**Conduction and Convection Heat Transfer**
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**Lecture- 44**
**Energy Integral Equation – II**

Now I will discuss about one very important point before moving on to the other case, Prandtl number much better than one.

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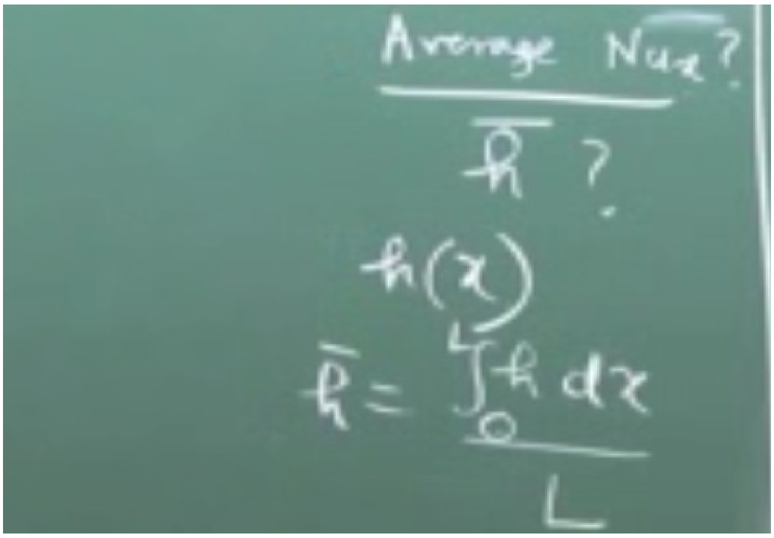


That important point is that how to get the average Nusselt number, why average Nusselt number is important because Nusselt number is a function of x. So, along the length of the plate it will vary. So, if somebody asked that what is the average heat transfer coefficient, how will you calculate it. So, in other words, what is the average heat transfer coefficient. So, you can see that age is a function of x, why h is a function of x?

You look at this expression, h it is a function of delta T and delta T is a function of x. So, you can write h as a function of x by substituting delta T as a function of x here. So, h average is equal to integral of h dx from x equal to zero to L divided by the total length. And Nusselt number average is based on this h average. Okay? So many times, this is a calculation that engineers need to predict what is the average heat transfer from the wall to the fluid or fluid to the wall depending on what is heated and what is cooled.

So, for that h as a function of x gives you the local variation, but you may also be satisfied with the average trend and for that you have to do that integration. So that being a trivial integration because delta being a polynomial symbol, so how delta will vary with x. So, delta square is equal to, delta square is proportional to x or delta T square is proportional to x. So, delta T will vary with square root of x, x to the power half.

So, the boundary layer profile that we draw is actually x to the power half. The profile of the boundary level, this one that we draw, it is actually function of the form x to the power half. Hydrodynamic boundary layer and thermal boundary layer for Prandtl number much less than one. So, delta T as a function of x when you substitute some constant into h to the power half that will give you h, will vary with x to the power minus half.

And then that integration you can do. Okay? Now we will do the next case which is a little bit more important than the previous case. “Professor - student conversation starts” No. That is alright. So, your question is that delta T is equal to zero if the heating or cooling starts from there also it is zero. But here the problem of growth of thermal boundary layer starts from here. So, your domain starts here.

This is not a domain of your interest because in this domain there is no heat transfer. You have to see that what is your domain of heat transfer. So, in this domain all temperatures are same. So, there is no heat transfer. So, heat transfer domain starts from where you have the wall temperature different from T infinity. So, within this part of the domain, the wall temperature and T infinity all are the same.

So, there is no heat transfer. So, you have to keep in mind that this equation is applicable in the domain where you have heat transfer. Okay? So that is why you apply with the domain starting from here. “Professor - student conversation ends” Prandtl number much greater than one means delta is much greater than delta T. So, you cannot say, that u is u infinity within the thermal boundary layer. So, we start with the energy integral equation,

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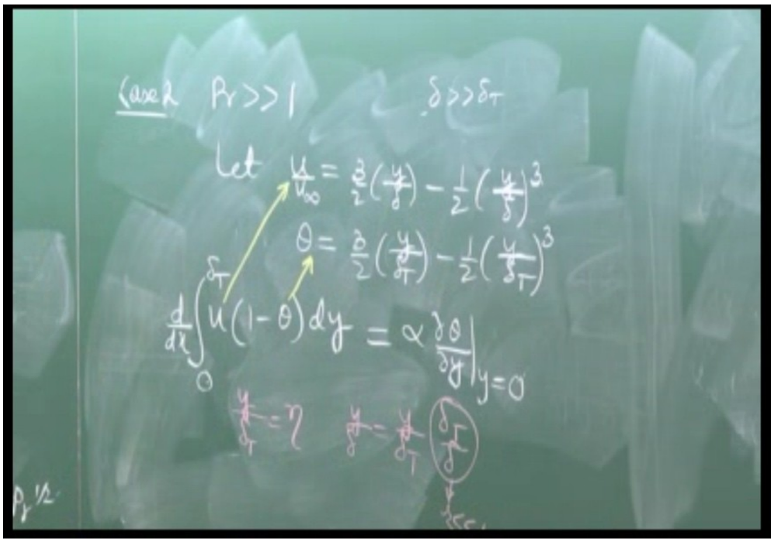


So, in this equation we will substitute this u and we will substitute this theta. So, we will assume that y by delta T equal to eta. So, y by delta is equal to y by delta T multiplied by delta T by delta. Let us say that delta T by delta is equal to a ratio r. What is the value of this r? Small or large? What is r, small or larger? It is small, right? So, this r must be much less than one. So, this we will use for algebraic simplification.

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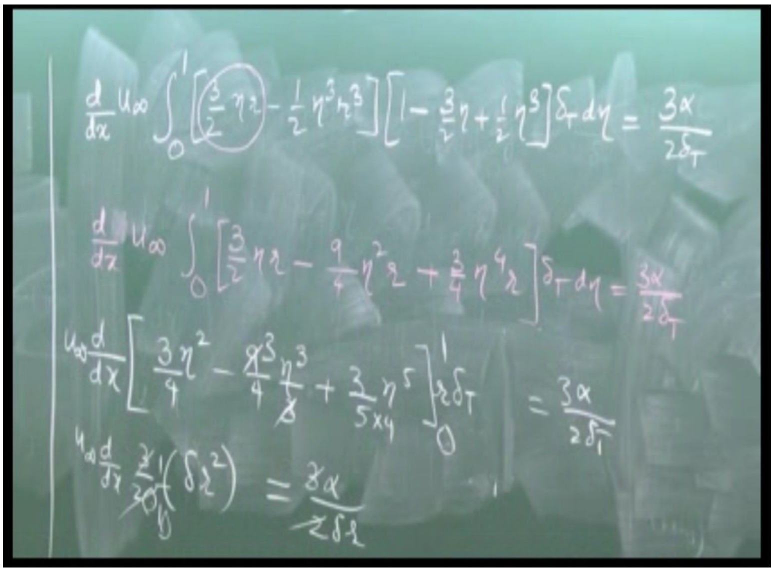

Now, when you multiply this term with this term, out of these two which one will be of dominance? This one will be of dominance because r is expected to be much greater than r cube for small r, right? So, you can write this. So, 3 eta square by four minus 9 by 4 eta cube by 3, plus 3 by 5, eta to the power 5 multiplied by 4. r multiplied by delta T becomes, so r is equal to delta T by delta. Can you quickly do it and tell me what is the value?

So, these two cancels, right? So, 3 by 20. There is one r, right? So, r multiplied by delta T. So, 3 by 20, in place of delta T we can write delta into r, right? So, delta r square. Why we are writing this is because delta we already know. We are interested to solve for r, that will give us what is delta T. So, 3 by 20 multiplied by delta r square is equal to 3… Delta T we have written delta into r, okay? So, these 3 gets cancelled and this becomes 1 by 10.

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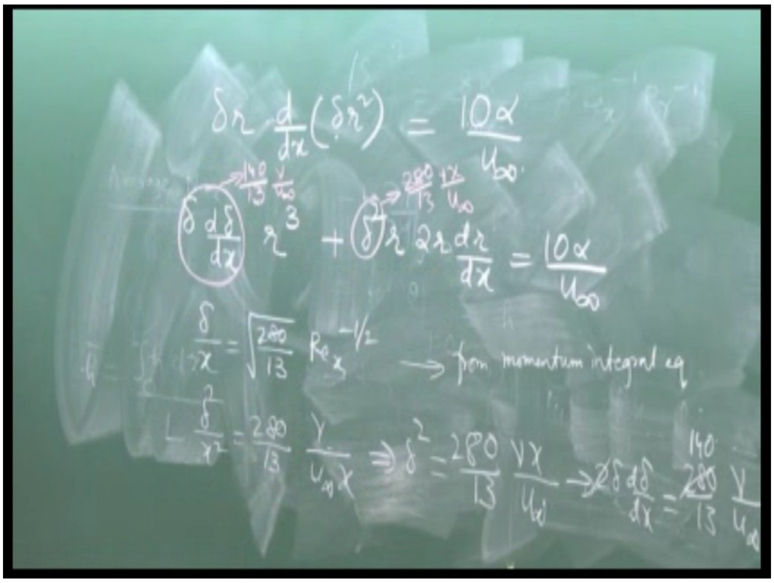


So, u infinity, may be u infinity we will take in the right-hand side. So, d dx of, so we can write delta r into d dx of delta r square is equal to 10 alpha by u infinity. So now we can write delta, d delta dx multiplied by r cube plus delta square r multiplied by 2r dr dx. Now this delta is a function of x, we can get from the hydrodynamic boundary layer equation. So, if you recall, if you had completed the exercise of finding delta as a function of x from momentum integral equation.

In the momentum integral equation, if you substitute u by u infinity is equal to 3 by 2 y by delta minus half y by delta whole cube and integrated you will get this delta as a function of x. So, we will use this, so delta square by x square is equal to 280 by 13 multiplied b Reynolds number to nu power minus one. So, nu by u infinity x, right? So, delta square is equal to 280 by 13 multiplied by nu x by u infinity.

So, you can clearly see that delta varies with x to the power half, the edge of the boundary layer. So, we require delta d delta dx, so if you now differentiate both sides with respect to x, so 2 deltas, d delta dx is equal to 280 by 13 nu by nu infinity, okay? That means delta d delta dx is 140 by 13 multiplied by nu by nu infinity. So, let us write it down there. Then this term delta square in place of delta square what we will write? 280 by 13 nu x by nu infinity. So here there is one r cube, here you have r square dr dx.

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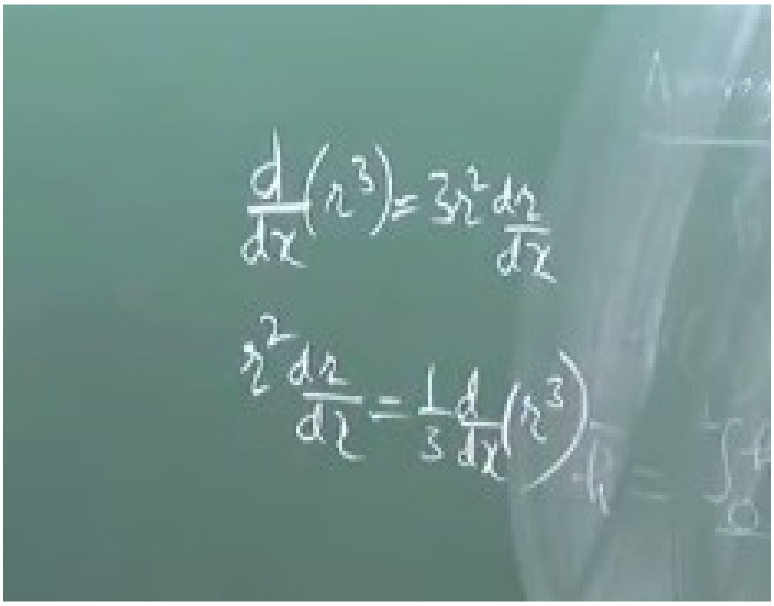

So, what is d dx of r cube? 3 r square dr dx. So, in place of r square dr dx we can write 1 by 3 d dx of r cube, right? So, let us move on to the next step.

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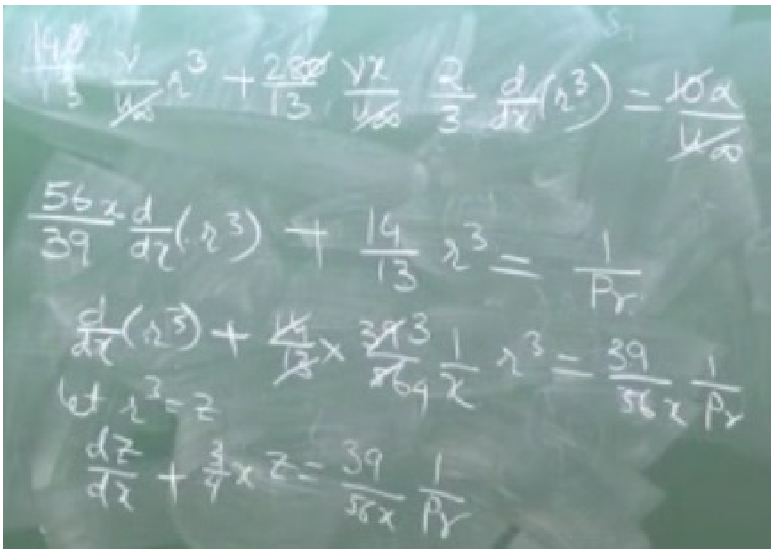

So, 140 by 13 nu by nu infinity r cube plus 280 by 13 nu x by u infinity multiplied by 2 multiplied by r square dr dx is 1 by 3 d dx of r cube, is equal to 10 alpha by u infinity, okay? So, u infinity gets cancelled from all sides, if you take the nu in the right-hand side, alpha by nu becomes one by Prandtl number. So, we can write, okay before that we can possibly simplify a little bit.

So, 56 by 39 d dx of r cube 56 by 39 x, oh sorry, 56 x by 39 d dx of r cube plus 14 by 13 r cube is equal to 1 by Prandtl number. Just check whether this is correct or not? So, we can write d dx of r cube plus 14 by 13 multiplied by 39 by 56, one by x, okay? So, this equation if you take let r cube equal to z, this equation you can write dz dx plus 3 by 4 x multiplied by z is equal to 39 by 56 x into one by Prandtl number. So, this equation is of the form dy dx plus py is equal to Q. So, the solution of this equation relies on multiplying with the integrating factor. So, let us identify the integrating factor.

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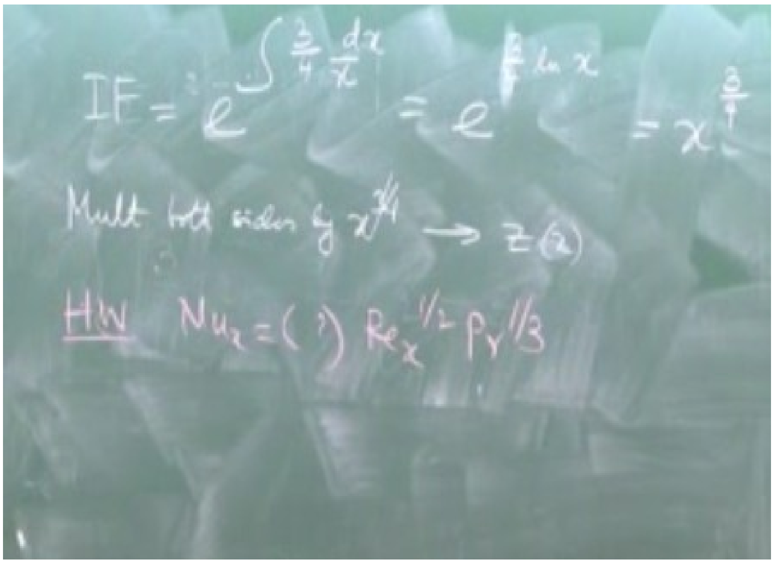



So, integrating factor, e to the power integral, right? So, e to the power 3 by 4 ln x. So x to the power, right? So, you multiply both sides by x to the power 3 by 4 and that should give you what is z as a function of x, remaining is straight forward. See it is important that you work out each and every individual step that gives you a lot of confidence. You will see, I mean many books will write from straight forward calculation it follows.

I mean, as a student I know, whenever I was a student never I found that it is such a straight forward calculation as it is mentioned in the book. So, you would see that the steps that we have gone through there are many steps which if you had tried to do by yourself it would have created a lot of problem and it is important that is why I have done it up to the stage from here.

It is really straight forward because then the remaining is just after putting the integrating factor you can get the solution. So, you please work it out completely and find out. So, this is like the homework that I am leaving on you. Find out the Nusselt number is equal to some constant into Reynolds number to the power half into panel number to the power one third. What is this constant?

It will - If you have done it correctly it will come out to be something in the range of 0.33 something like that. So please do that. Please complete this, but I have done enough number of steps so that I mean the remaining should be easy for you to complete. I mean, essentially for deriving all this as I told you that these derivations are, these are the problems from this part of the course and you must learn how to do these derivations.

No formula based study please. So if you know, or if you learn how to do these derivations maybe exactly these derivations may not come in the exam but if you have followed this you will see that some difference in velocity profile, some different temperature profile, but the method will be very similar. So please gain some confidence and as I want to repeatedly tell you that I do not want to discuss anything in the undergraduate class which I cannot derive in the board.

So, thousands of formula, which I cannot myself derive in the board, I cannot expect that you will remember this and you will derive this and all this. So, whatever are the essential fundamental things which can be derived in the scope of an undergraduate course, that part of convection we are only covering. So, this will give you the foundation for learning the advanced topics in convection. Okay, we will stop here today.

In the next lecture, so what we have done up to today is, we have discussed about force convection over a flat plate. But in engineering maybe another sort of important problem is what happens for force convection inside a pipe or inside a duct or inside a channel, not in an open space but in a confinement. So that is called as internal force convection. So, from our next lecture we will start discussing on internal force convection. Thank you.