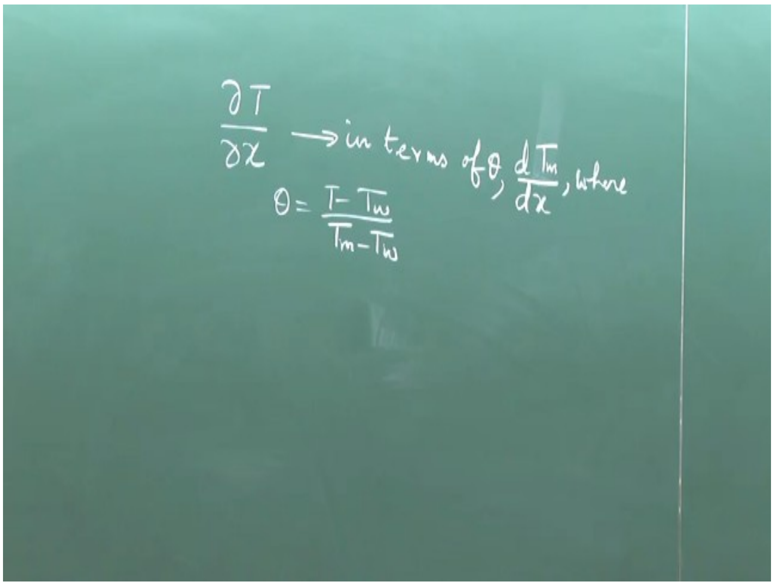
Conduction and Convection Heat Transfer
Prof. S.K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 47
Internal Force Convection – III

In the previous lecture, we were discussing about the concept of hydrodynamically and thermally fully developed flow. We discussed about the consequences of hydrodynamic and thermally fully developed flow in terms of expressing a non-dimensional temperature and a non-dimensional velocity. Now, we have seen the term delta T delta x can be expressed in terms of theta and d T m d x, where theta is T minus T wall by T m minus T wall.

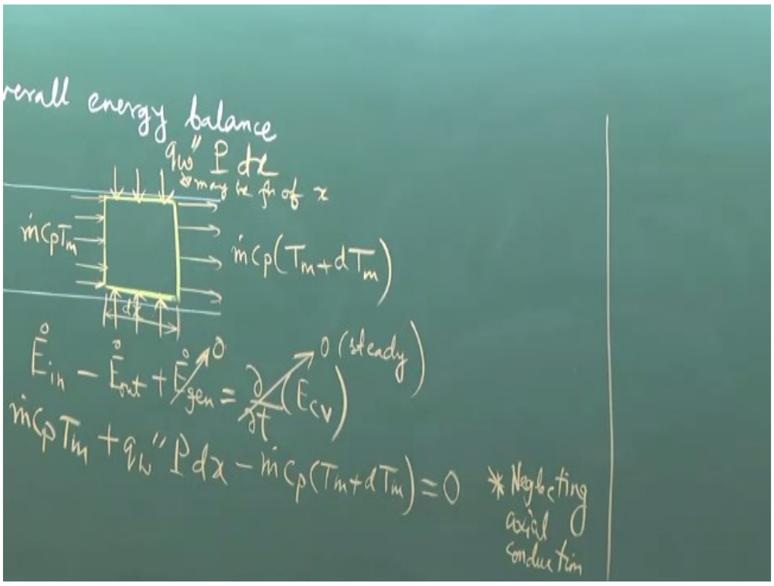
(Refer Slide Time: 00:54)



So, for constant wall temperature it is theta into d T m d x and for constant wall heat flux, it is just d T m d x without involving theta that much we discussed in the previous class. Now, irrespective of the boundary condition, this parameter is always involved. So, one thing we have been successful in reducing the dimensionality of the problem that when we are expressing delta T delta x in terms of d T m d x.

We are converting like a 2-dimensional or a 3-dimensional problem to (()) (02:37) 1-dimensional problem. But the question is how to calculate this d T m d x, so for that we will make an overall energy balance.

(Refer Slide Time: 02:52)



So, let us say that this is the channel and we take a control volume like this. Across this control volume, we are going to write an energy balance, so when we are going to write an energy balance, there is some energy that is entering. Let us say that this length is d x. There is some energy that is leaving, so what is the energy that is entering in terms of the bulk mean temperature.

The bulk mean temperature is an equivalent temperature which would have existed uniformly across the cross section to make the flow of the same energy as that of the actual case. So, if T m is the bulk mean temperature then what is the rate at which energy is transferred across this section, it is m dot into H, m dot into H is m dot into C p into T m. We are considering incompressible fluid and let us say this one is m dot C p T m plus d T m.

Then, we have heat transfer at the wall, what is the heat transfer at the wall? If the wall heat flux is q double prime this may be a constant or it may be a variable. Let us say that the wall heat flux is q double prime. So, what is the heat transfer rate across the wall. Let us say this is a circular pipe of radius r. So, if the wall heat flux is q double prime, what is the rate of heat transfer? q double prime into 2 pi r d x, right. 2 pi r d x is the surface area of the pipe.

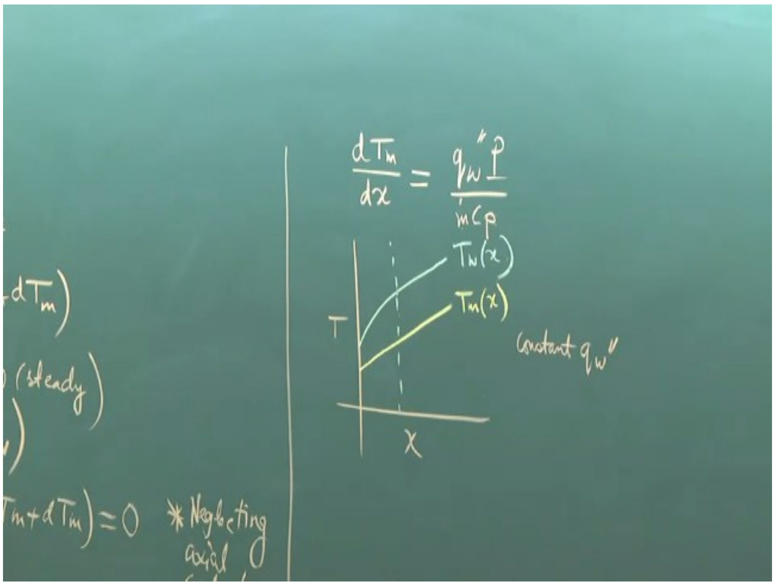
So, 2 pi r d x is nothing but perimeter into d x. So, to generalize it for any section, we will write it as q double prime into perimeter into d x. So, we can write for this control volume, rate of energy in minus rate of energy out plus rate of energy generated. So, this is zero because it is steady flow and steady state. Rate of energy generation is zero that we are not considering in this problem. If there is some rate of energy generation, we can accommodate this in this formulation very easily.

Rate of energy in is m dot C p T m plus q double prime Pd x. What is the rate of energy out m dot C p T m plus d T m. Now, this equation is not perfect, but it has an approximation. My question is what is that approximation that is there in this equation. This is not exactly correct. "Professor - student conversation starts" Yes, no, no, no, it is not taken as constant. "Professor - student conversation ends".

Even if it is a function of x it is true because we have taken a small element over which it will be constant. Q double dash, this may be function of x. So, in the axial direction we have considered that there is heat transfer due to fluid flow, but we have not considered that there is heat transfer due to axial conduction. So, we have neglected axial conduction, so ideally, we should have taken a heat flux minus K d T m d x here and minus K d T m d x at x plus d x here.

So, we had not taken any axial conduction. So, we have neglected axial conduction and that is valid in many practical problems where advection is much more dominating than axial conduction. So, this very importantly is valid neglecting axial conduction.

(Refer Slide Time: 09:33)



So now from this equation, you can write d T m d x is equal to q double dash P by m dot C p. Now, can you tell from here that d T m d x is constant if wall heat flux is constant, yes or no, yes. Because for steady flow rate is m dot is constant, we assume that the properties are constant then the perimeter of the cross section, it is the constant for the geometry that we are considering. So, if the wall heat flux is constant then the d T m d x is constant.

So, we had earlier shown that for the constant wall heat flux, delta T delta x is equal to d T m d x is equal to d T w d x is equal to constant, ok. But not for any general constant wall heat flux. For constant wall heat flux and thermally fully developed flow, ok. So now, we come to an inference that this is the constant for constant wall heat flux not just in the thermally fully developed flow region, but about the entire region.

Because this overall energy balance theory is very general it does not take into account whether it is thermally fully developed or not. It is just the simple energy balance using the definition of bulk mean temperature that is converting the multidimensional problem to 1 dimensional problem, but it does not take into account whether it is thermally fully developed or not. So, the conclusion is that even if the flow is not thermally fully developed for constant wall heat flux d T m d x is the constant, but this is still an approximation.

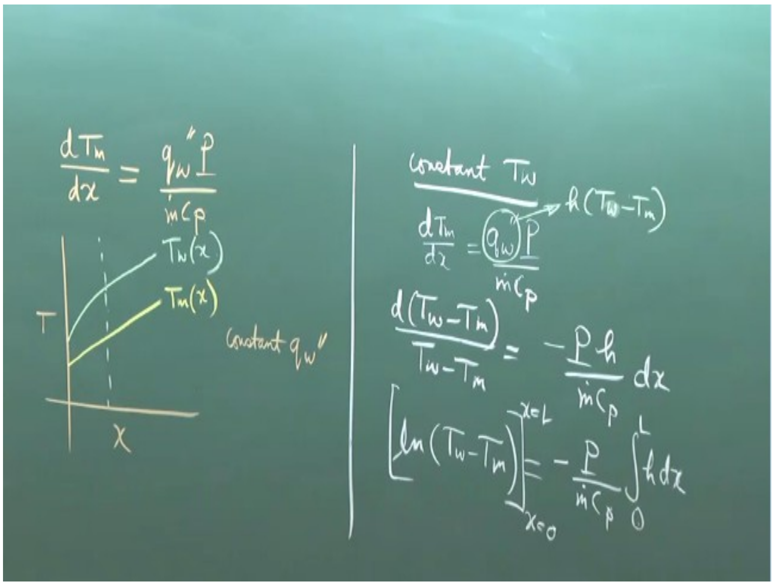
Because it has neglected axial conduction. If you do not neglect axial conduction that is not true. Whereas this is exactly true, does not matter whether axial conduction is there or not, ok. So, we can make a graph of say T versus x for constant wall heat flux. So, when we write plot T versus x, we essentially want to plot basically T m and T wall because we are now converting it to 1 dimensional problems where the functions of x are T m and T wall.

So, let us say that T wall is greater than T m now what is the graph of T m versus x, it will be a straight line because d T m d x is a constant assuming that axial conduction is negligible it is a single straight line throughout otherwise in the thermally developing region this may not be a constant, so it may be a curve and then in the thermally fully developed region, it will be a constant that formula.

So, let us make as sketch of this is T m versus x assuming that the wall is heated, now what will be the graph of T wall versus x assuming T wall is greater than T m. So first that graph should be above this if it is heated then how will the graph look like, see d T wall d x is a constant only if it is a thermally fully developed flow. So, let us say that the flow becomes thermally fully developed from here.

So, up to this it will be a curve beyond this it will be a straight line. What straight line, straight line parallel to this because d T m d x is equal to d T wall d x. So, the slopes of these 2 straight lines are equal in the thermally fully developed region. So, this is T wall as a function of x, ok.

(Refer Slide Time: 16:00)



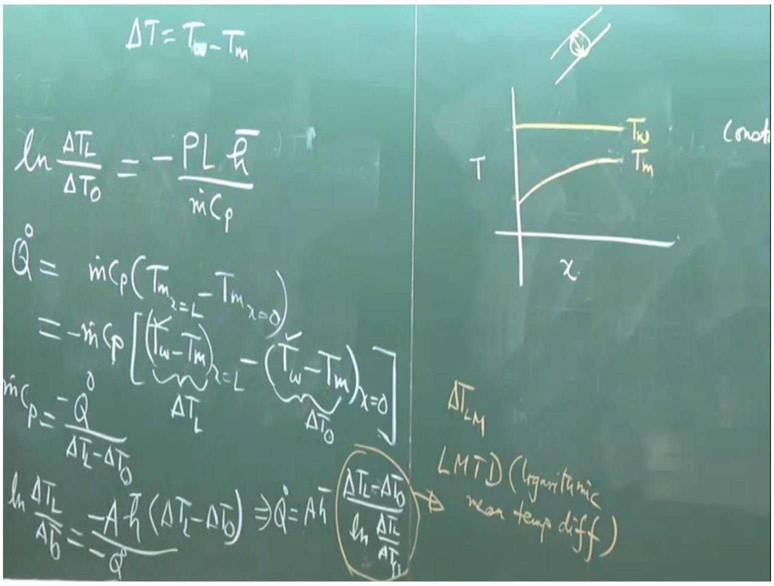
Now, this is the case of constant wall heat flux. Let us stud y the qualitative behavior of constant wall temperature may be first we studied quantitatively and then we show it in a plot. So, the case of constant wall temperature. So, d T m d x is equal to q double prime p by M dot C p. How can you write q double prime in terms of the heat transfer coefficient h, q double prime equal to what h, h into T m minus T w or T w minus T m.

So, when you are considering q double prime, look at this figure, here the sign convention is the heat transfer is from the wall to the fluid that means T wall minus T m, right. If it is the opposite the sign itself will take care. So, in place of q double prime, we will take h into T wall minus T m, so we can write d of T wall minus T m divided by T wall minus T m is equal to minus p into h by m dot C p d x.

So, what we have done is, we have written d T m d x as T of d x of T m minus T wall because T wall is a constant it does not matter whether we take it inside the derivative or not. It will make no difference because d T wall d x is 0 for constant T wall, ok and then this minus sign is observed because we have converted T m minus T wall to T wall minus T m.

So, it is of the form (()) (18:21) it will be log of this, so Ln of T wall minus T m is equal to minus p by m dot C p integral of h d x from x equal to 0 to x equal to L, where L is the total length of the channel or the pipe. This of course we have to put a definite limit from x equal to 0 to x equal to L. X equal to 0 is the starting and x equal to L is the ending of the channel.

(Refer Slide Time: 19:48)



So, we can write Ln of delta T L by delta T 0 where delta T is T wall minus T m is equal to, so what is this, this is the average heat transfer coefficient times L. So, minus P into L into h average by m dot C p. Now what is the total heat transfer rate, Q dot. Let us say you transfer some heat from the wall to the fluid, so how can you measure what heat has been transferred from the wall to the fluid.

Let us say you are doing an experiment, so the situation is that you have some heat let us say you have a heating coil at the wall, you have a heater at the wall, you are transferring heat to the fluid. So how do you measure that what heat actually has been transferred to the fluid. So, you can measure the thermal energy at the inlet and you can measure the thermal energy at the outlet. So, the difference between these 2-thermal energy is the heat that is supplied from the wall.

So, you can say Q dot is nothing but m dot C p into T m at x equal to L minus T m at x equal to 0, right. This is just simple energy balance. So, this heat transfer this is nothing but the change in enthalpy. So, if you write the first law of thermodynamics for the control volume this is what you will get as a heat transfer. There is no (()) (22:18) done in this case. So, all the heat that is supplied is used to change this m dot C p into T that is m dot into h.

Of course, we neglect the changes in kinetic energy and potential energy. So, you can write these as m dot C p, see we can write this because this T wall is the constant so it is just adding and subtracting the same constant from the 2 terms, but why we have done is because this is the delta T at x equal to L and this is the delta T at x equal to 0. So, we can write m dot C p is equal to Q dot minus Q dot by delta T L minus delta T 0.

So, you can substitute that here and write Ln of delta T L by delta T 0, perimeter into length is what the total surface area, so this A is not cross-sectional area this is the surface area that is 2 pi r into L for a circular pipe of length L. So, minus area into h then in place of M this one you can write Q dot by delta T L minus delta T 0. So, Q dot is equal to A into h average into delta T L minus delta T 0 by Ln of delta T L by delta T 0.

So, you can see that normally what is Q dot A into h into delta T, the temperature difference between the 2 systems across which the heat transfer is taking place here one system is the wall another is the fluid. So, it would have been ideally A into h into T wall minus T m, but it is not A h into T wall minus T m. It is h into these, the reason is that T wall minus T m is not a constant. It is continuously varying with x.

So, if you make a plot of say T wall so T versus x. So, for constant T wall, so this is constant T wall. For constant T wall, what is T wall as the function of x, this is T wall constant. What about the T m, see look at this equation d T m d x, so d of T wall minus T m by T wall minus T m is this. So that means these T wall minus T m where is with e to the power minus x, right. Because log of these varies with minus x so this T wall minus T m varies with e to the power minus x.

So, that means T m has an exponential variation with x. So, for constant wall temperature you get assuming that the wall is again heated, this is T m as the function of x. This is an exponential curve. Why exponential curve you can look from the analytical expression. So, if you want to say that the rate of heat transfer is equal to A into h into delta T. Now, the delta T is different. Delta T at x equal to 0 is this, then it becomes this, it becomes this, it becomes this, like this.

So, delta T is continuously varying, so you require some equivalent average delta T. We have shown here by your calculation that average delta T is not this delta T plus this delta T by 2, not, it is these one delta T L by minus delta T 0 by Ln delta T L by delta T 0. This is called as LMTD or logarithmic mean temperature difference. so, why do we require a logarithmic mean temperature difference because the temperature difference itself is continuously varying.

So, what would be the logarithmic mean temperature difference, if the temperature difference is constant let us say that this T wall is the constant and T m is the constant, then what would be the logarithmic mean temperature difference or T wall like this T m like this. This difference is the constant. Then what would be the logarithmic mean temperature difference, no. The logarithmic mean temperature difference physically represents the equivalent temperature difference.

So, if this difference is a constant this constant value itself is the logarithmic mean temperature difference. See if or anything you can do it mathematically. How do you do it mathematically, see if a constant temperature difference is there. Then, it is delta T L equal to delta T 0. So, it is 0 by Ln 1, 0 by 0. You can use (()) (30:16) rule to find out what is the logarithmic mean temperature difference.

But my question is why should we do that from physical understanding we understand that the logarithmic mean temperature difference is the effective average temperature difference between the 2 fluids. If the 2 fluids temperature are continuously varying, you have to judiciously use that formula, but if the temperature difference itself is the constant then the logarithmic mean temperature difference will be that constant itself because it is the equivalent temperature difference.

So, we can say that to summarize so sometimes in short form this is written as delta T LM, logarithmic mean temperature difference, so this is just a short form of doing it, but it is the very important parameter and in one of the latter chapters on heat exchange here, what we learn which is a very important topic for practical engineering applications. This terminology will come over and again. So please make a note this very important terminology.