is symmetric in the 2 sides. So, that 2 and this 2 will cancel. So, this will become integral of f theta into T m minus T wall d y plus T wall, right. We have absorbed the y by H within the derivative.

So, if that be the case then can you tell what is integral of f d y, nondimensional, see f is what f is the velocity profile. So, integral of f d y non-dimensionally it will become 1 from zero to 1, right. Because integral of u d y is u average into the height, so u by u average into integral of d y nondimensional is 1. So, this is from the condition that u integrally u d y is equal to u average into 2 H, so u by u average integral, this will become 1. So, this is equal to 1.

So, we can say that T m minus T wall is equal to T m minus T wall into integral of f theta d y that means integral of f theta d y equal to 1, this is the constraint. So, theta will be now you can write f 2 of Nusselt number and y bar. The functions f and f 2 are completely known, so this will tell you what is Nusselt number based on H. Now normally we can calculate the Nusselt number on the basis of H, but engineers refer to use a length scale which is called as hydraulic diameter.

So, what is hydraulic diameter, if have a circular pipe you have the actual diameter, but you do not have a circular pipe, there is nothing called a physical diameter, but something equivalent to the physical diameter is called as the hydraulic diameter. So, for as circular pipe what is the physical diameter.

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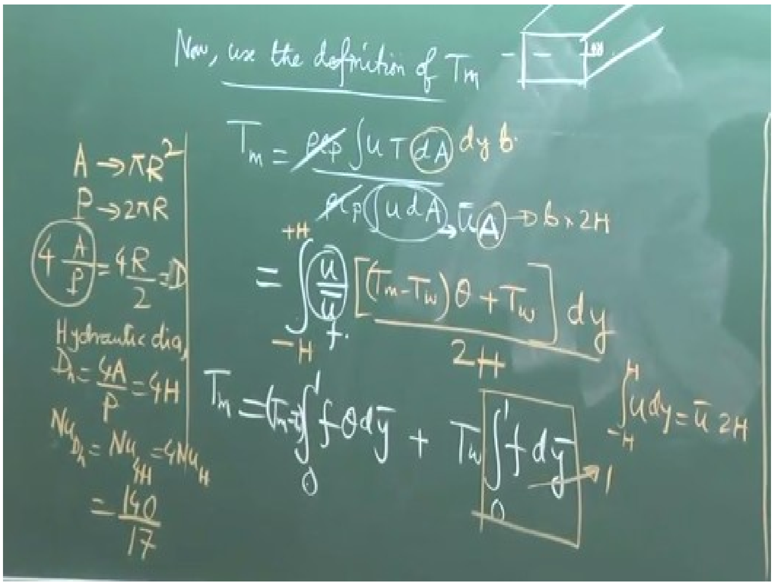

So, what is the area, area is pi r square. What is the perimeter 2 pi r, so what is area by perimeter? So how can you get diameter from here, multiply by 4, so these becomes diameter, ok. So, 4 into area by perimeter for a circular pipe is the actual diameter. For a noncircular geometry that is some effective diameter which is called hydraulic diameter. So, hydraulic diameter let us write it here is 4 into area by perimeter.

So, what will be the hydraulic diameter for a parallel plate channel the half height is H. Area by perimeter we have already calculated what was that H, so 4 area by perimeter is 4 H. So, the Nusselt number based on hydraulic diameter is the Nusselt number based on 4 H that is 4 times the Nusselt number based on H and if you evaluate it all these numbers I have given the outline, but if you now evaluate the value this value will come out to be 140 by 17.

So, this is your homework 140 by 17, okay. So, I have given the full frame work, in this frame work just you have to do the integrations, integral A f 1 f 2 all this and once you substitute, you will get the value of this, so we can see that the Nusselt number is the constant for thermally fully developed flow for this example, this is the value of the constant. We will stop here and we will continue in next lecture.