Conduction and Convection Heat Transfer
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Lecture- 43
Energy Integral Equation – I

Just like for the hydrodynamic boundary layer we have solved the hydrodynamic boundary layer equations by using the integral methods, we will see the corresponding integral method for the thermal boundary layer equation or the energy equation for the thermal boundary layer. So, we will begin with that, Integral method.

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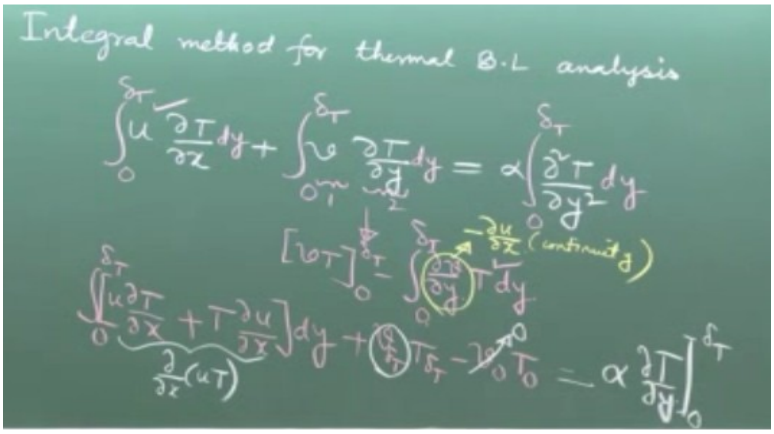
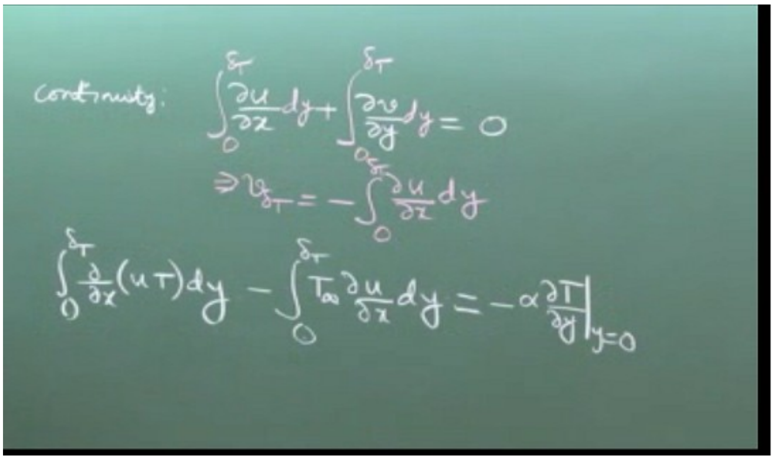


So, we will begin with the thermal boundary layer equation. As a first tip what we should do, we have already done the same thing for the hydrodynamic boundary layer equations. Now you tell what should we do for the thermal boundary layer equation. Integral method, what we are doing what we are doing basically we are integrating it with respect to y across what? Across the thermal boundary layer for the energy equation.

Now, we will integrate this by parts. This is the first function and this is the second function. So, this will be first function into integral of the second minus integral of derivative of the first into integral of the second. And we can make a further simplification in place of del v del y, you can write as minus of del u del x from the continuity equation. Therefore, this equation becomes integral of u del T del x zero to delta T. We can club this term and this term together, plus T del u del x, plus v at delta T multiplied by T at delta T minus v at zero multiplied by T at zero. That is the stuff. This is equal to the right-hand side. Now these two terms together becomes del del x of T multiplied by u or u multiplied by T. This term is zero because v zero is zero, no penetration boundary condition at y equal to zero.

So, we must estimate what is v at delta T. So how do we estimate what is v at delta T? We will use the continuity equation and integrate the continuity equation across the thermal boundary layer. **(Refer Slide Time: 05:58)**



So, continuity- So from here we get v at delta T is equal to minus, what is T at delta T, what is this? This is practically T infinity. So, we can write… Right hand side what is the temperature gradient at delta T, zero, because there is no further variation in temperature. So, this becomes minus. See, just like the momentum integral equation give the wall shear stress directly, this equation gives the wall heat flux because minus k del T del y at y equal to zero is the heat flux at the wall.

It gives it through some integral expression and as we have seen, that if we use an approximate temperature profile, just like approximate velocity profile, then the result may be erroneous for the velocity or temperature but integral of velocity and temperature it is not so erroneous, typically because we are using some complimentary, like some function multiplied by one minus that function something like that. So, with this understanding:

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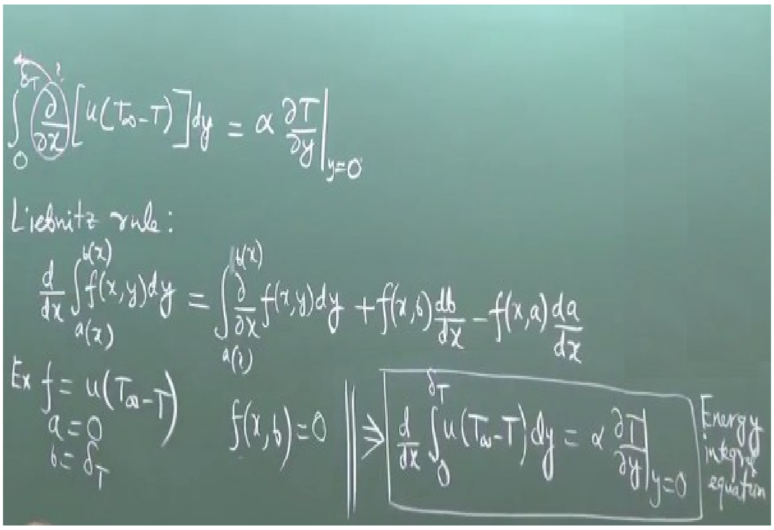

we are combining these two terms because T infinity is a constant, we can take it easily within and outside the derivative without any problem. Next, our issue will be whether we can bring this derivative out of the integral or not. And for that we will use the Leibniz rule to check. So, in this example f is u multiplied by T infinity minus T, a is zero and b is delta T. So, what is f (x, b)? That is f at y equal to delta T, zero because T is T infinity.

At y equal to delta T, T is T infinity. So, this term become zero. What is the other correction term? This is zero because a is zero, so da, dx is zero So that means here also we can just like what we could do for the momentum equation we can take this out of the integral without any problem because the correction terms are zero. So, we can write, this equation is called as energy integral equation.

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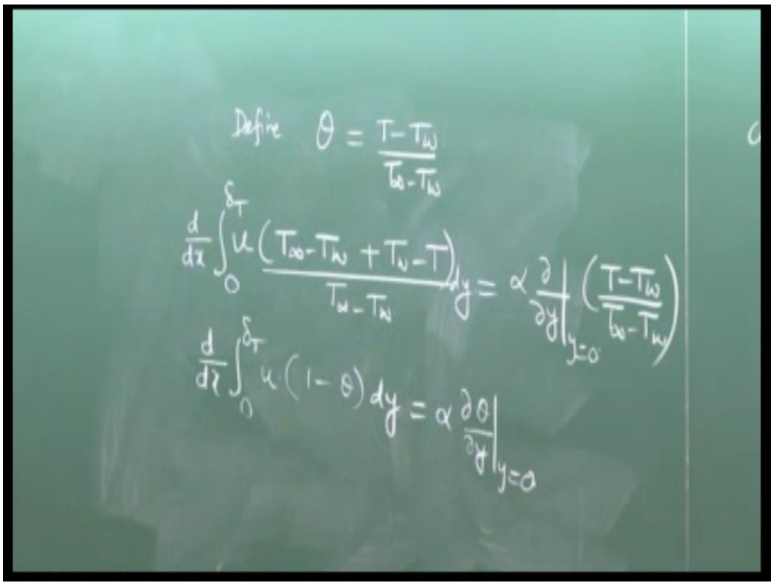

So, we can write this equation in terms of non-dimensional velocity and non-dimensional temperature. So, if you define theta is equal to T minus T wall by T infinity minus T wall. The purpose of defining theta in this way is that this has similar scaling as u y u infinity. See, u y u infinity, zero at the wall and one at the edge of the hydrodynamic boundary layer. This is zero at the wall and one at the edge of the thermal boundary layer.

So, u y u infinity and theta have the same scaling. So, if you use this non-dimensional temperature, then you can write d dx of u…So we have added and subtracted T wall. So, this becomes d dx of zero to delta T, U multiplied by one minus theta, one dy is there. This is the energy integral equation in terms of the normalized temperature. So, we will make an assessment of the situation using the two limiting cases as we had done for the similarity solution, one is Prandtl number much less than one and the other is Prandtl number much greater than one.

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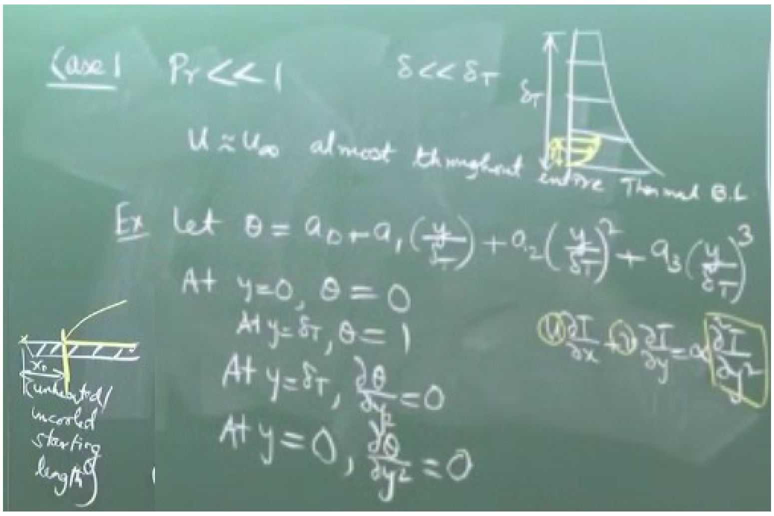

So Prandtl number much less than one means, delta is much less than delta T. So, if this is the delta T then your delta is much less. So almost throughout the thermal boundary layer, u is u infinity. So, u is approximately equal to u infinity, almost throughout entire thermal boundary layer. So that u part is there, what about theta?

So, theta just like we could use approximate velocity profiles for the velocity for the momentum integral equation similarly we can make use of approximate temperature profiles. So as an example, this kind of velocity profile we had taken for the momentum integral equation, similar thing we are taking for the temperature. You could take other profiles but you have to find the constants based on the essential boundary conditions.

So, what are the boundary conditions in terms of priority. What is the most important boundary condition or what are the most important boundary conditions? See, at least two constants if those are there, those should match the values at y equal to zero, and y equal to delta T, that much should be there. So, at y equal to zero, what is theta? Theta is zero. At y equal to delta T theta is one.

Then at y equal to delta T, del theta del y is equal to zero. And the fourth one at y equal to zero, so if you look into the equation, u del T, del x, plus v del T del y is equal to alpha del two T del y two. So, at y equal to zero, both u and v are zero. So therefore, this must be equal to zero. So, in terms of theta. So, you can see, if you cast it in proper non-dimensional form, the boundary conditions look like exactly the same as those where for velocity.

When we approximated the velocity, profile using this formula. Therefore, the constant will also be the same because the similarity is a mathematical similarity. Mathematics doesn't understand that one is heat transfer, one is fluid mechanics. If you bring exact similarity into picture the values will also be similar. So, you will get theta is equal to what was the velocity profile? 3 by 2.

Now instead of y by delta it will be y by delta T for the temperature profile minus half y by delta T whole cube. So, if you work out this by substituting these conditions you will find out exactly these.

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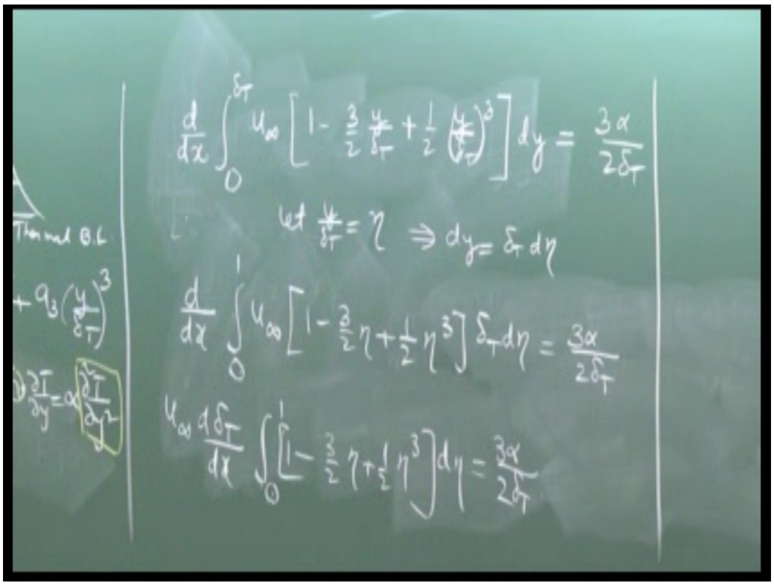

So, we will substitute that here in this equation, d dx of integral zero to delta T, u becomes u infinity, multiplied by one minus theta, okay? So, we have substituted the temperature profile in an energy integral equation. Then let us assume y by delta T is equal to eta. So dy delta T multiplied by d eta. So, d dx of… This is equal to 3 alpha by 2 delta T.

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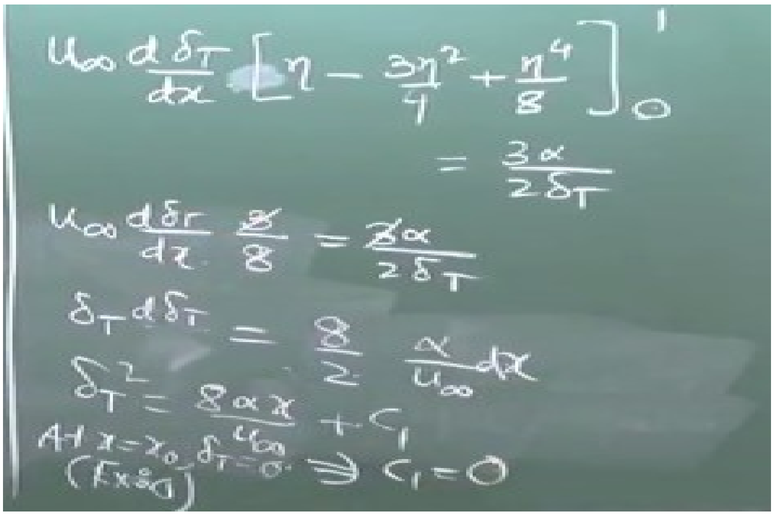

Now we will integrate this. So, if we do it quickly, u infinity, d delta T dx. So, this becomes 8 minus 3, 5; 8 minus 6 two, plus one. So, 3 by 8. So, delta T, d delta T, dx. Now what is the boundary condition to get. This is dx. So, if we integrate this, if we integrate this, then delta T square is equal to 8 alpha x by u infinity plus some constant c1. So, question is what is the boundary condition to get this value of c1?

What is the boundary condition at x equal to zero? See, this is a subtle distinction between the thermal and hydrodynamic boundary layer, at x equal to zero, the hydrodynamic boundary layer thickness must tend to zero. But at x equal to zero, the thermal boundary layer thickness may not be zero. I will give you one example. Let us say, that this is the solid boundary and this plate is heated starting from here.

So, this zone is only heated. So, the thermal boundary layer will develop from here and not from here. Okay? So, the hydrodynamic boundary layer the frictional effect will be failed from, starting from here itself. But thermal boundary layer, the heating or cooling effect of the wall will be faced only from that location from where you start hitting on cooling. So, you may have a length x zero which you may say as unheated or uncooled starting length. So, we can say that at x equal to x0, delta T equal to zero. As a special example, let us take x zero equal to zero. But this is a special example. Do not think that it is a generalization, x0 could be any value but as a special case we are considering that the entire plate is heated or cooled. So that means, you will get c1 equal to 0.

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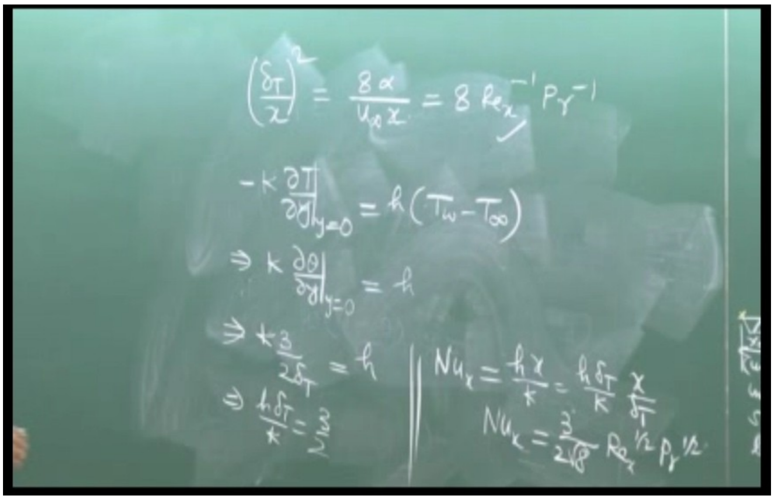

So, delta T by x, is equal to 8 alpha by u infinity x, right? So, this is 8 multiplied by Reynolds number to the core, minus one into Prandtl number to the pr minus one, because this is u infinity x by nu into alpha by nu. Alpha by nu is one by Prandtl number. Okay? Now our objective is to get the Nusselt number. So, we will use the boundary condition at the wall. So, theta is T-T wall by T infinity by T wall.

So, we have brought this within this derivative. Is it true if the wall temperature is a function of this? Yes, or no? Is the transformation from this form to this form valid if T wall is a function of x or not? Doesn't matter because this is derivative with respect to y. This is not derivative with respect to x. So, if T wall varies utmost it will vary with x, but wall is along x. So, wall temperature cannot vary with y.

So, with respect to y derivative T wall may be a function of x but still it does not have any dependence on y. So, k del theta del y at y is equal to zero is this. This means h delta T by k is equal to 3 by 2. Now what is required is Nusselt number. That is hx by k, Nusselt number at x. So, this is 3 by 2 and x by delta T from this expression, so 3 by 2 root 8 Reynolds number to the power half multiplied by Prandtl number to the power half.

If you recall in the similarity solution it was 1 by square root of pie, right? So, you can compare that how accurate this is as compared to that. Find a numerical values and you will see that the difference is not that much. So actually, the integration has smoothened out many sources of air and makes the result more or less acceptable.