Conduction and Convection Heat Transfer

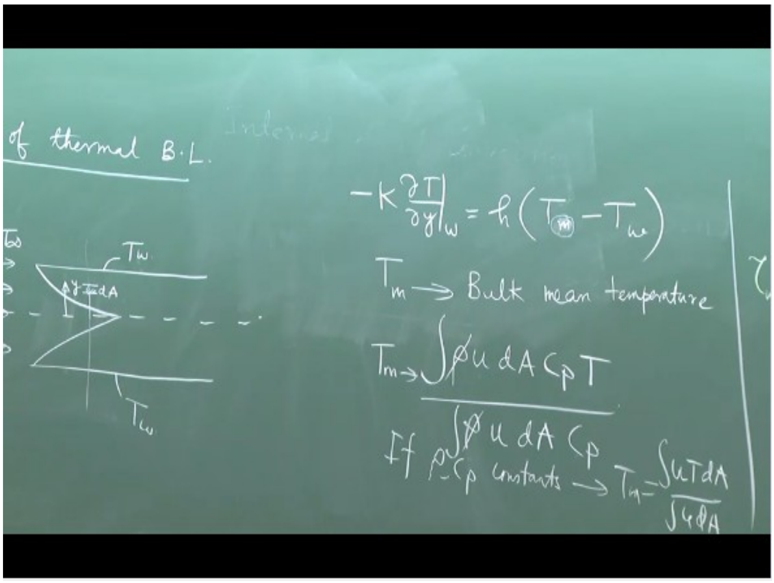
Prof. S.K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 46
Internal Forced Convection – II

So, when we are interested about the hydrodynamics in parallel we should also look into the issues of heat transfer. So, we will look in to that now. Just like the hydrodynamic boundary layer growing.

We will also have the thermal boundary layer growing. So, important take home message from this part of the analysis is that for the hydrodynamically fully developed flow the velocity profile is not a function of x, it is a function of y only. In the developing flow, the core velocity continuously varies with x, there is nothing called as u infinity.

(Refer Slide Time: 30:59)



Growth of thermal boundary layer. So, let us say that this fluid which was coming with a velocity u infinity was also coming with a temperature of T infinity and the wall temperature T wall. You may argue that the 2 walls may not be at the same temperature at the undergraduate level, we will not be entering into that complication. We will assume that it is symmetric with respect to the thermal problem also.

That means the 2 walls are at same temperature. But the temperature need not be constant, it may be a function of x. So just like the growth of hydrodynamic boundary layer, the thermal boundary layer also grows. Now, we have already learned that there is nothing called as u infinity in these type sub problems and analogously there is also nothing called as T infinity. T infinity is only outside the channel.

Once the flow enters the channel, the effective velocity of the fluid, the core temperature continuously changes and it is not T infinity. So, question is for internal flows when you write minus K del T del y at the wall, this is equal to h. So, we are writing a heat transfer equation say from fluid to the wall. So, we are writing the wall temperature. So, it is different from that of the fluid temperature.

So, we are writing the heat flux minus k del T del y at the wall is equal to h into this temperature difference. So, T, this is not T infinity. What is this? That we will discuss. What is this? So here just because the y axis is directed positively upwards, that is why this is this difference, otherwise if the y axis was from the plate down then it would have been T wall minus this. So, it is just because of the axis issue.

But the more important is, what is this T? Now this T is not T infinity because T infinity is no more the referenced temperature in the internal flow. Just like u infinity is no more the referenced velocity for internal flow. T infinity is no more the reference temperature for internal flow. So, if T infinity is no more the reference temperature for internal flow then what is the reference temperature?

That reference temperature is called as T m, which is bulk mean temperature. What is bulk mean temperature? Let us say that you have a fluid in a particular section. This section has non-uniform temperature. Now if you mix this entire fluid in a particular section very thoroughly so that the entire fluid comes to a uniform temperature in a particular section with the same thermal energy as that of the previous case. Then that equivalent uniform temperature is called as the bulk mean temperature.

So, the entire fluid in a particular section mix to come to a cross-sectionally uniform temperature. So, what is the utility of this concept? See, actually this problem is like a 2-dimensional problem. If it is a parallel plate the third dimension is not important. By introducing this bulk mean temperature, we are converting it into a cross-sectionally average problem or a 1 dimensional problem.

Because if we are using a bulk mean temperature that means we are no more interested with the variation in the y direction. We are using a temperature which is already cross-sectionally averaged out. So, that will now depend only on x. So, what is the definition of this? Now, what is the thermal energy of the fluid? Think of the first law of thermodynamics. If you have a flowing system, if you have a fluid that is flowing,

What is the thermal energy of it? In a first law energy balance equation, what is the thermal energy component that you write? M dot into h mass flow rate into specific enthalpy, right? So, for a, incompressible fluid that we are discussing here the h we can write as c p into T. So, now what is it? the T is not uniform over the entire cross sections, so what you do is that at location y, you take a small area d A.

So, what is the mass flow rate through this d A? rho u d A. This is m dot into c p into T. This integrated is the total thermal energy. So, that divided by m dot C p will give you some p average. This is integral of m dot C p T. This is integral of m dot C p. So, this will give you some sort of cross-sectionally average T. So, cross-sectionally average T is not integral of T d a by integral of d a. It is weighted with the velocity.

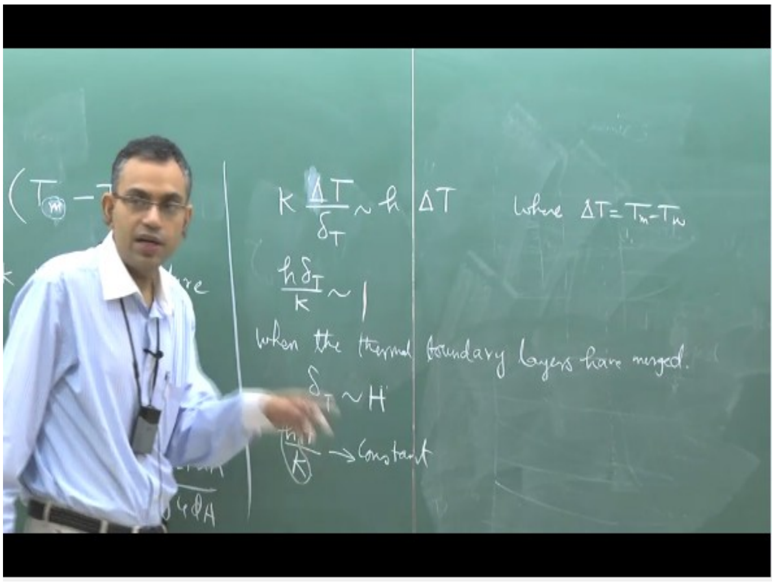
Because when you are calling it as a bulk mean temperature, it is also called as mixing cup temperature. The factor that is creating the mixing is the fluid flow. So, it has to be weighted by the velocity. So, when you write the thermal energy of a flowing system, it is not independent of velocity that is where convection is related to fluid mechanics. So, when you write m dot into h, in that m dot, the velocity is there.

So, you cannot write the average temperature without referring to the velocity. So, this is called as the bulk mean temperature, but if you have rho constant for this particular problem and C p also not changing. So, if rho C p are constants then T m is equal to integral of u T d A by integral of u d A. So, now so we can understand this T m is a function of what? X or y? it is a function of x because it is already integrated over y, so this is a function of x.

T wall is a function of x. T is a function of both x and y. T is a function of both x and y. but T m is a function of x, t wall is the function of x. So, h can be a function of whatever. So, del T del y at wall when you calculate so this T is a function of x y. So, del T del y at the wall is the function of what? x. So, this is the function of x that means h in general is the function of x, right.

Now, let us try to write an order of magnitude expression for this. So, let us say that delta T at a given x is the thermal boundary layer thickness.

(Refer Slide Time: 42:00)

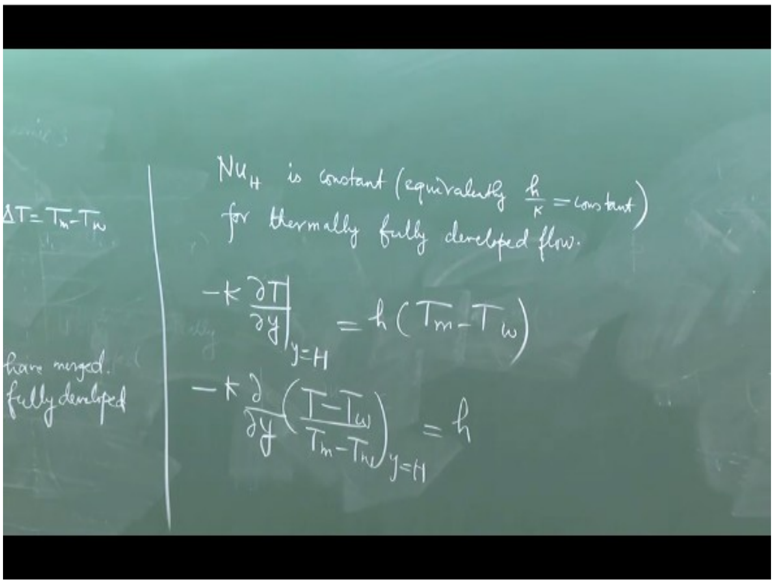


So, we can write K into h delta T by K is of the order of 1, right. It is of the order of 1, but it is not a constant. When it will become a constant? When delta T does not change anymore. That means when the 2 thermal boundary layers have merged. When the 2 thermal boundary layers are merged then delta T is half height of the channel right. Or for a circular pipe, it is the radius of the pipe.

So, when the thermal boundary layers are merged or met this is the half height of the channel H. So, then you can H by K, this is a constant. See why it is so? Because H by K is of the order of 1 by delta T and when the delta T becomes a constant? H by K becomes a constant and H by K becomes a constant means H by K into a constant length is also a constant. This is a non-dimensional way of representing H by K.

So, it is basically this H by K, that is a matter of interest, but non-dimensionally when expressed this is actually what? This is the Nusselt number. So, the Nusselt number becomes constant for a thermally fully developed flow. Thermally fully developed flow means the 2 thermal boundary layers are merged. So, this is called as thermally fully developed flow.

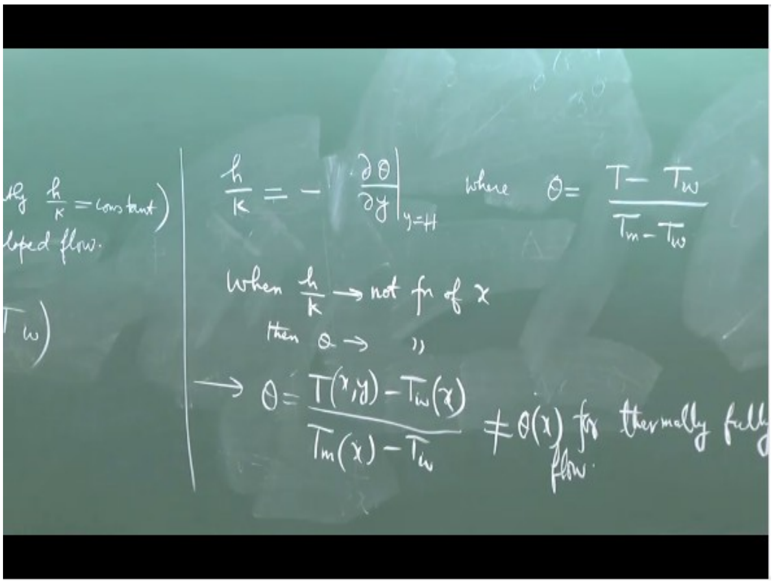
(Refer Slide Time: 45:11)



So Nusselt number based on H is constant equivalently H by K is constant for thermally fully developed flow. Now, we will try to manipulate with this equation minus k del T del y at y is equal to H is equal to H into T m minus T wall. So, in place of this we can write minus K del del y of T minus T wall. See, this is a y derivative because this is a y derivative any function of x is like a constant with respect to it.

So, we have taken this within the derivative. These are all functions of x and T wall is also a function of x.

(Refer Slide Time: 46:40)



So, we can write that h by k is equal to minus k del minus theta del y at y is equal to h. Where theta is T minus T wall by T m minus T wall, right. So, when H by K is not a function of x then theta is also not a function of x, right. Because this may be function of y, because at y equal to h, you will get a value which is not dependent on y in any way, but it is a function of x in general.

But for thermally fully developed flow this is not a function of x therefore this also should not be a function of x. If del theta del y is not a function of x that means theta is also not a function of x. So, in general to generalize it we should not think of special functions. But generalizing, that we can say that if del theta del y is not a function of x then in general, theta should also not be a function of x.

So, theta is not a function of x, this is another hallmark of thermally fully developed flow. So, this is 1 point Nusselt number is constant. The second point is theta is equal to T which is a function of x y minus T wall, which is a function of x by T m minus T wall. This is not equal to a function of x for thermally fully developed flow.

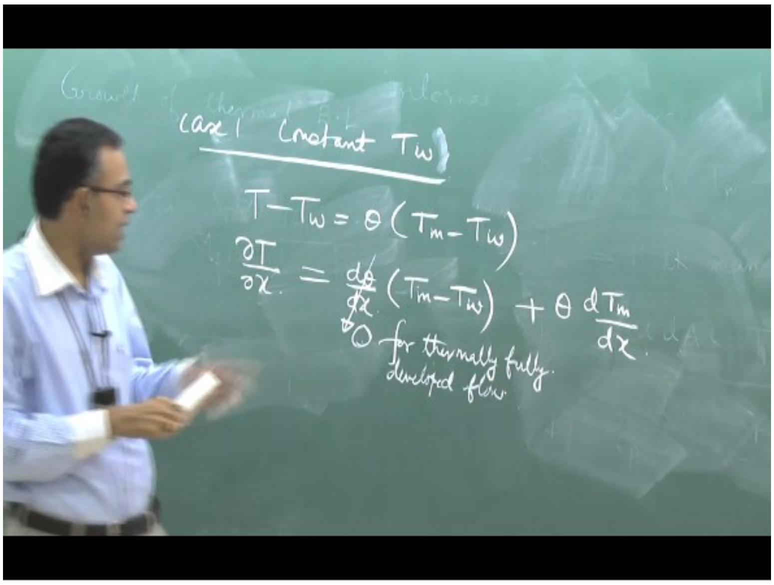
Now, you can clearly see that this does not mean that T is not a function of x. So, the reason is quiet clear. Let us say that you have 2 parallel plates like this. You have thermal boundary layers growing and now you go on say heating the plate. This is heat flux. So, if you go on heating the plate, how can it happen that the temperature has become a constant? So even if the thermal boundary layers have merged, temperature can still vary with x.

So, this is a very important difference between hydrodynamically and thermally fully developed flow. For hydrodynamically fully developed flow, u is not a function of x, but for thermally fully developed flow, T still remains a function of x, but this definition of non-dimensional temperature theta that is not a function of x. So, it is not a very intuitive thing because all these are: this has function of x y.

This is function of x, this is function of x, this is also a function of x, but this ratio is no more a f unction of x for thermally fully developed flow and why we have just proved it. Now, we will have some observations with regard to 2 different boundary conditions which we will discuss in this course: one is a constant wall temperature, another is a constant wall heat flux. So, constant T wall what is the consequence?

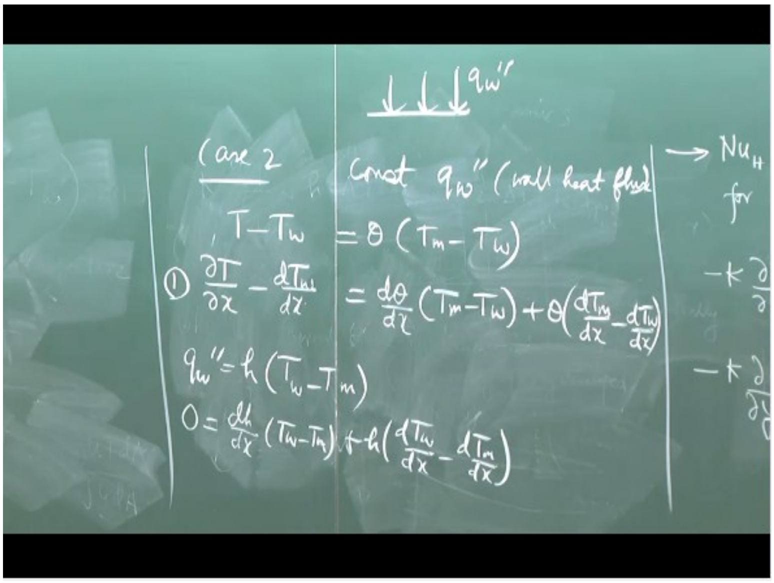
So, this is case one. Constant T wall, you have T minus T wall is equal to theta into T m minus T wall. Differentiate both sides with respect to x. So, this becomes del T del x right. What is the derivative with respect to x? 0 because T wall is constant is equal to d theta d x. So, this d theta d x is 0 for thermally fully developed flow.

(Refer Slide Time: 51:14)



So, you can write del T del x is equal to theta into d T m d x. When we will solve the energy equation, you will see that in the energy equation, you have term mu del T del x. So, in that del T del x, we will substitute theta into d T m d x. Then, the next important case which we will discuss in length over this course is the constant wall heat flux boundary condition.

(Refer Slide Time: 53:04)



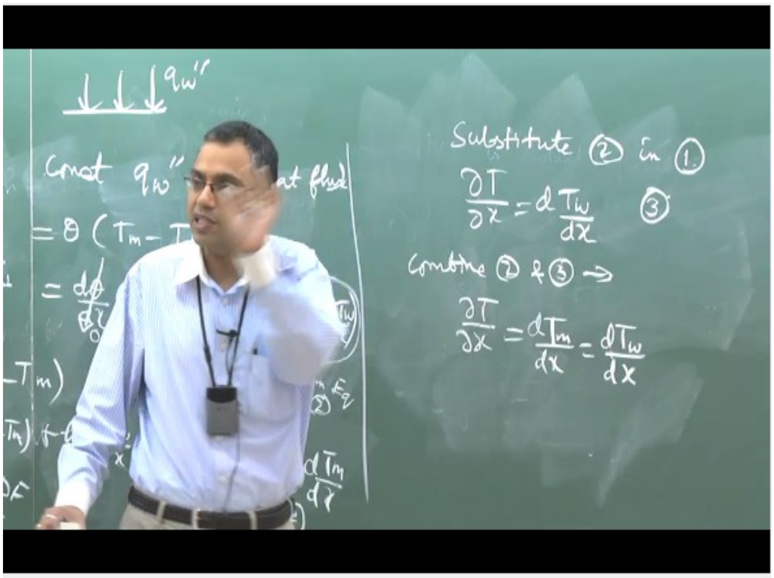
So, here you can write again T minus T wall is equal to theta into T m minus T wall. So, del T del x minus d T wall d x. Now, T wall it may be a function of x. If always remember that on a surface you cannot control the temperature and heat flux simultaneously. So, if you are controlling the heat flux, temperature will vary in its own way. It will be some function of x. Not that you can maintain it both isothermal and constant flux that is not physically possible.

So, you have del T del x minus d T wall d x is equal to. Now, what is the wall heat flux? Let us say, this is equation number 1. What is the wall heat flux? This is equal to h into. So, if you are directing the heat flux in this direction at the top wall then it is h into T wall minus T m, right. So, constant wall heat flux if you differentiate with respect to x, 0 is equal to d h d x into T wall minus T m plus h into d T wall d x minus d T m d x, right.

Now for thermally fully developed flow, one important thing. See these are very subtle concepts. For thermally fully developed flow, can we tell h is a constant? We can tell h is a constant provided case also a constant because the very basic premise of the thermally fully developed flow is H by K is a constant. But often when h is constant we say the flow is thermally fully developed and that is grossly valid, provided k is a constant.

So, if we assume that k is a constant then this is 0 for thermally fully developed flow if k is constant, right. So, if this 0 then that means you have d T wall d x is equal to d T m d x. This is equation number 2. Now, you substitute equation number 2 in equation number 1.

(Refer Slide Time: 56:30)



So, d T wall d x equal to d T m d x means this term will be 0 and this term is 0 for thermally fully developed flow. Because theta is not a function of x, right and this term becomes 0 from equation number 2. So, what is left? The left-hand side is 0 that means del T del x is equal to d T wall d x. This is equation 3. If you combine equation 2 and 3, you will get del T del x equal to d T m d x is equal to d T wall d x, right.

This is plain and simple comparing a (()) (57:42) combination of 2 and 3. Can we tell anything more about this? Just these 3 are equal. Can we go beyond that and tell something? These are not merely equal, but this shows that this is a constant, why? This is a function of x and y and this and this are functions of x. In general, if you have a general function of x and y is equal to a general function of x.

General function you have to keep that in mind, not specific function. So, this could be any arbitrary function of x y. A general function of x and y is the general function of x. This equality is possible only if h is a constant, ok. Otherwise, that equality may be valid only for special cases, but not for general functions. So that means not only these 3 are equal, but h is equal to a constant.

So, to summarize our discussion today for the constant wall temperature you can write del T del x is equal to theta into d T m d x for thermally fully developed flow with constant wall temperature. With constant wall heat flux, this is the condition. So, we have got expressions for del T del x in terms of the bulk mean temperature gradient. So that essentially has made one thing possible, mathematically what heat has given us?

That heat has given us an equivalent 1 dimensional formulation of the problem. Instead of del T del x, now we can use d T m d x or d T wall d x. Those are 1-dimensional parameters and similarly for constant wall temperature. So, we will take it up from here and we will continue in the next lecture. Thank you.